1. Introduction

2. Few Electron Dots

3. Double Quantum Dots

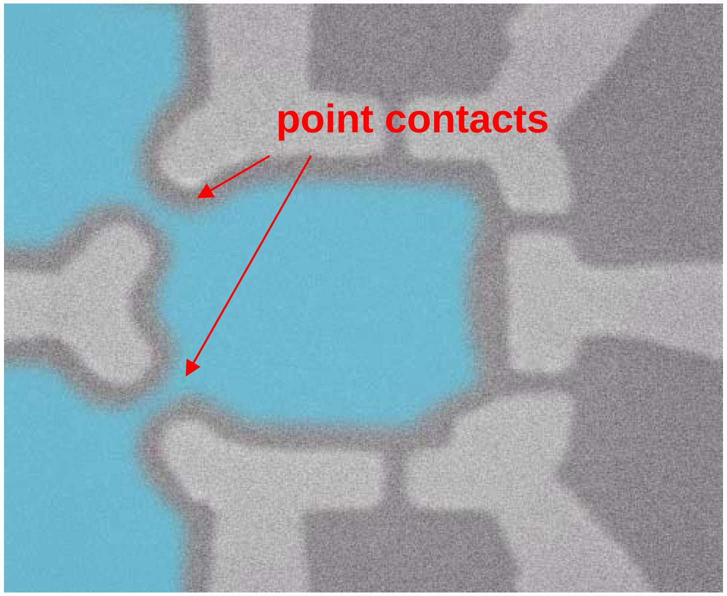
4. Kondo Effect

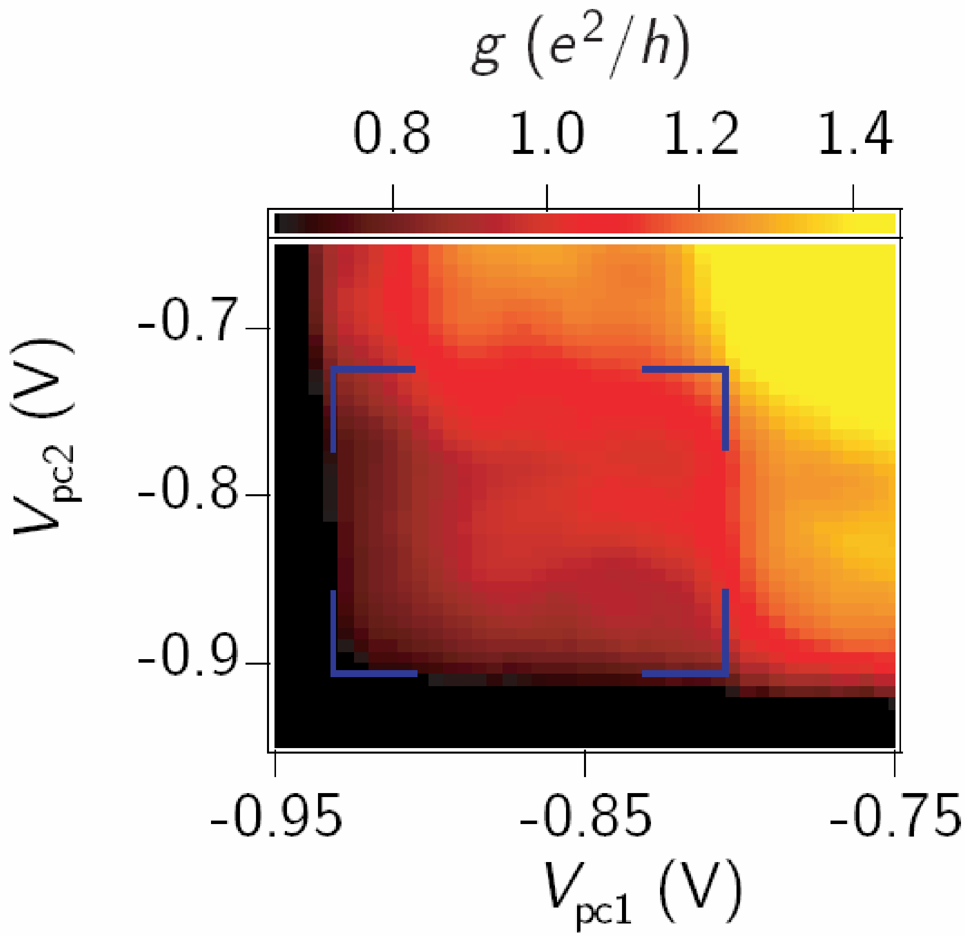
**5. Open Dot Experiments**

Huibers, Ph.D. Thesis (1999)
Huibers et al., PRL83, 5090 (1999)

# Open Dot Regime

## Open Dot



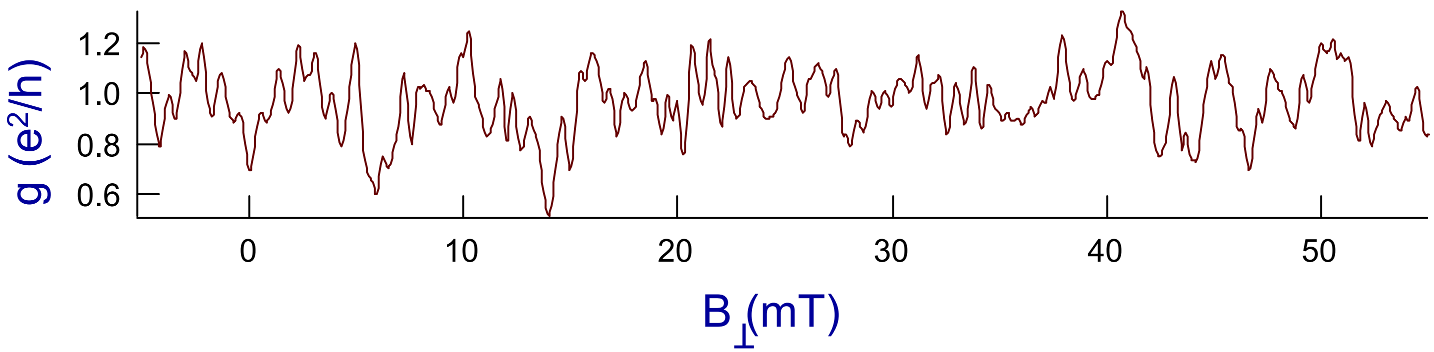



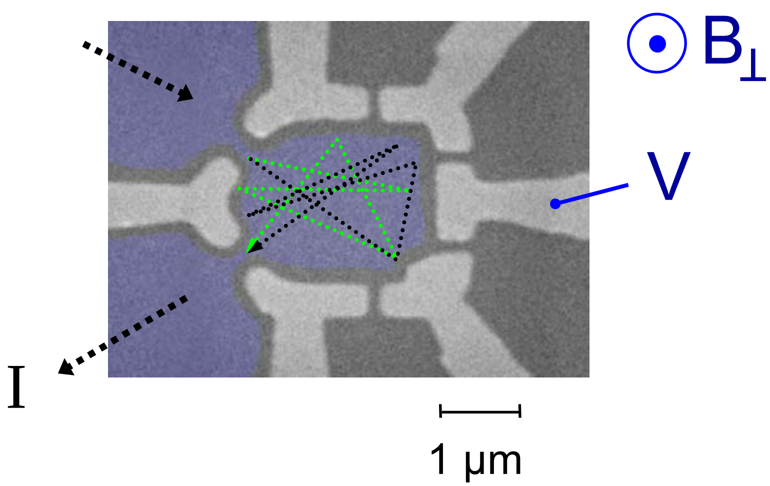

- $V_{gate}$ set to allow ≥ $2e^2/h$ conductance through each point contact
- Dot is well-connected to reservoirs
- Transport measurements exhibit CF and Weak Localization

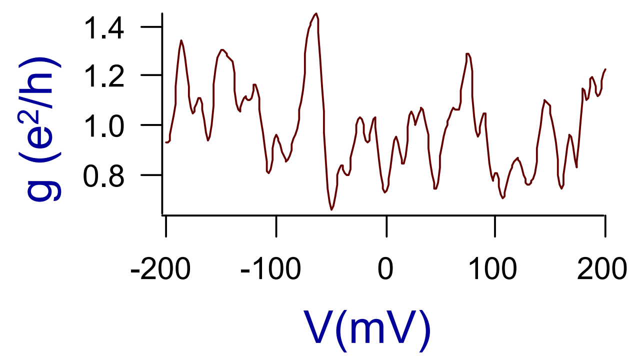
many open dot slides: A. Huibers and J. Folk

