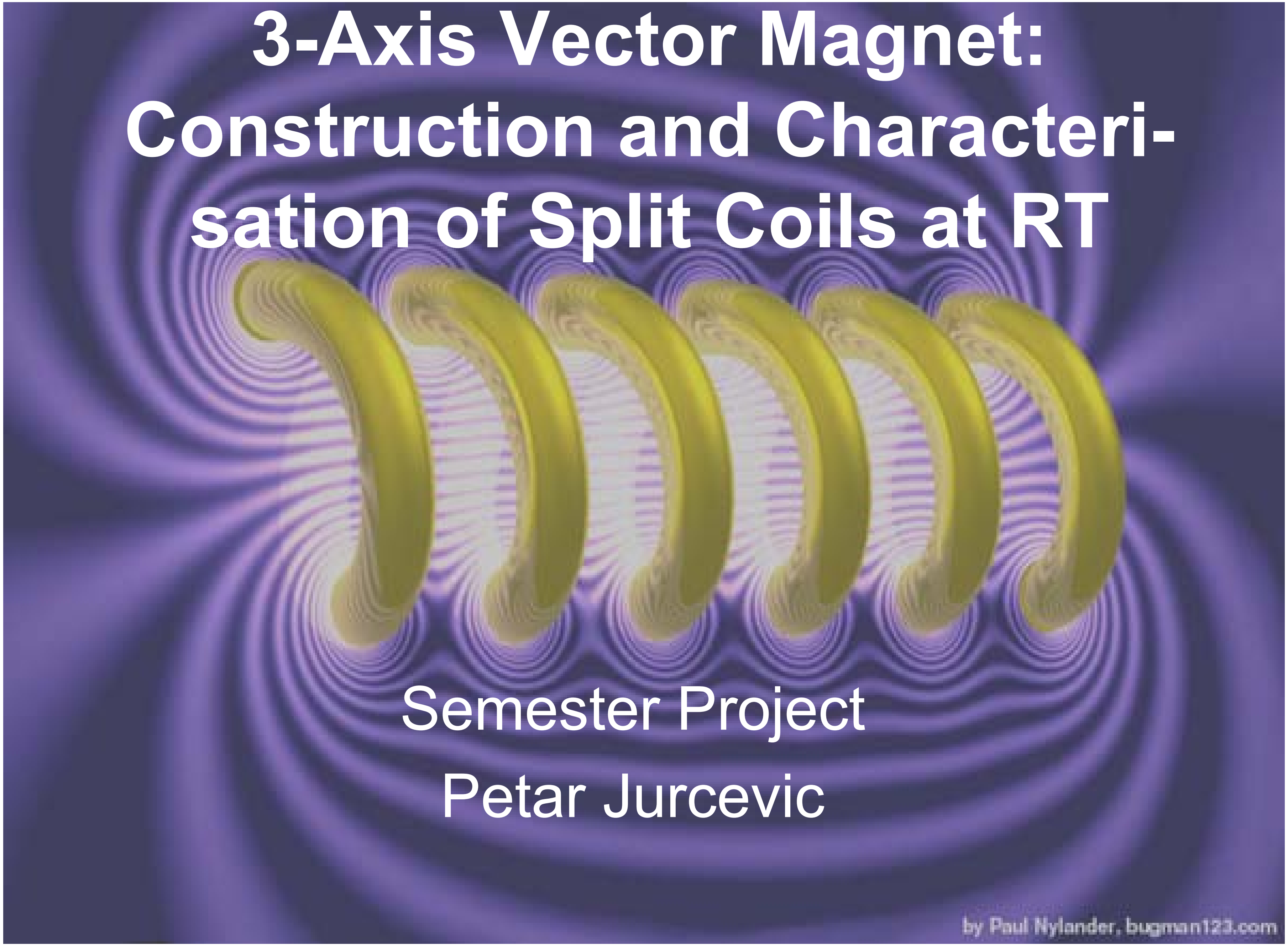
3-Axis Vector Magnet: Construction and Characteri-sation of Split Coils at RT



Semester Project

Petar Jurcevic

Outline

- Field Calculation and Simulation
- Construction Details
- Field Calculations
- Characterization at RT
- Summary

A short repetition

Maxwell‘s equations (SI units)

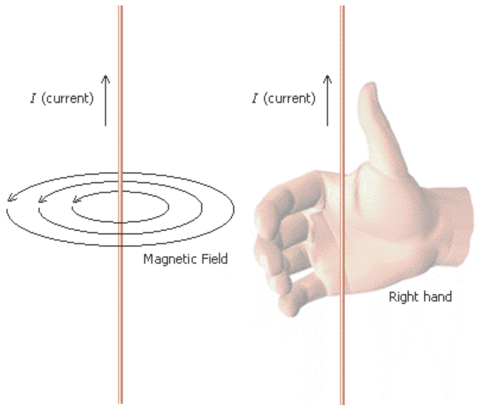
$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} = 4\pi k \rho \qquad \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0 \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\nabla \times \vec{B} = \frac{\vec{J}}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 i + \frac{1}{c^2}\frac{\partial}{\partial t}\int \vec{E} \cdot d\vec{A}$$

Right-hand rule

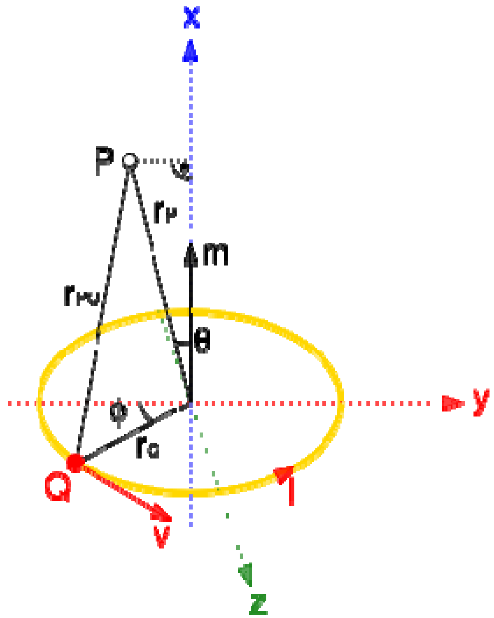


Magnetic field of a circular loop

Biot-Savart Law

$$dB(r) = \frac{\mu_0}{4\pi} I dl(r') \times \frac{r - r'}{|r - r'|^3}$$

μ_0: Permeability
I: Current in Ampere
dl(r´): infinitesimal length of the conductor at position r´
r: Point where the field is computed