# Open Dot Regime: Conductance Fluctuations

$N_L = N_R = 1$



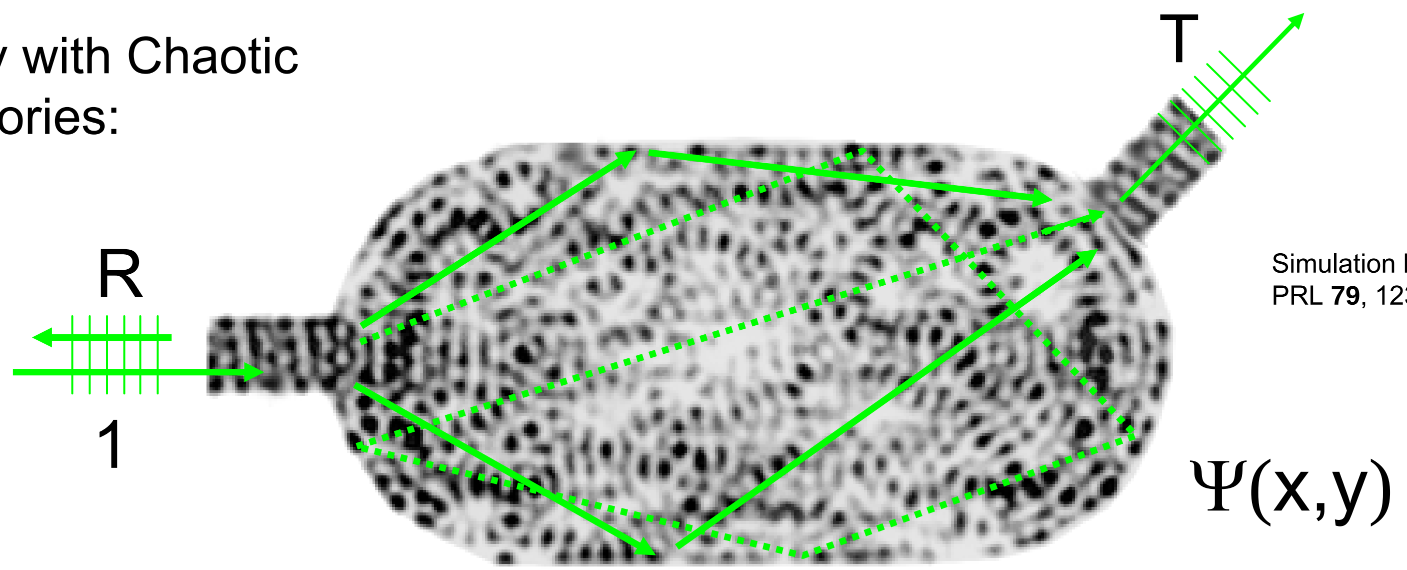



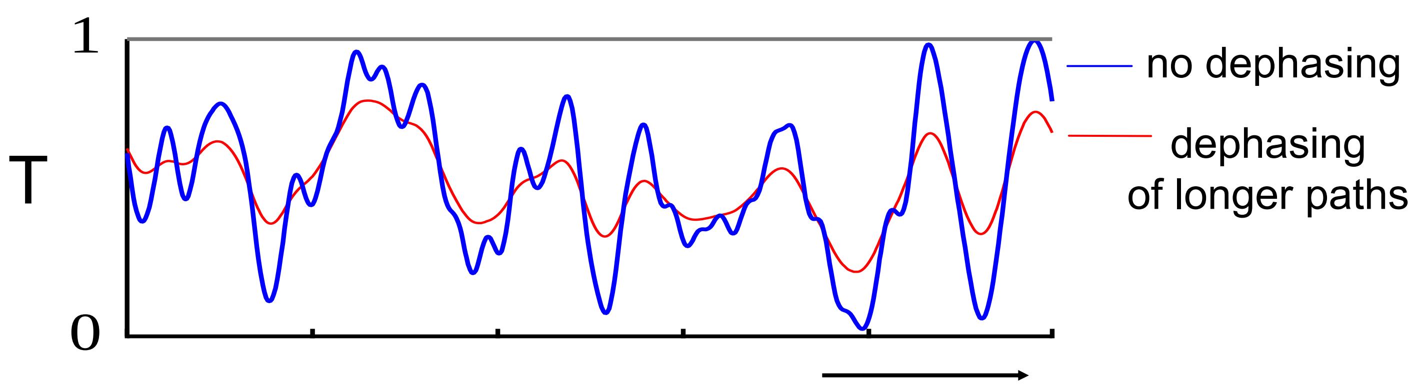

<u>Repeatable</u> random
intereference fluctuations
as function of dot parameters

# Two-Dimensional Quantum Dot

2D Cavity with Chaotic trajectories:




Simulation by R. Akis, PRL **79**, 123 (1997)

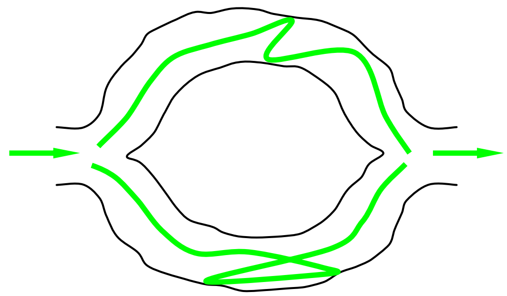



Any parameter that changes path accumulated phase

Goal: use quantum dot as a probe of quantum phase coherence

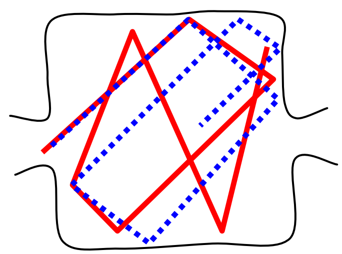
# Interferometers

Two-arm:



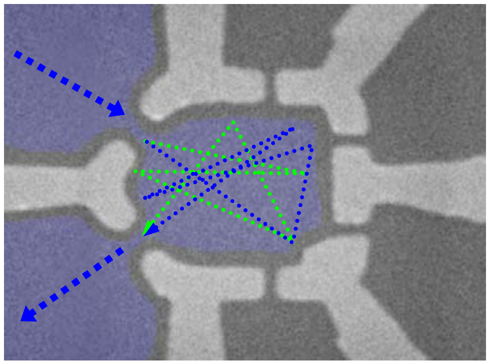
sometimes:
reflections or
small signal

Regular/
Integrable:



Problem:
<u>partially</u> chaotic

Chaotic:

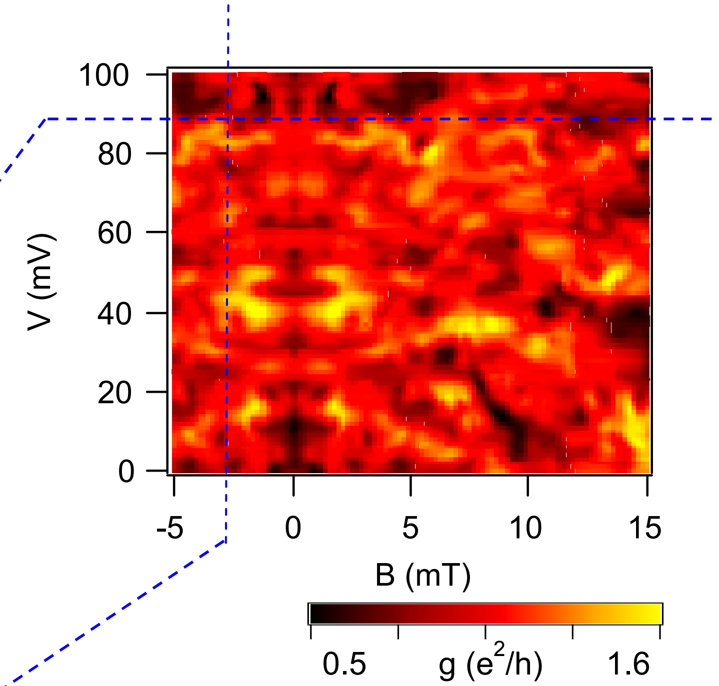


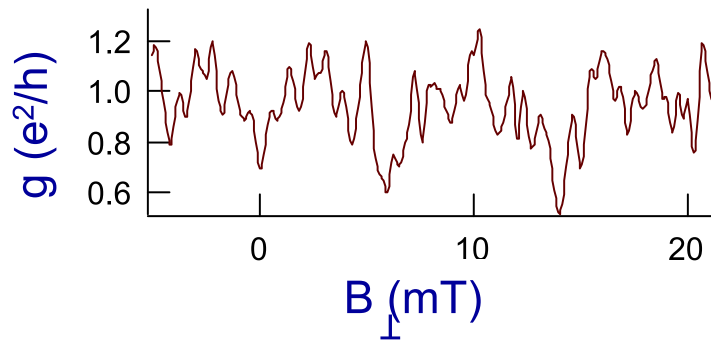
1. Mostly chaotic/ergodic

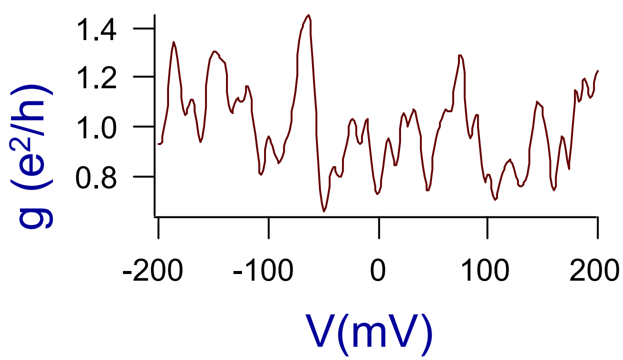
2. Interesting physics
& complete description

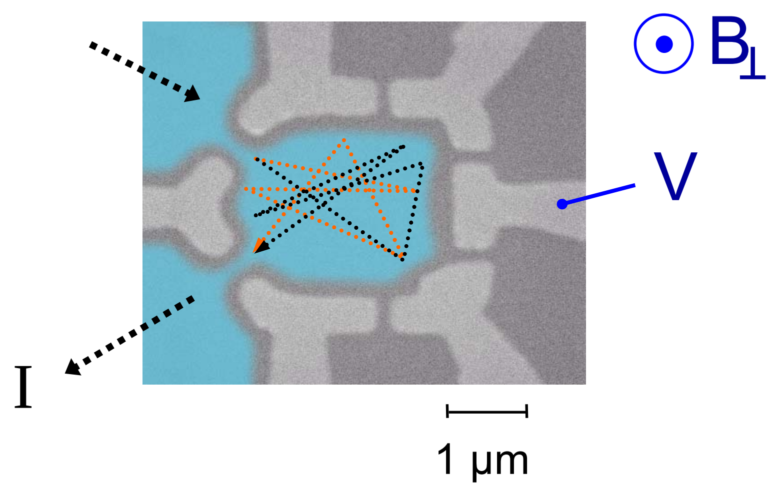
# Quantum Interference in Open Dots

Interference between all possible trajectories
gives rise to repeatable random
intereference fluctuations
as function of dot parameters

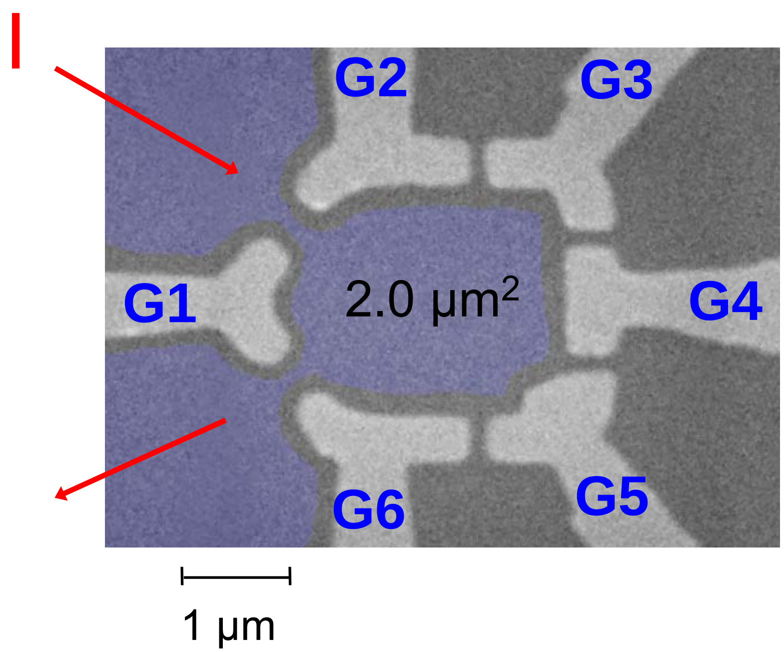


# Typical Quantum Dot




2D conductor:

area = 2.0 μm$^2$

charge density = 2 10$^{11}$ e/cm$^2$

$\lambda_F$ = Fermi wavelength = 50 nm

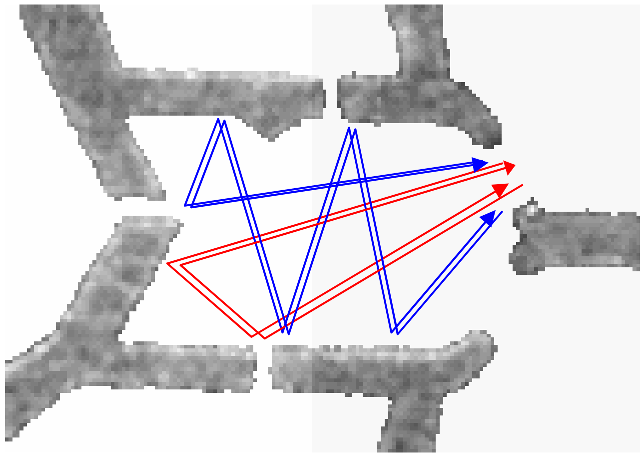
$v_F$ = Fermi velocity = 200 μm/ns

$E_F$ = Fermi energy = 7 meV

| | |
|---|---|
| Dwell time in dot: | 200 ps |
| Crossing time: | 7 ps |
| | 30 bounces |

bulk mean free path $\ell_e$ ~ 2- 10 μm

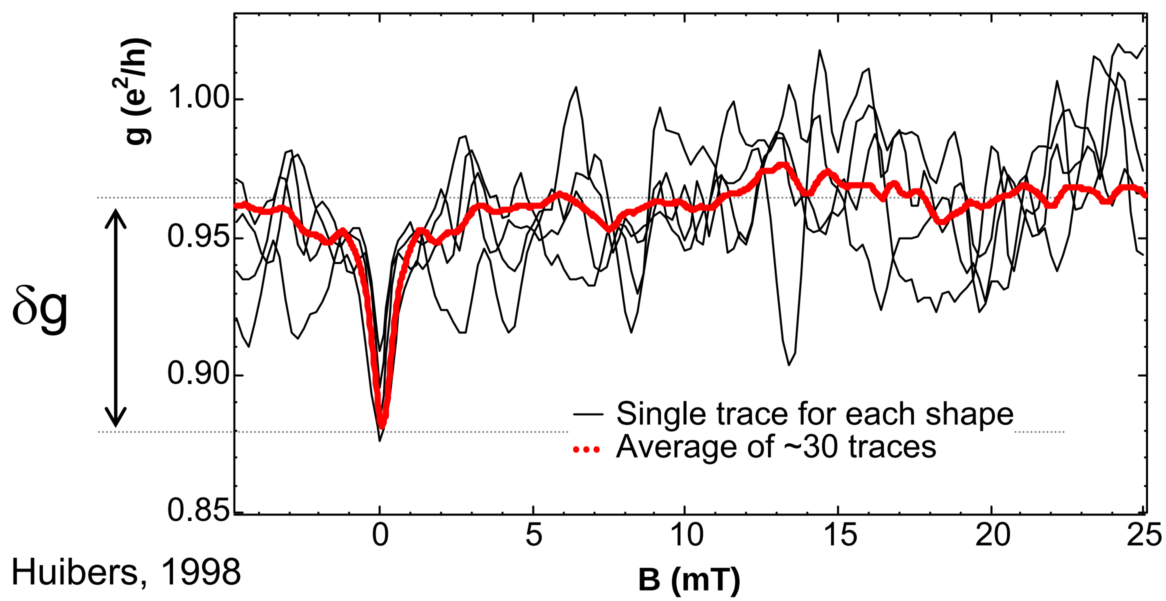
# Weak Localization



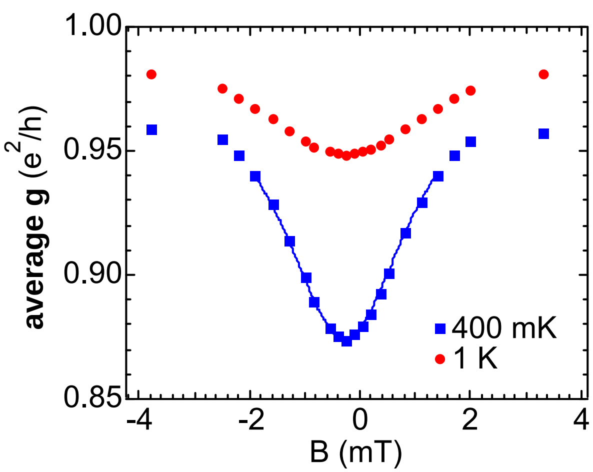
At B=0, phase-coherent backscattering results in “weak localization”