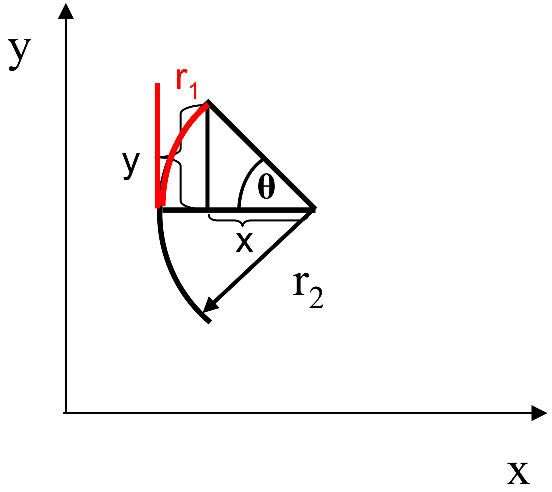
Parametrization of the loop

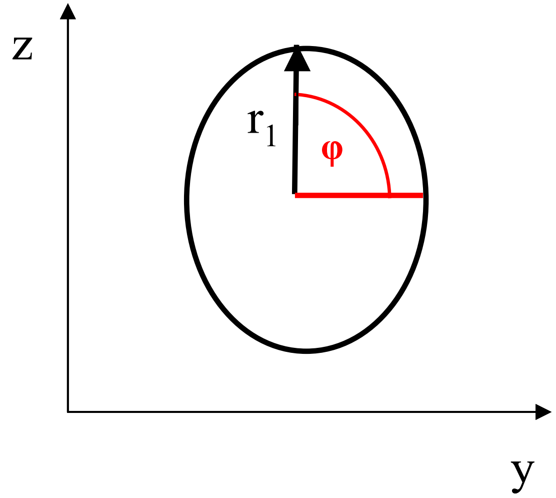
$$r' = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ r_Q \cos(\phi) \\ r_Q \sin(\phi) \end{pmatrix}$$

$$B(r_P) = \frac{I \mu_0 N}{2} \frac{r_Q^2}{\left(r_Q^2 + r_P^2\right)^{3/2}}$$

N: Number of windings

Curved circular loop: Parametrization I





$$y = r_2 \sin(\theta)$$
$$x = r_2 \cos(\theta)$$
$$z = r_1 \cos(\varphi)$$

$$\theta = \frac{r_1'}{r_2} = \frac{r_1 \sin(\varphi)}{r_2}$$

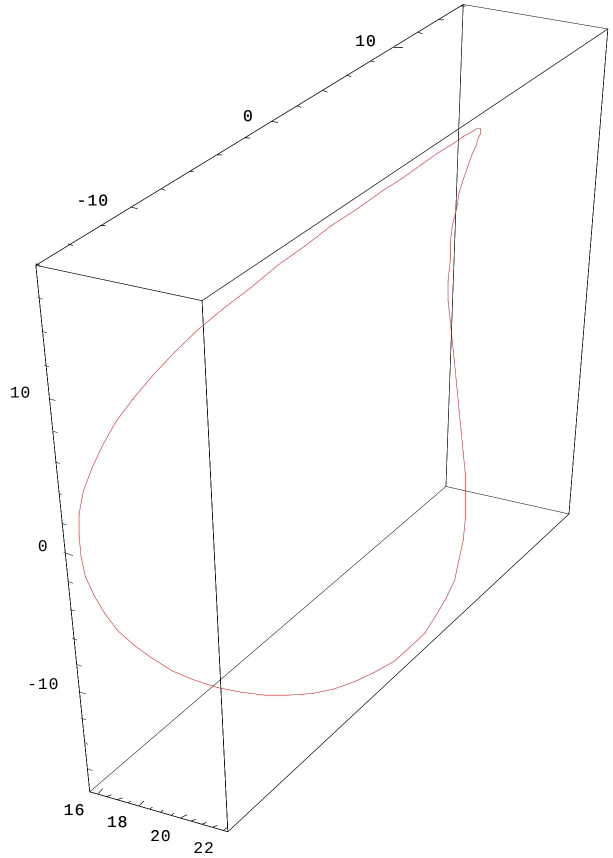
$$y = r_2 \sin\left(\frac{r_1 \sin(\varphi)}{r_2}\right)$$

$$\Rightarrow \quad x = r_2 \cos\left(\frac{r_1 \sin(\varphi)}{r_2}\right)$$

$$z = r_1 \cos(\varphi)$$

Curved circular loop: Parametrization II

$$r(\varphi) = \begin{pmatrix} r_2 \sin\left(\dfrac{r_1 \sin(\varphi)}{r_2}\right) \\ r_2 \cos\left(\dfrac{r_1 \sin(\varphi)}{r_2}\right) \\ r_1 \cos(\varphi) \end{pmatrix}$$

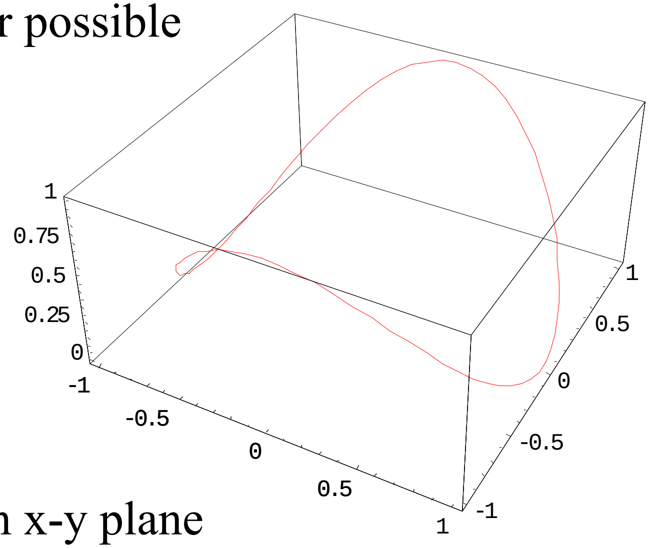


$$r_1 = 0.017m$$
$$r_2 = 0.022m$$

Curved circular loop: Parametrization III

There are a lot of other possible parametrizations like:

$$r(\varphi) = \begin{pmatrix} \rho \sin(\varphi) \\ \rho \cos(\varphi) \\ a \cos^{2n}(\varphi) \end{pmatrix}$$



with n=1,2,3....

ρ: radius of the loop in x-y plane

a: difference between the highest and the lowest point of the loop

Advantage

- Calculations can be simplified
- The term $a\cos^{2n}(\varphi)$ can be substituted for any symmetric and periodic function

Disadvantage

- The field is not calculated in the centre of the IVC, but in the centre of the curved loop -> need of a additional parameter

Field Calculations

Remember: Bio-Savart Law

$$dB(r) = \frac{\mu 0}{4\pi} Idl(r') \times \frac{r - r'}{|r - r'|^3}$$

$$r(\varphi) = \begin{pmatrix} r_2 \sin\left(\frac{r_1 \sin(\varphi)}{r_2}\right) \\ r_2 \cos\left(\frac{r_1 \sin(\varphi)}{r_2}\right) \\ r_1 \cos(\varphi) \end{pmatrix} \frac{1}{\sqrt{r_2^2 + r_1^2 \cos^2(\varphi)}}$$

How to proceed:

- r(φ) has to be normalized
- To get the infinitesimal length dl of the conductor take the derivative of r(φ): **dl(φ)= dr(φ)/dφ**
- Calculate only the x-component of the cross-product

$$\Rightarrow dB_x = \frac{\mu_0 I}{4\pi} \frac{r_1^2 \cos^2(\varphi) \cos\left(\frac{r_1}{r_2}\sin(\varphi)\right) + r_1 r_2 \sin(\varphi) \sin\left(\frac{r_1}{r_2}\sin(\varphi)\right)}{\left(r_2^2 + r_1^2 \cos^2(\varphi)\right)^{3/2}}$$

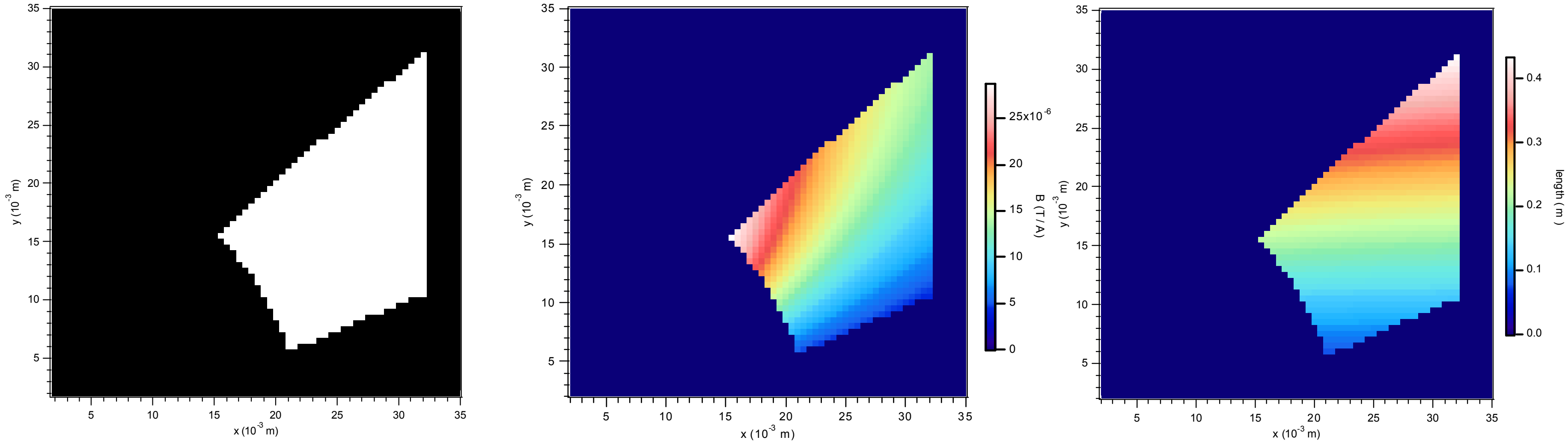
Field Simulation I

Integration of dB_x not easy → Numerical approach

Real coils have a certain thickness → Separation of the curvature from distance in x-direction → Introduction of a new parameter ξ which describes the distance in x-direction

$$dB_x = \frac{\mu_0 I}{4\pi} \frac{r_1^{\,2}\cos^2(\varphi)\cos\left(\frac{r_1}{r_2}\sin(\varphi)\right) + r_1 r_2 \sin(\varphi)\sin\left(\frac{r_1}{r_2}\sin(\varphi)\right)}{\left[\xi^2\cos^2\left(\frac{r_1}{r_2}\sin(\varphi)\right) r_2^2 \sin^2\left(\frac{r_1}{r_2}\sin(\varphi)\right) + r_1^{\,2}\cos^2(\varphi)\right]^{3/2}}$$