↓

Conductance dip at B=0

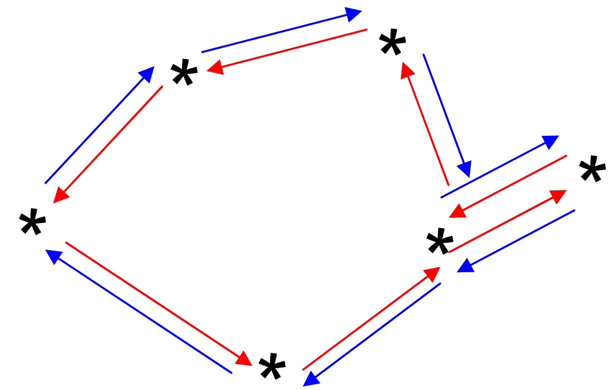



Huibers, 1998



# Quantum Correction: Weak Localization

constructive interference of coherently backscattered, time reversed trajectories decreases conductivity



$$\frac{\delta\sigma_{loc}}{\sigma} \propto -\frac{1}{k_F\ell}\ln\left(1+\frac{\tau_\varphi}{\tau}\right)$$ 2D ($L_\varphi$<<W)

$$\frac{\delta\sigma_{loc}}{\sigma} \propto -\frac{L_\varphi}{W}\frac{1}{k_F\ell}\left(1-\left(1+\frac{\tau_\varphi}{\tau}\right)^{-1/2}\right)$$ 1D ($L_\varphi$>>W)

magnetic field: AB-flux, cut off trajectories of area A>$\phi_0$B
magnetoconductance

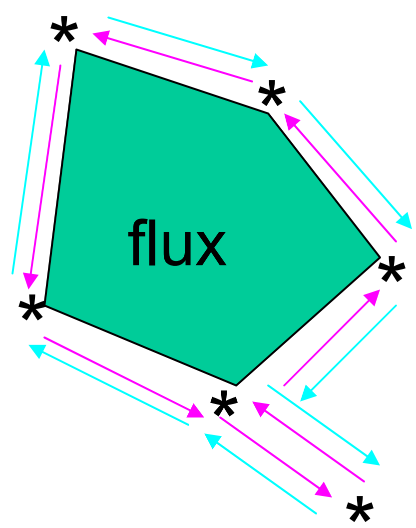
(assuming spinless electrons)

# Weak Localization in Magnetic Fields

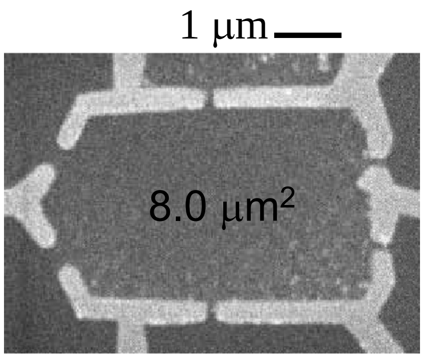
in a given magnetic field B, trajectories enclosing flux acquire additional Aharonov-Bohm phase:

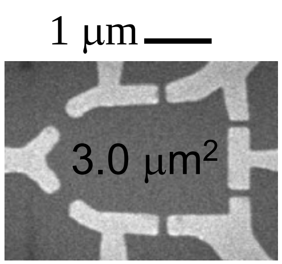
$$\phi = \frac{2e}{\hbar}\int(\nabla \times A)\cdot d\vec{S} = \frac{2eBS}{\hbar}$$

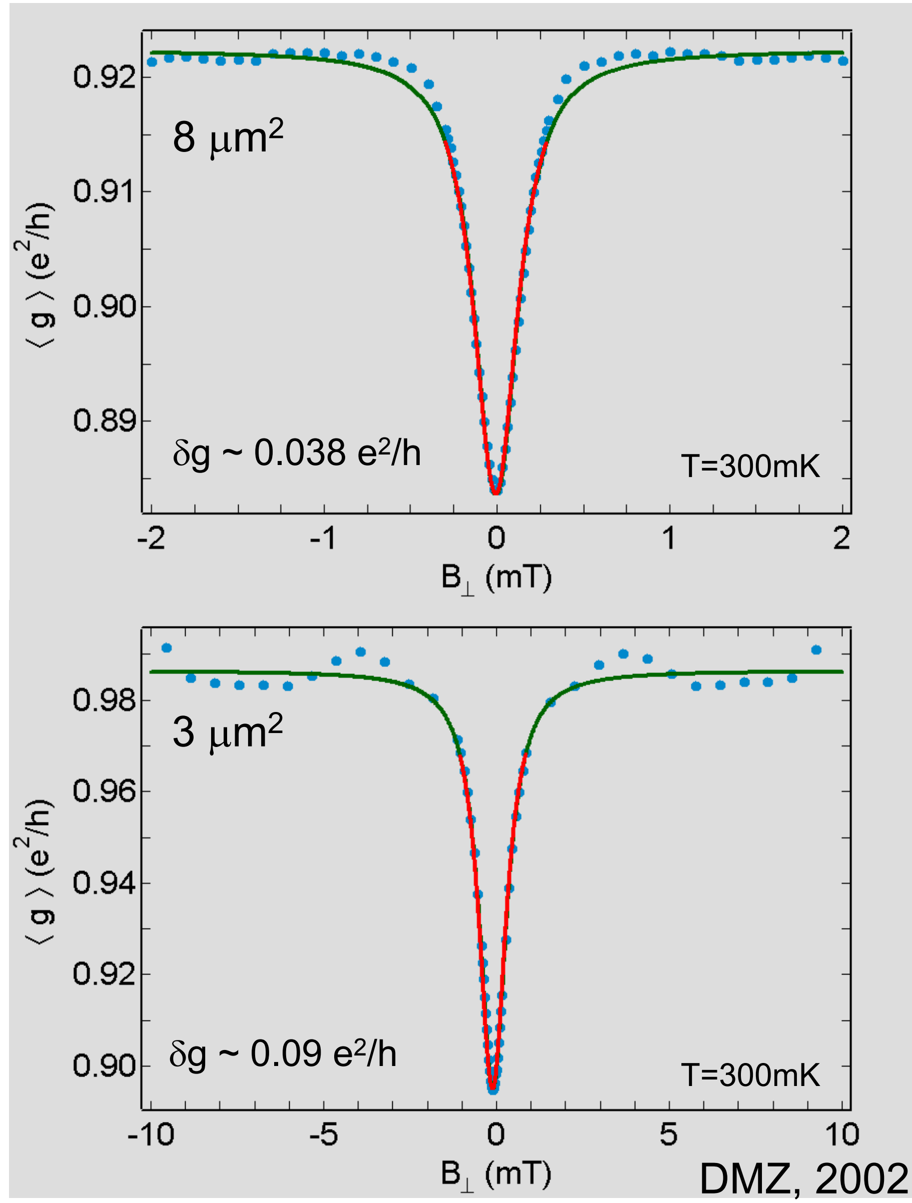
when summing over all trajectories, this $\phi$ will effectively eliminate trajectories of area $A >> \phi_0/B$. ($\phi_0 = h/e$)