Field Simulation I



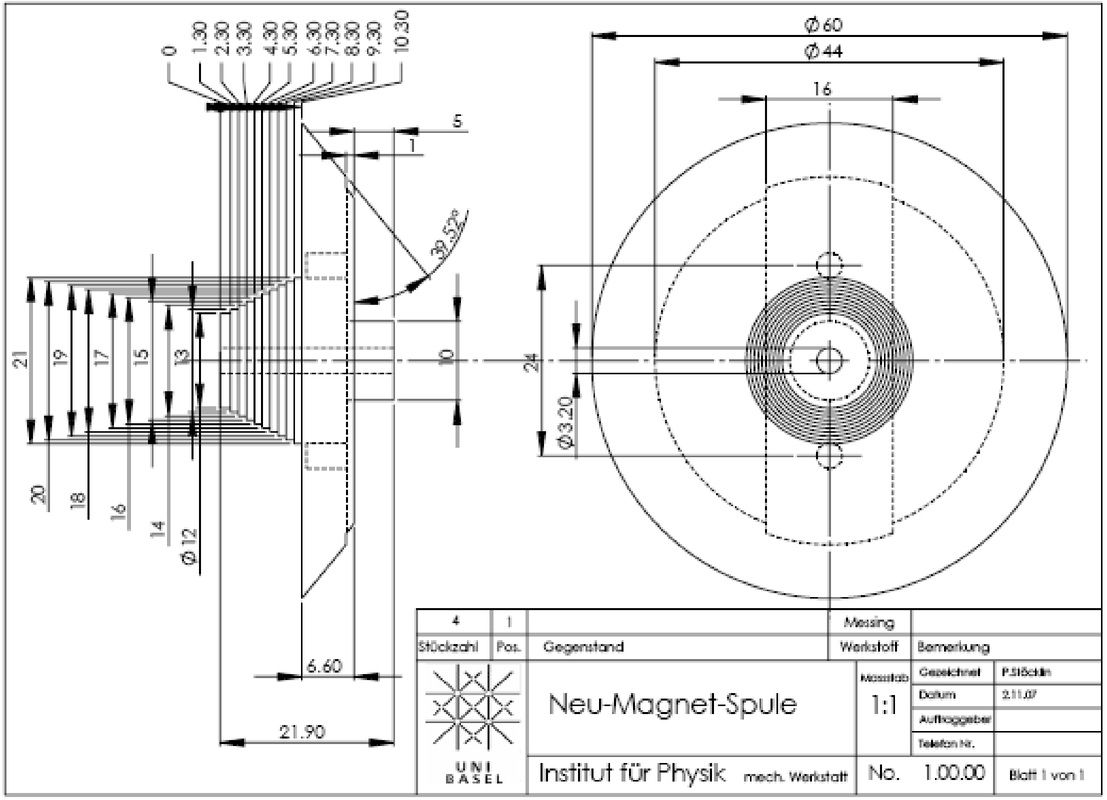
Grid 5mm x 5mm

2200 windings

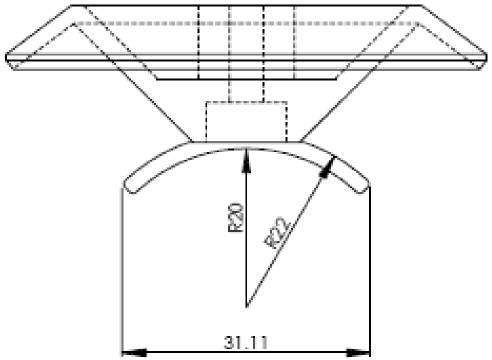
B-field at 1 Ampere: 0.023 T; B-field at 50 Ampere: 1,16 T

Estimated wire length: 250.5 m

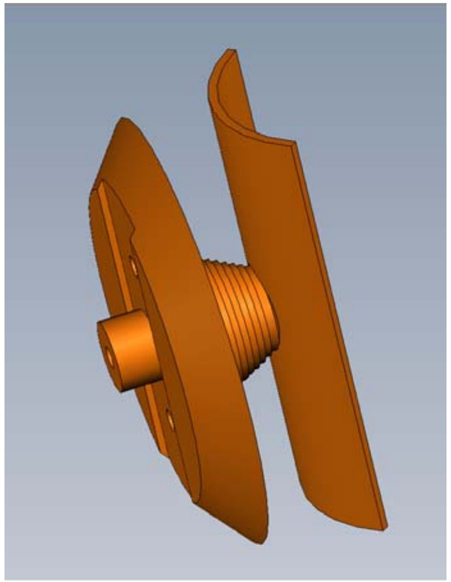
The Coil



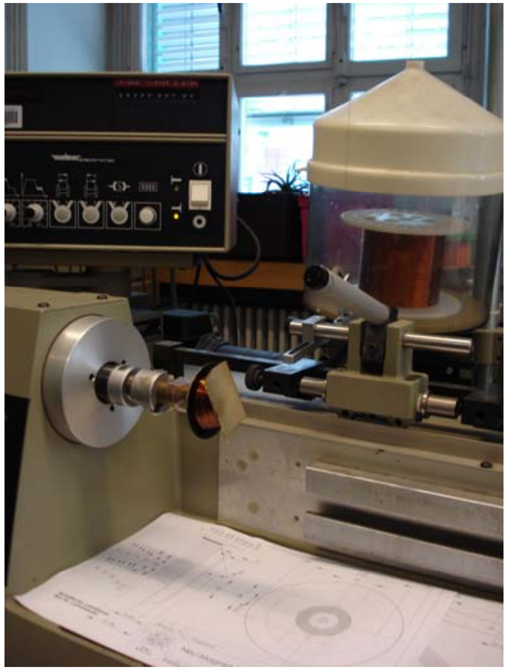
Ø 60
Ø 44
16
0
1.30
2.30
3.30
4.30
5.30
6.30
7.30
8.30
9.30
10.30
5
1
39.52°
21
19
17
15
13
10
24
Ø 3.20
20
18
16
14
Ø 12
6.60
21.90
4
1
Messing
Stückzahl
Pos.
Gegenstand
Werkstoff
Bemerkung
Massstab
1:1
Gezeichnet
P.Stöcklin
Datum
2.11.07
Auftraggeber
Telefon Nr.
Neu-Magnet-Spule
UNI BASEL
Institut für Physik
mech. Werkstatt
No.
1.00.00
Blatt 1 von 1

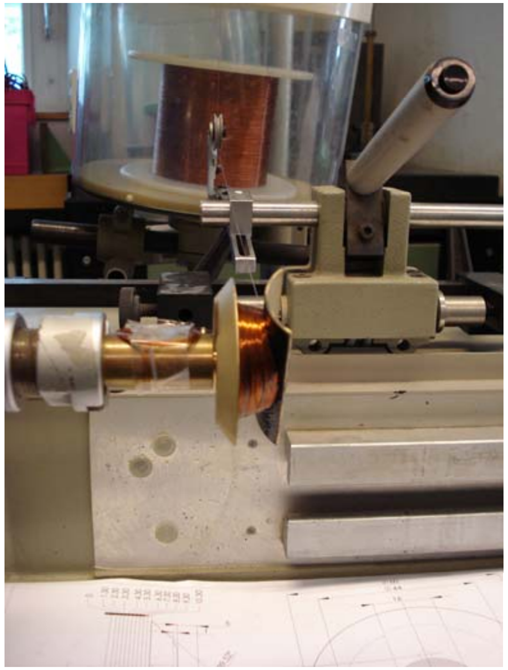


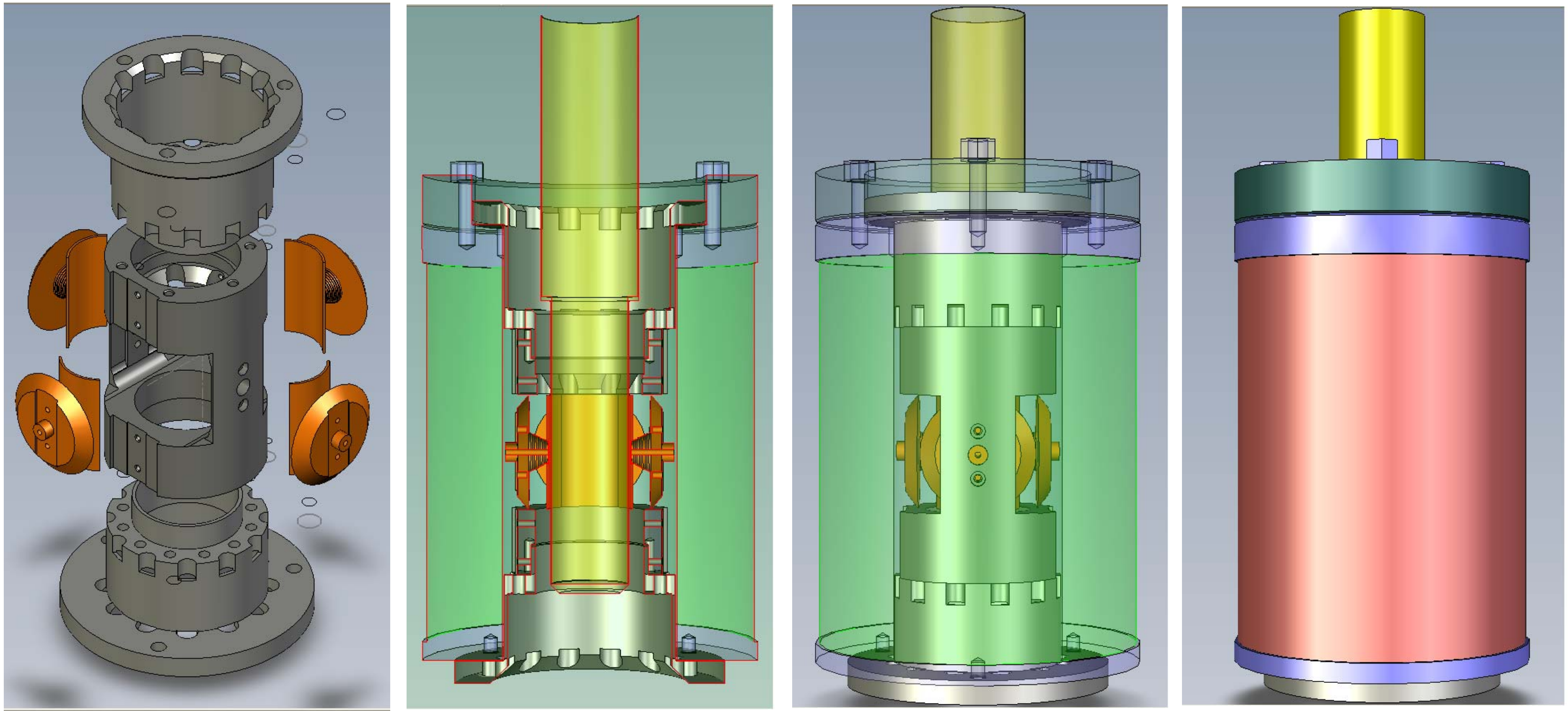
R20
R22
31.11



Winding





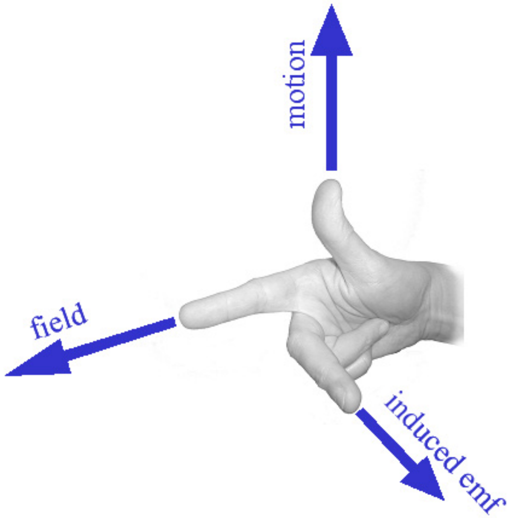


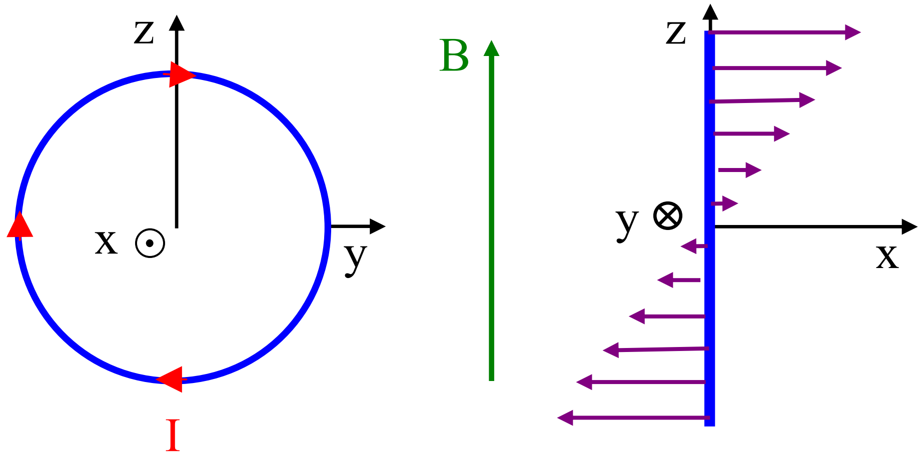
Force Calculations I

Lorentz Force

$$\vec{F} = I \oint d\vec{l} \times \vec{B}$$

Right Hand Rule





Integration over the 2π → no net force
Integration over the semi circle (-π/2, π/2)

$$F_x^{\cap} = 2\,NIBr$$

⇨ **33.6 kN** ≈ 34*10^3 chocolate bars

B = 9T; I = 50A; r = 0.017m; N = 2200

Force Calculations II

For the curved circular loop:

$$F_x^{\cap} = 2NIBr\sin\left(\frac{r_1}{r_2}\right)$$

⇒ **30.4 kN**

I = 50A; B = 9T;
N = 2200; r1 = 0.017m
r2 = 0.022m

The force over the whole circle is zero, but points in opposit direction over the two semi circles