# Weak Localization: Measure of Dephasing



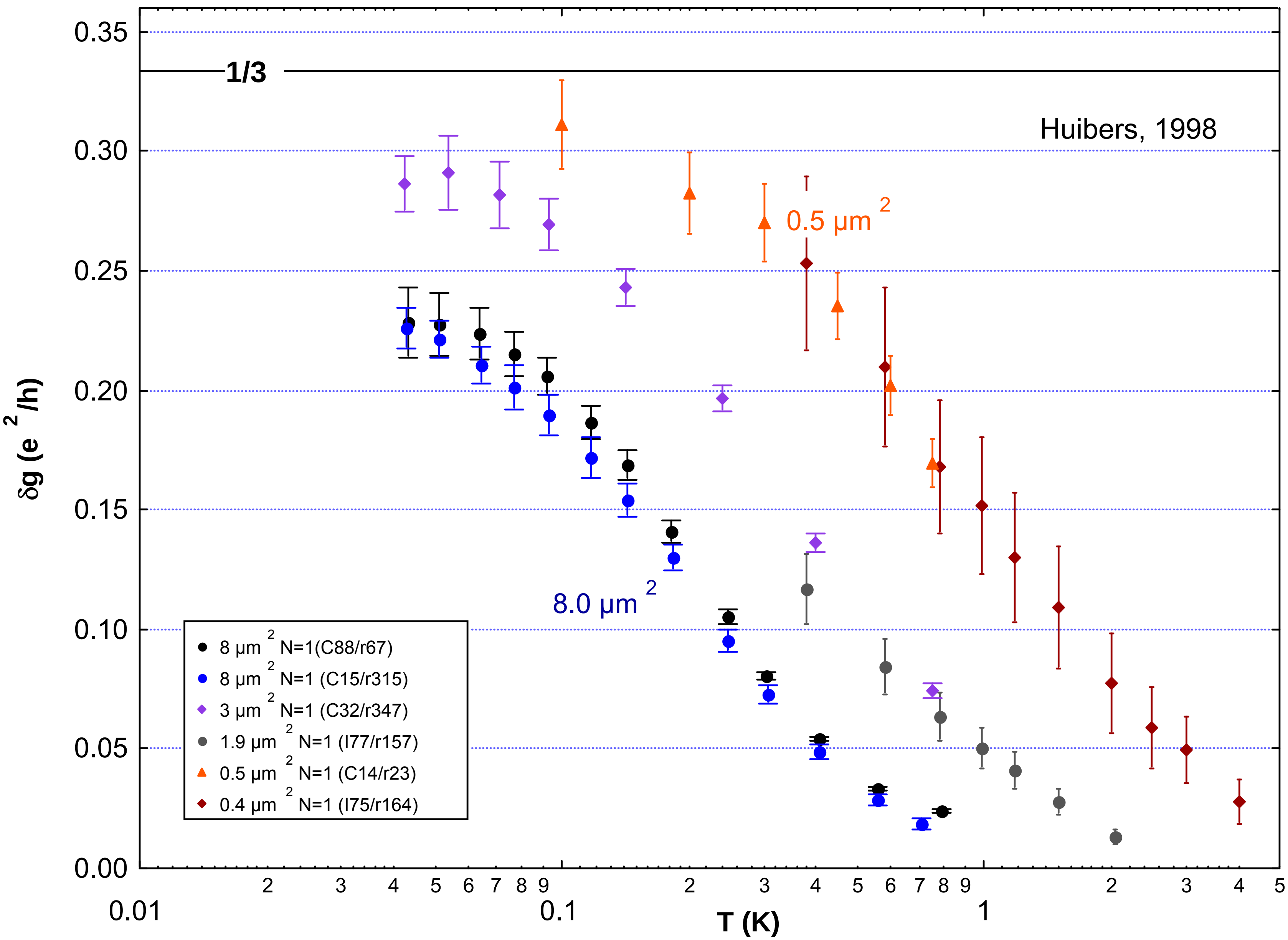




random matrix theory

$$\delta g = \frac{1}{2N + 1 + \gamma_\phi} \qquad \tau_\phi^{-1} = \frac{\Delta}{h}\gamma_\phi$$

# Weak Localization vs T



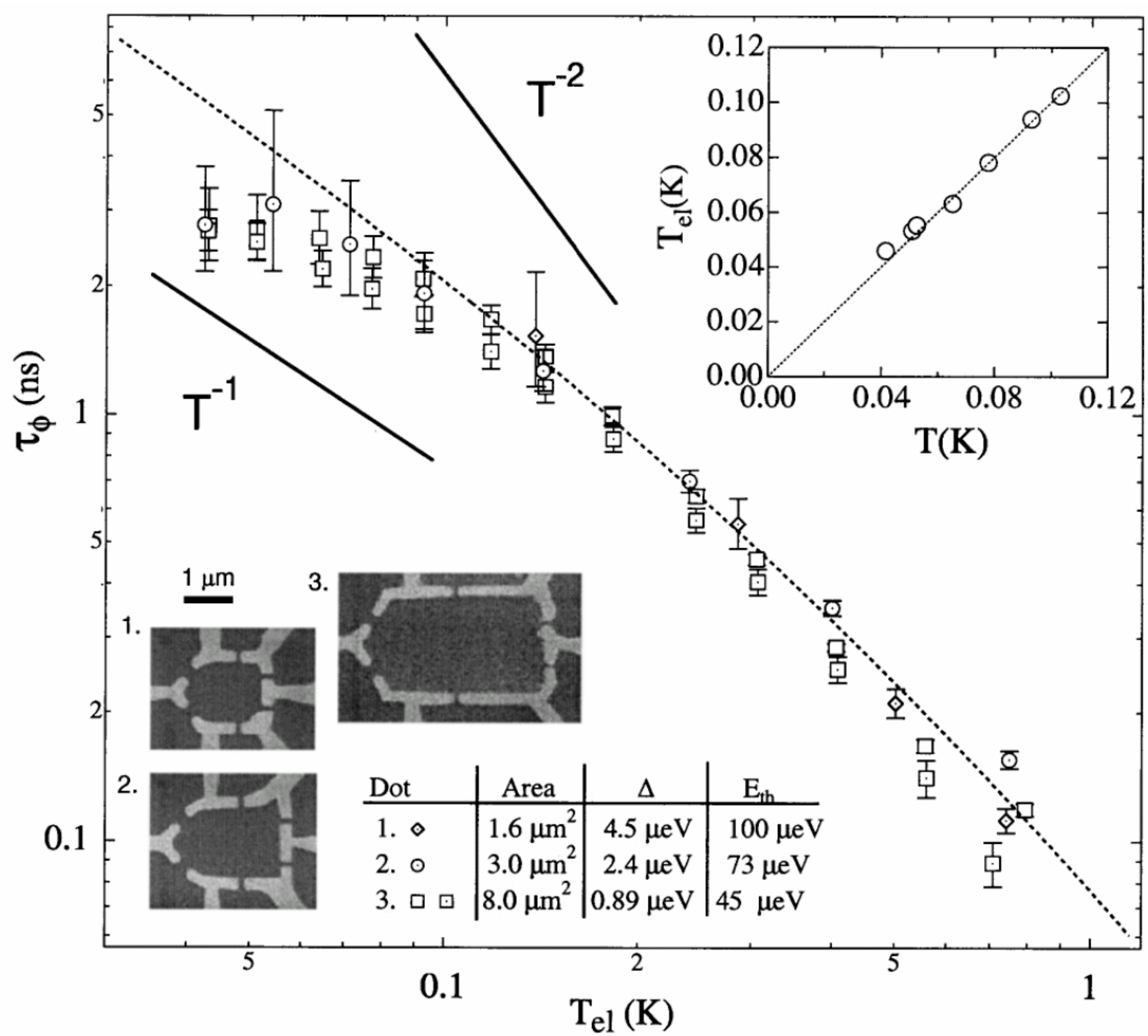
# Low Temperature Saturation?

using

$$\delta g = \frac{1}{2N + 1 + \gamma_\phi}$$

obtain

$$\tau_\phi^{-1} = \frac{\Delta}{h}\gamma_\phi$$




| Dot | Area | Δ | $E_{th}$ |
|---|---|---|---|
| 1. ◇ | 1.6 µm² | 4.5 µeV | 100 µeV |
| 2. ⊙ | 3.0 µm² | 2.4 µeV | 73 µeV |
| 3. □ ⊡ | 8.0 µm² | 0.89 µeV | 45 µeV |

Huibers et al., PRL83, 5090 (1999)

# Spin-Orbit Coupling

electrons move with the Fermi velocity, electric fields in material appear as magnetic fields in the rest frame of the electron

these magnetic fields

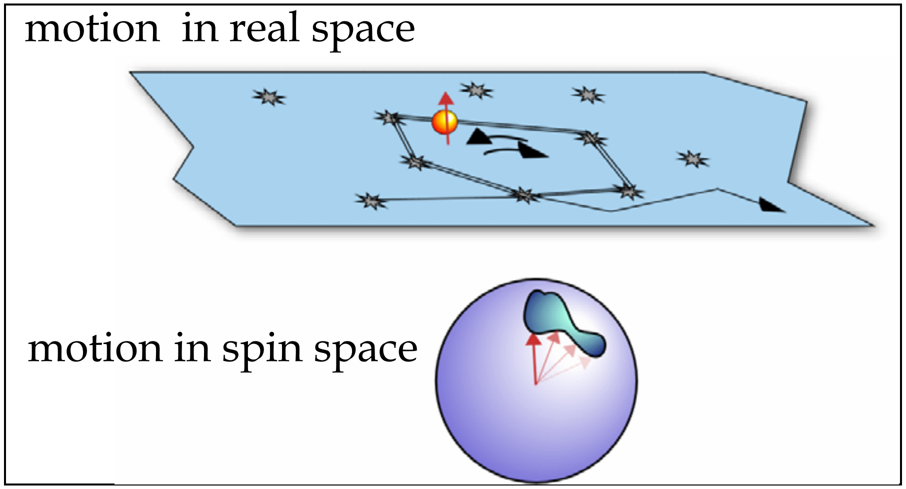
- depend on magnitude of electron velocity (density dependence)
- couple to the electron spin via Zeeman coupling

→ spin-precessions

electric fields due to:

- heterointerface (***Rashba***)
- crystalline anisotropy in III-V zincblende crystal (***Dresselhaus***)

*spin precession affects phase interference*
*(2π in spin space gives -1 to phase)*