⇒ Emergence of a torque

We just consider the flat case

$$\vec{M} = \vec{\mu} \times \vec{B}$$

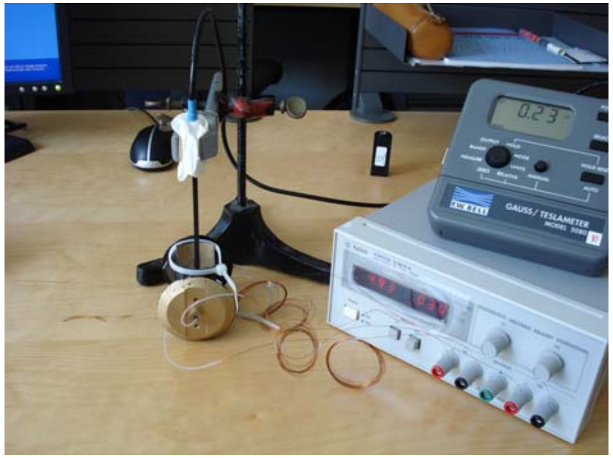
$$\vec{\mu} = \vec{A}I$$

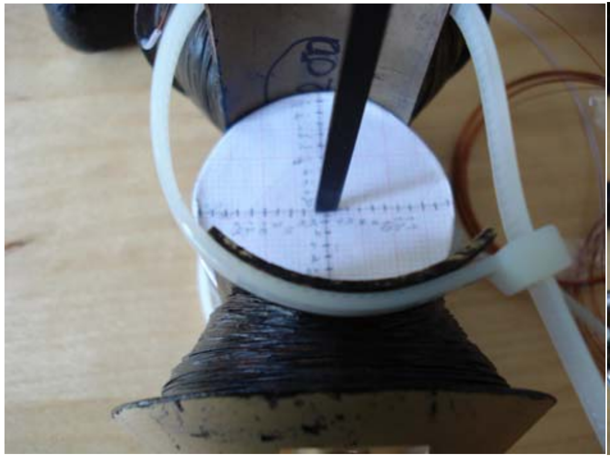
$$\vec{A} = r^2\pi\vec{n}$$

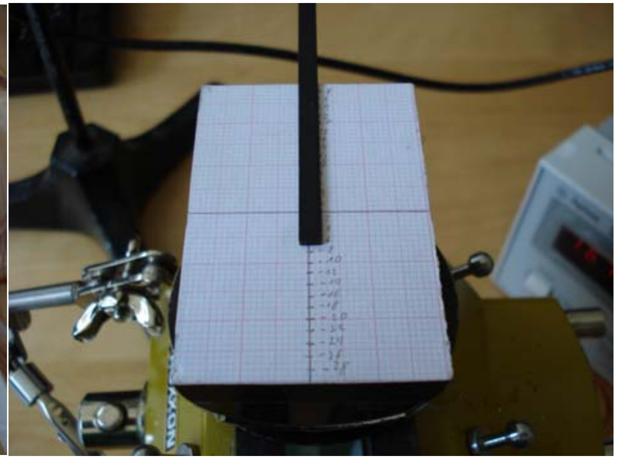
$$M_y = 1244\,Nm$$

712 Nm

Measurement setup

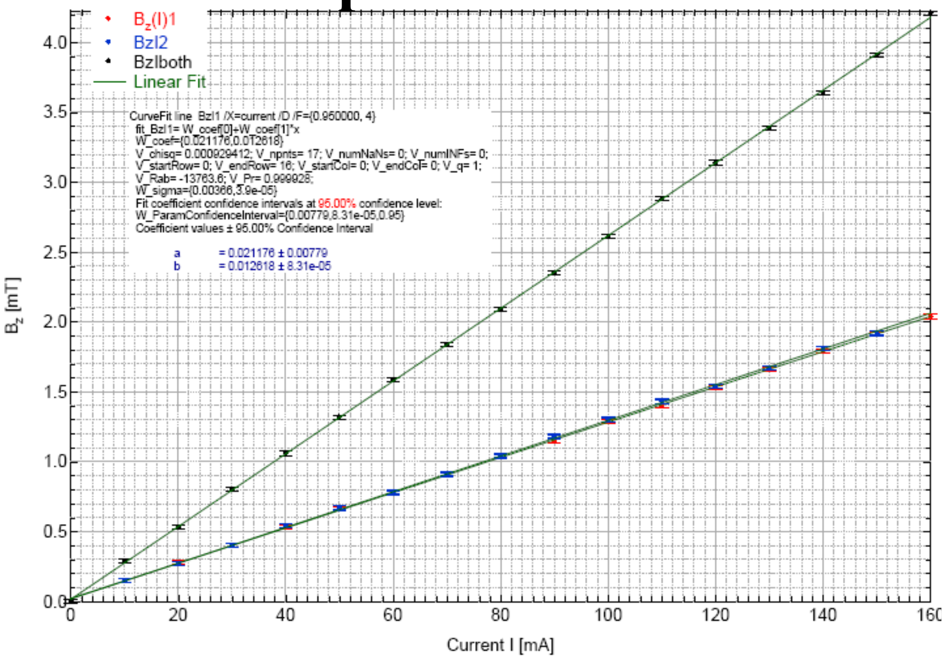






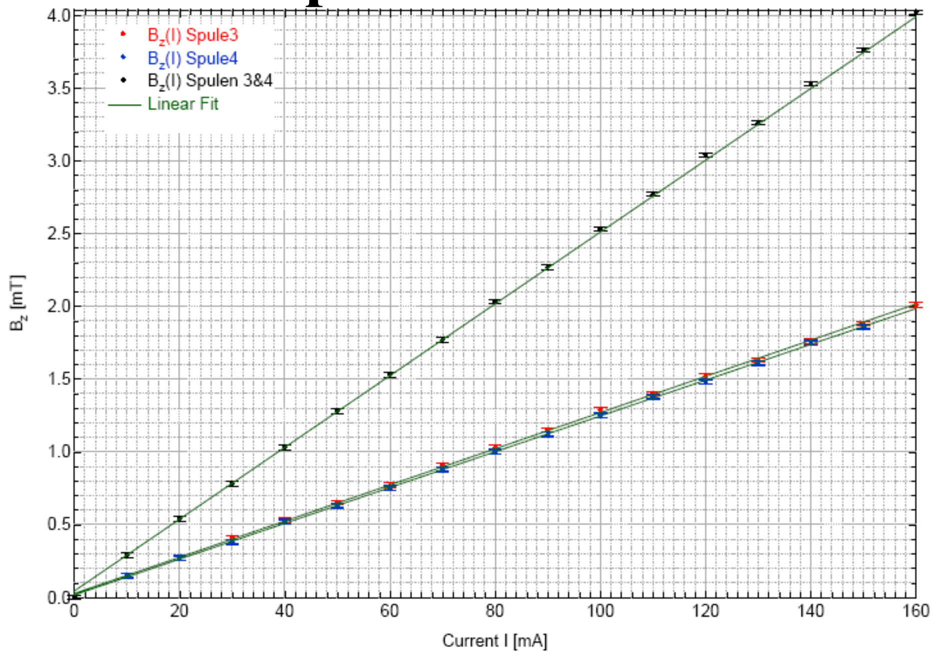
Measurements I

Spools 1 & 2



2.53 mT @ 100 mA (1&2)

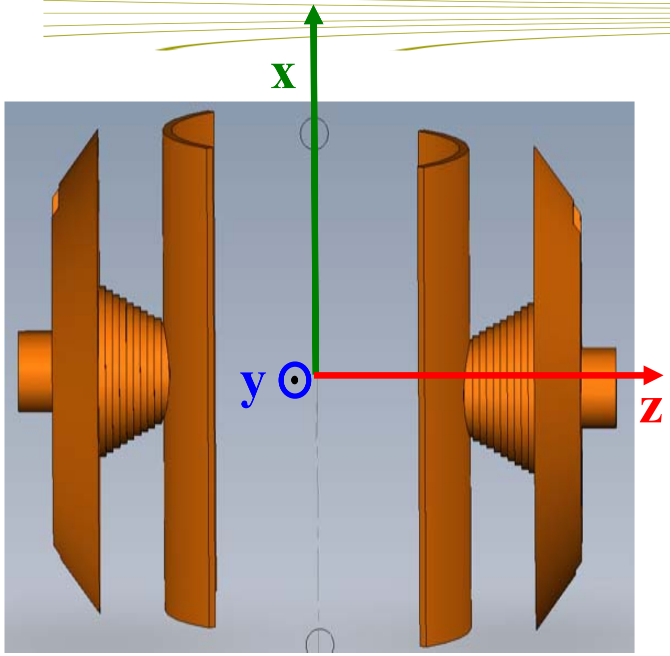
Spools 3 & 4

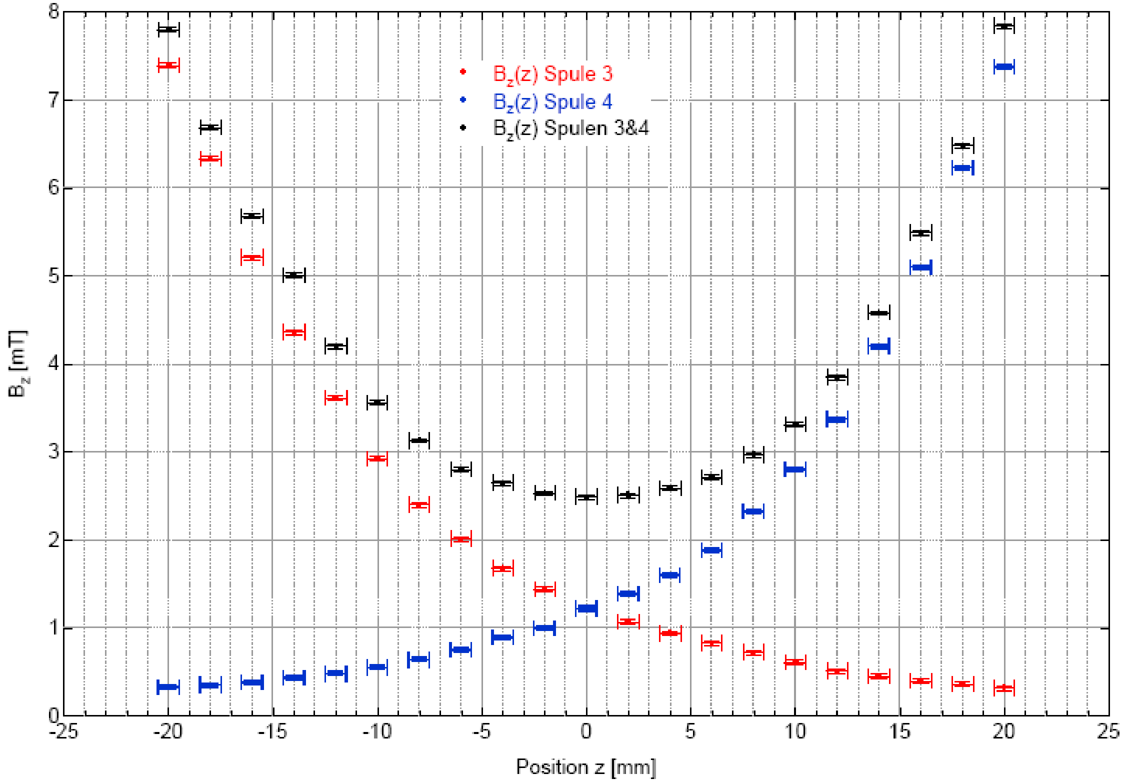


2.38 mT @ 100 mA (3&4)