# Spin-Orbit Coupling

presence of electric fields $\vec{E} = -\frac{1}{e}\vec{\nabla}V$

electrons are moving in these electric fields

rest frame of electrons: effective magnetic field

$$\vec{B}_{so} = -\frac{\vec{v}}{c} \times \vec{E}$$

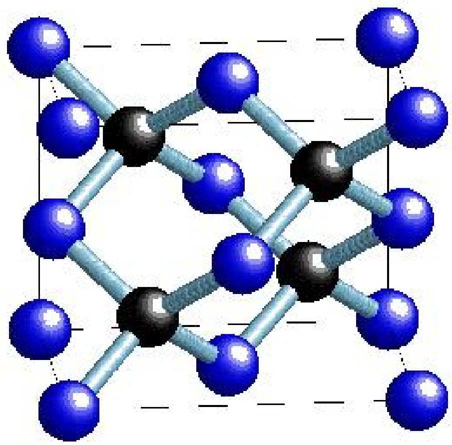
magnetic moment $\vec{\mu} = \frac{e\vec{S}}{mc}$ of electron couples to $\vec{B}_{so}$

$$H_{so} = -\vec{\mu} \cdot \vec{B}_{so}$$

electrons precess around $B_{so}$
$B_{so}$ depends on the electron momentum

spin rotation symmetry is broken,
time reversal symmetry is NOT broken

# Spin-Orbit Coupling due to Crystal Anisotropy



Conventional cell

III-V Semiconductor

Zinkblende crystall structure:
two interpenetrating fcc lattices
with only Ga atoms on one lattice,
only As on the other

absence of inversion symmetry

symmetry considerations:

$$H_{so} = \gamma(\sigma_x k_x (k_y^2 - k_z^2) + \text{cycl.})$$

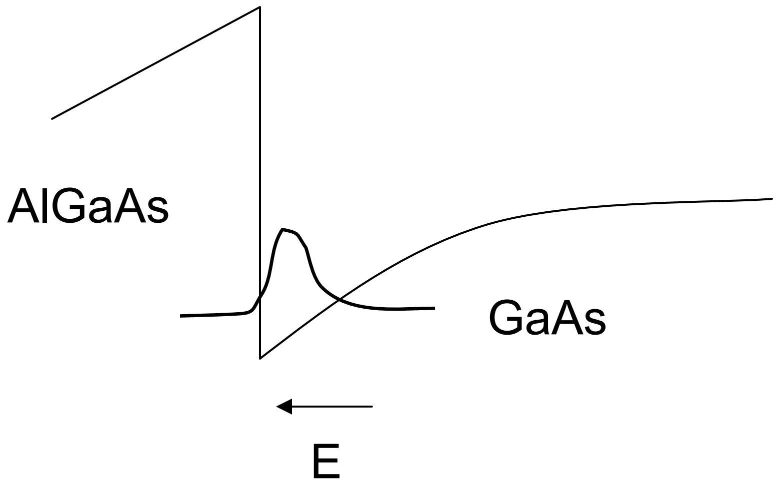
G. Dresselhaus,
Phys. Rev. 100, 580 (1955)

after size quantization (2D): $\langle k_z \rangle = 0 \qquad \alpha = \gamma \langle k_z^2 \rangle$

$$H_D^{(1)} = \alpha(\sigma_x k_x - \sigma_y k_y)$$ k-linear Dresselhaus term

$$H_D^{(3)} = \gamma(\sigma_y k_y k_x^2 - \sigma_x k_x k_y^2)$$ k-cubic Dresselhaus term

# Spin-Orbit Coupling due to Heterointerface




electric field at heterointerface perpendicular to 2D plane

$$\vec{B}_{so} \propto (k_y E, -k_x E, 0) \perp \vec{k}$$

$$H_R = \beta(\sigma_x k_y - \sigma_y k_x)$$ Rashba term (k-linear)

coupling strength parameters $\beta$ and $\gamma$ can be determined from Band structure, for example in $k \cdot p$ approximation

# Weak Antilocalization

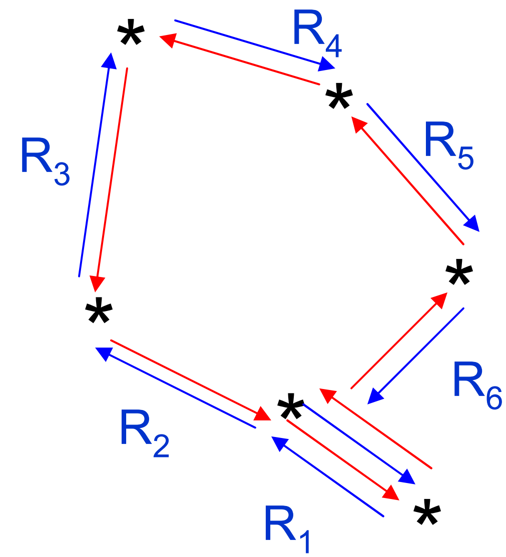
initial state: $|i\rangle$

final (forward): $|f_f\rangle = R_N \dots R_2 R_1 |i\rangle = R|i\rangle$

final (backward): $|f_b\rangle = R_1^{-1} R_2^{-1} \dots R_N^{-1} |i\rangle = R^{-1}|i\rangle$ (TRS)

$R_i$: spin rotations $R = R_N \dots R_2 R_1$ $R^\dagger R = 1$ $R^{-1} = R^\dagger$

interference term $\langle f_b | f_f \rangle = \langle i | R^2 | i \rangle$



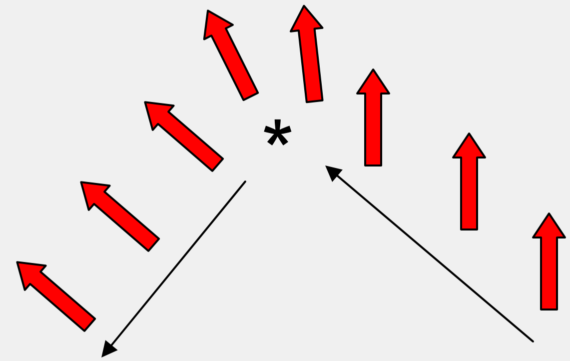

assuming strong spin-orbit coupling, summing over all trajectories is equivalent to averaging $R^2$ over sphere

$\overline{\langle f_f | f_b \rangle} = -\frac{1}{2}$ **destructive interference**
**opposite sign for Magnetoconductance**

# Mechanisms of Spin-Orbit Coupling

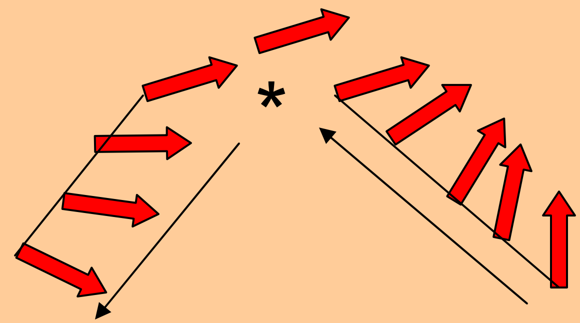
## Elliott-Yafet

- electric field of atoms/impurities felt during scattering events
- spins precess during scattering events
- spins are invariant between scattering
- spin obrbit spin relaxation time
- dominant mechanism in Au $\tau_{so} \propto \tau_{tr}$

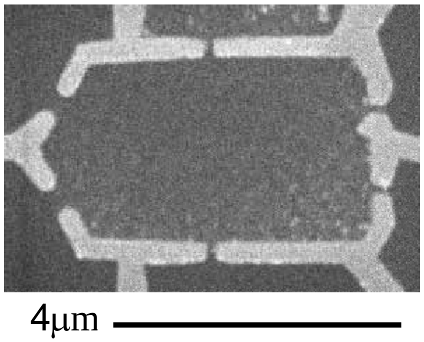


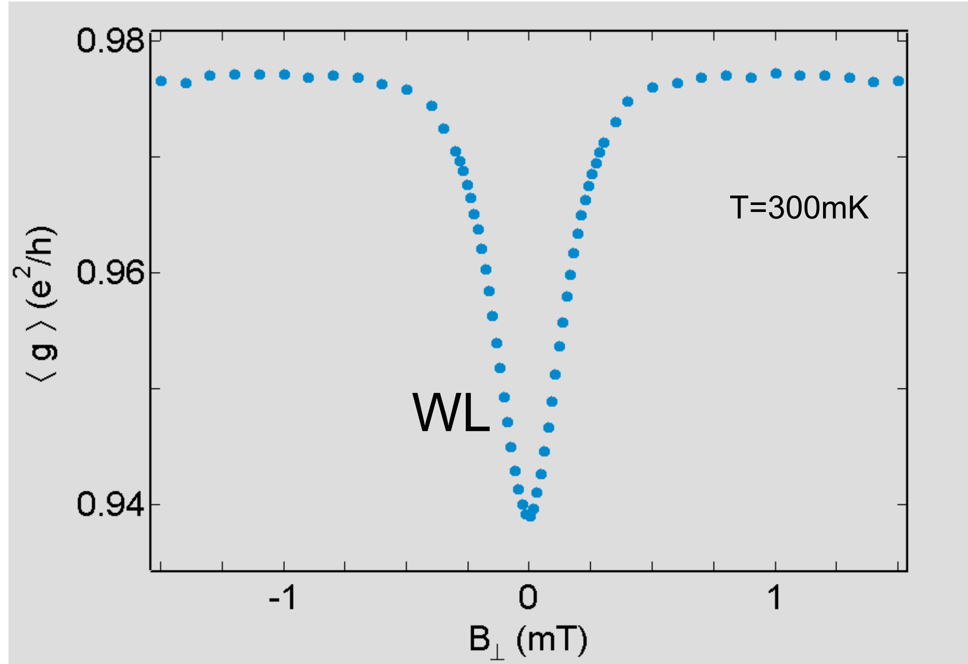
## Dyakonov-Perel

- built in electric fields of material
- spin precessions during ballistic travel
- elastic scattering leaves spin invariant
- usually small rotations between scattering assumed, giving a random spin walk, $\tau_{so}^{-1} = \Omega^2 \tau$
- dominant in GaAs heterostructures

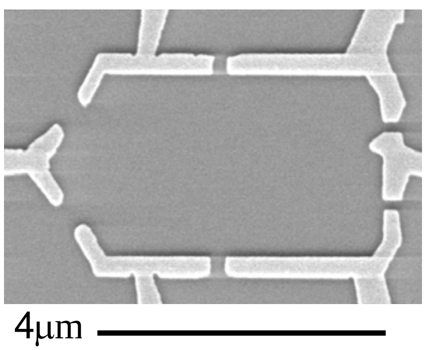


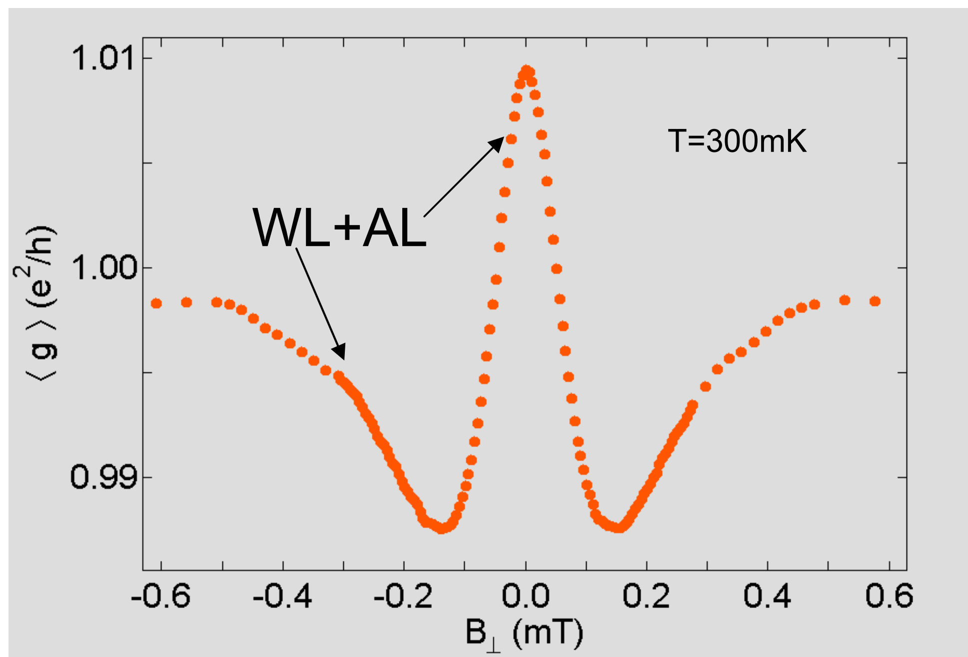
low density
weaker SO coupling
weak localization (WL)



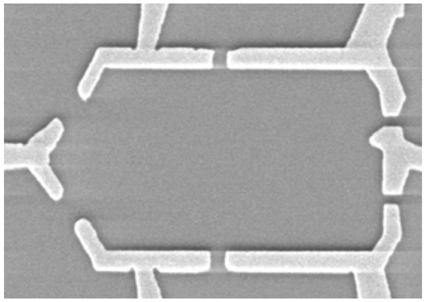

dots are on **different** wafers



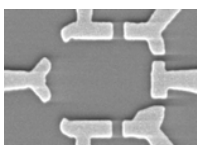

high density
stronger SO coupling
antilocalization (AL)

**large dot**
**antilocalization (AL)**
due to SO coupling



4µm

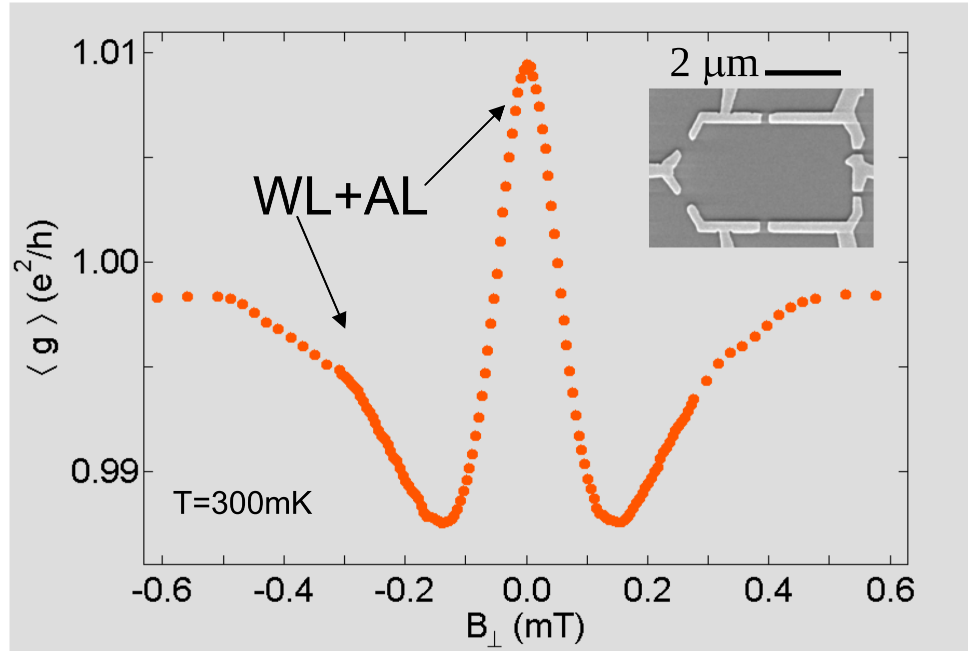
dots on the **same** wafer

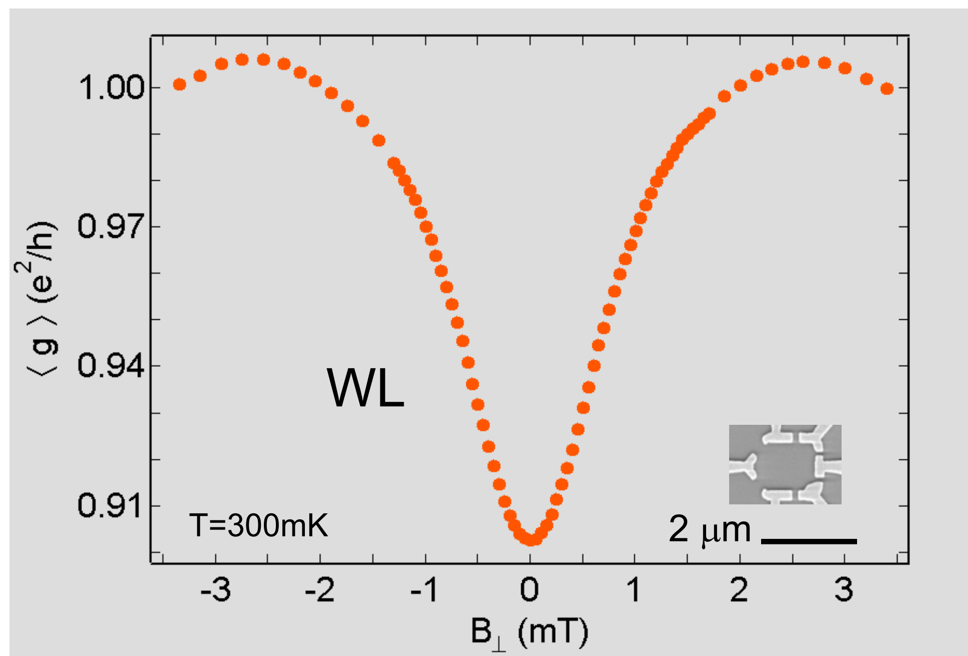


1 µm

**small dot**
**weak localization (WL)**
SO coupling **supressed**

predicted theoretically by:
A. Khaetskii and Y. Nazarov, PRB **61**, 12639 (2000).
B. Halperin et al., PRL **86**, 2106 (2001).
I. Aleiner and V. Fal'ko, PRL **87**, 256801 (2001).