Comparison with simulation ($B_z(0)$=23 mT): Accuracy 3.4% - 8.9%

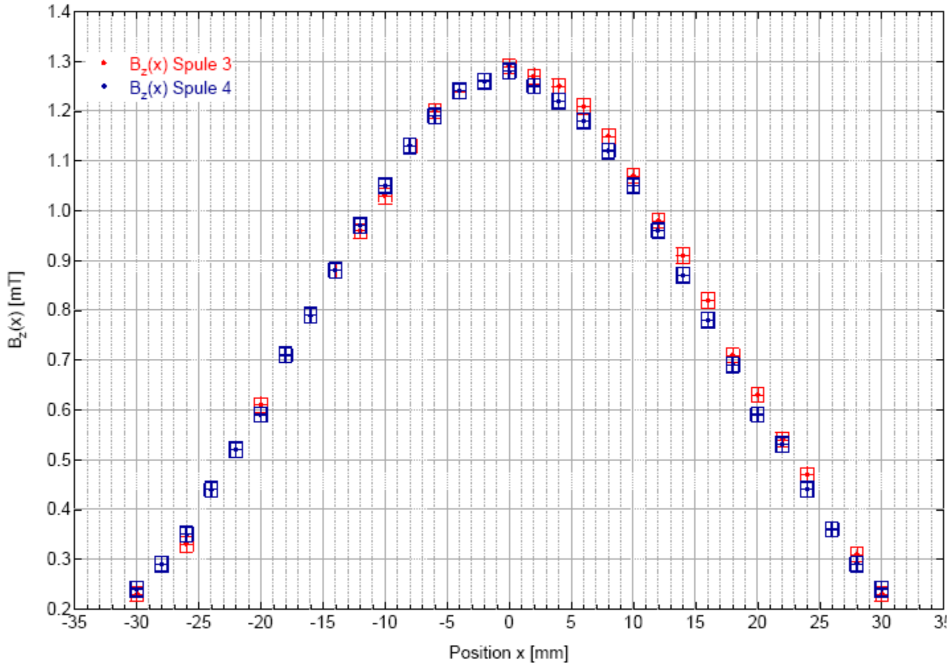
Measurements II



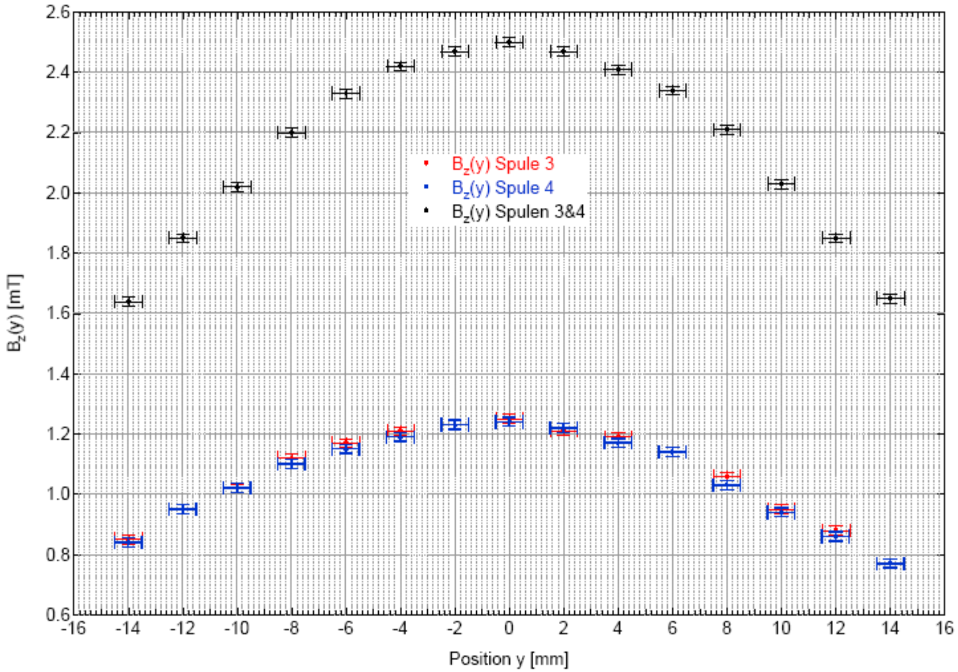


$B_z(z)$ @ 100 mA

Measurements III



$B_z(x)$ @ 100 mA



$B_z(y)$ @ 100 mA

Measurements IV

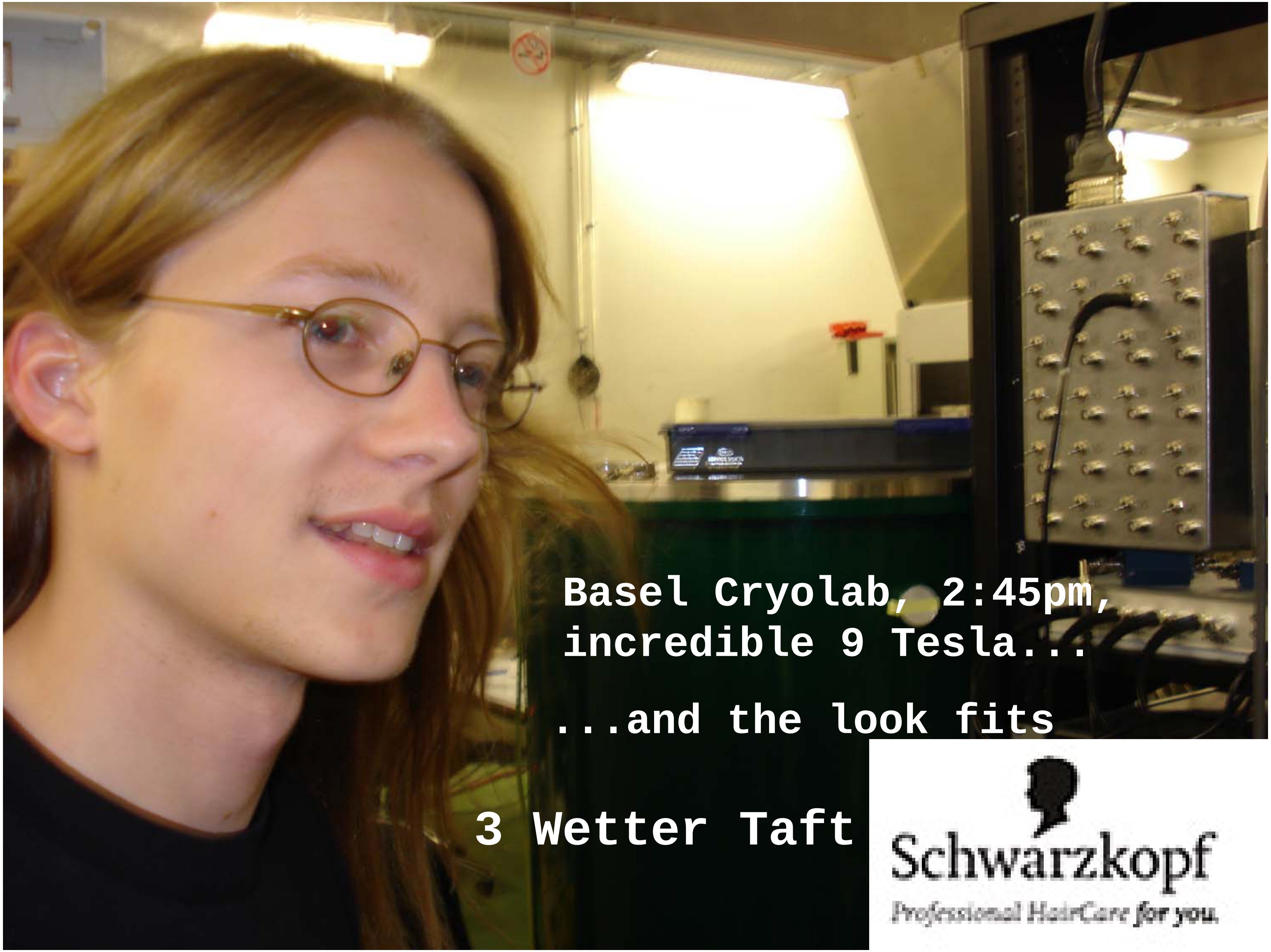
	Coil 1	Coil 2	Coil 3	Coil 4
Number of windings	2200	2200	2202	2200
Bz @ 100mA	1.29mT	1.26mT	1.26mT	1.25mT
Resistance R DC	169.4Ω	163.4Ω	162.2Ω	163.6Ω
Resistance R @ 120Hz	184.1Ω	174.2Ω	175.5Ω	176.9Ω
Resistance R @ 1kHz	338.7Ω	322.8Ω	325.4Ω	328.6Ω
Inductance L @ 120Hz	107.8mH	101.9mH	104.6mH	104.6mH
Inductance L @ 1kHz	62.1mH	59.8mH	60.6mH	59.9mH
Inductance L @ 10kHz	41.5mH	40.5mH	41.2mH	40.5mH
Phase θ @ 120Hz	23.84°	23.83°	24.12°	24.03°
Phase θ @ 1kHz	49.03°	49.36°	49.42°	48.89
Phase θ @ 10kHz	71.47°	70.74°	71.75°	71.59°

Next Steps

- Install the coils into the cage
- Insert and install the 2-axis split pair system into the solenoid
- Characterize the system at liquid helium temperature

Summary

- High forces and a high torque can act on the system
- The calculated magnetic field is comparable with the measured field
- In the dimension of the device the magnetic field is almost homogeneous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3 Wetter Taft
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Professional HairCare for you.