# New Random Matrix Theory (SO coupling & $B_{||}$)

$$\langle \delta g \rangle = \frac{e^2}{h}\frac{N}{4}\left\{ -\frac{4b^2 + 2G_C F_C}{\left(4G_C b^2 + G_C^2 F_C - 16 a_x^2 F_C x^2\right)} - \frac{4G_C b^2}{F_C\left(4G_C b^2 + G_C^2 F_C - 16 a_x^2 F_C x^2\right)} + \frac{4a^2}{F_C\left(4a^2 + F_C\right)} \right\}$$

## spin-orbit parameters

$$a_x^2 = \pi\kappa \frac{E_T}{\Delta}\left(\frac{A}{\lambda_1 \lambda_2}\right)^2$$ (AB like SO term)

$$a^2 = \left(\left(\frac{L_1}{\lambda_1}\right)^2 + \left(\frac{L_2}{\lambda_2}\right)^2\right) a_x^2$$ (spin flips)

$$h^2 = \frac{\pi}{2}\left(\frac{E_Z}{\Delta}\right)^2 \left(\frac{\Delta}{E_T}\right)\left(\frac{L}{\lambda_{so}}\right)^2$$ (SO + $B_{||}$)

## magnetic fields

$$x^2 = \pi\kappa\left(\frac{E_T}{\Delta}\right)\left(\frac{2eB_\perp A}{h}\right)^2$$ perpendicular

$$b = \pi \frac{g\mu_B B_{||}}{\Delta}$$ parallel

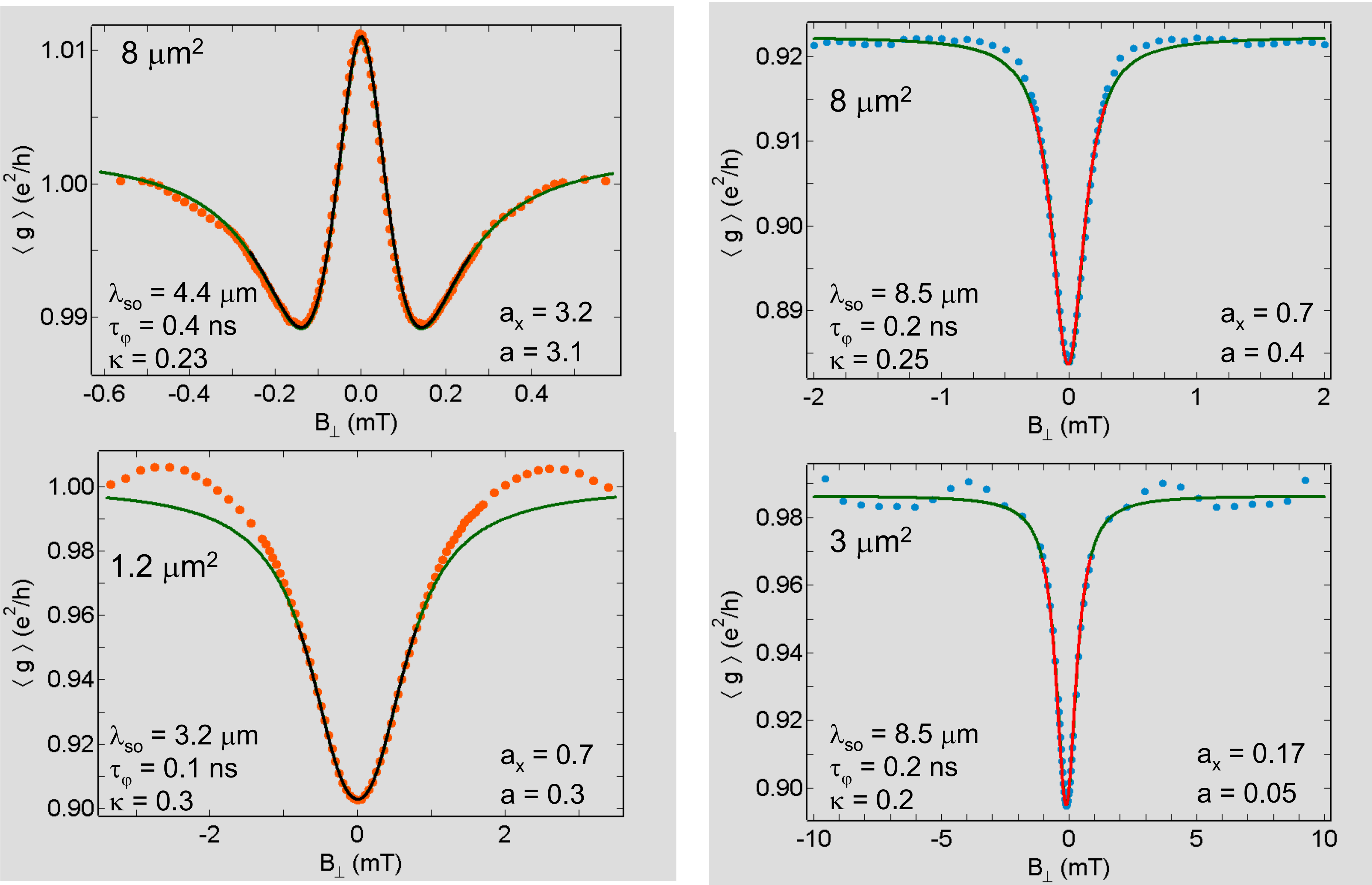
$$N_C = N + 2h^2 + \gamma_\varphi \qquad F_C = N_C + h^2$$

$$G_C = N_C + 2\left(a^2 + a_x^2\right) - h^2$$

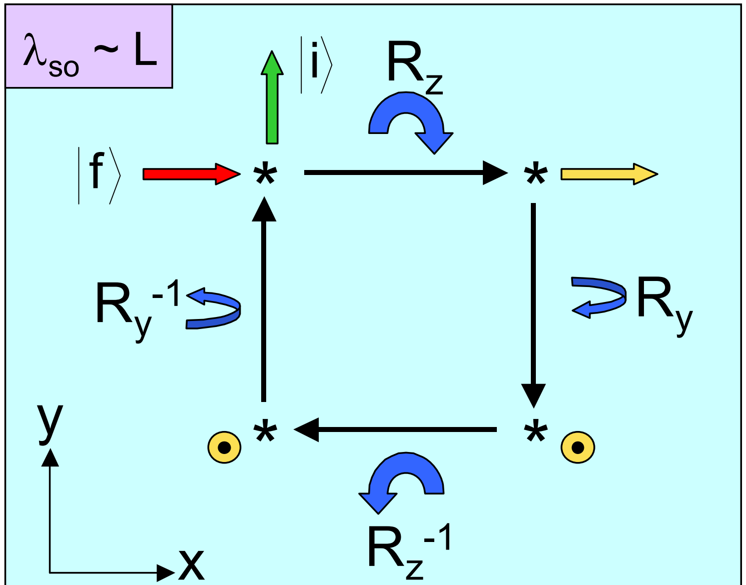
| | |
|---|---|
| $\lambda_1, \lambda_2$ | SO length along crystal axes |
| $\lambda_{so} = \sqrt{\lambda_1 \cdot \lambda_2}$ | average SO length |
| $\nu_{so} = \sqrt{\lambda_1 / \lambda_2}$ | SO anisotropy |
| $\gamma_\varphi$ | decoherence rate |
| $\kappa$ | geometry dependent constant |

Jan-Hein Cremers, Piet Brouwer, Vladimir Fal'ko, Phys. Rev. B**68**, 125329 (2003)

# high density material (SY4) low density material (CEM)

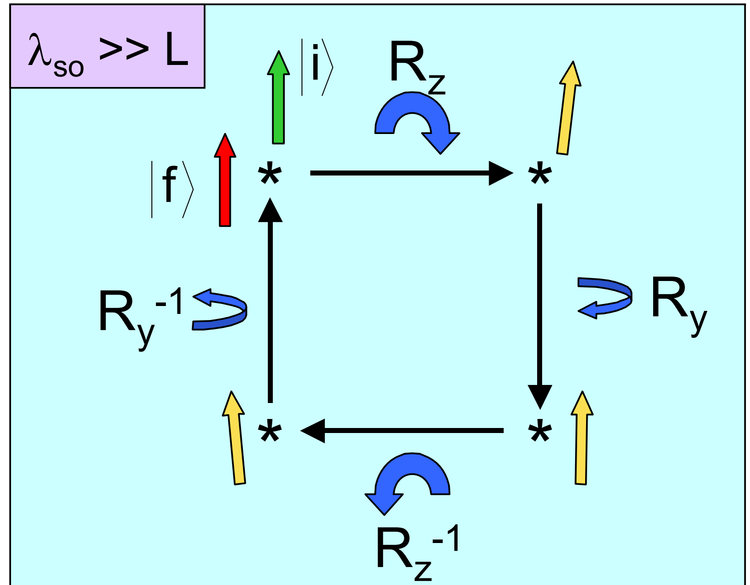


# Suppression of Spin-Orbit Coupling in Dots




$$|f\rangle = R_y^{-1}R_z^{-1}R_yR_z|i\rangle \neq |i\rangle$$

large rotations,
order of rotations matter




$$|f\rangle = R_y^{-1}R_z^{-1}R_yR_z|i\rangle$$

$$\approx R_y^{-1}R_yR_z^{-1}R_z|i\rangle = |i\rangle$$

small rotations,
commute, cancelation