1. Open Dot Experiments

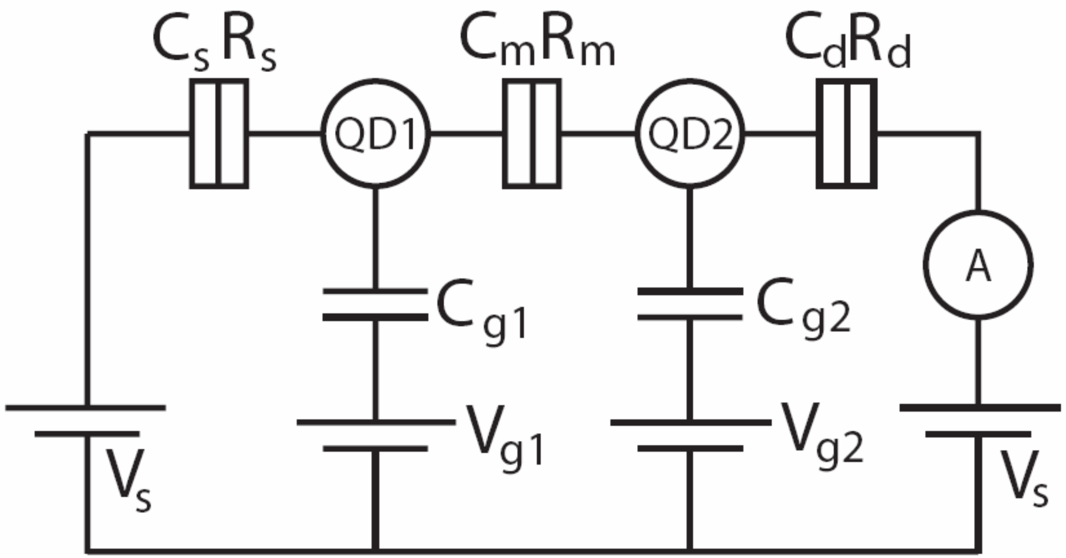
2. Kondo effect

3. Few Electron Dots

4. Double Quantum Dots

van der Wiel et al., RMP75, 1 (2003)
A. C. Johnson, Ph. D. Thesis (2005)

Double Quantum Dots

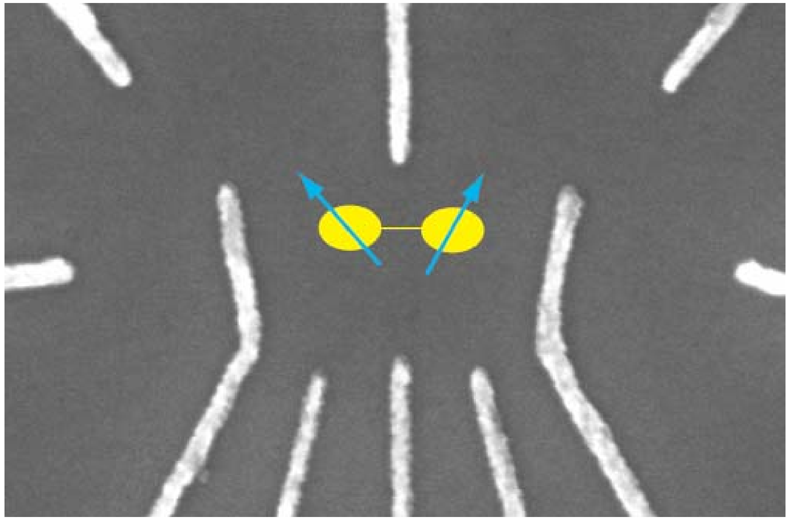


mutual charging energy

$$E_m = \frac{e^2}{C_m}\left(\frac{C_1 C_2}{C_m^2} - 1\right)^{-1}$$

interdot tunneling t

$$G_m = 4\pi \frac{e^2}{h}\left(\frac{t}{\Delta}\right)^2$$

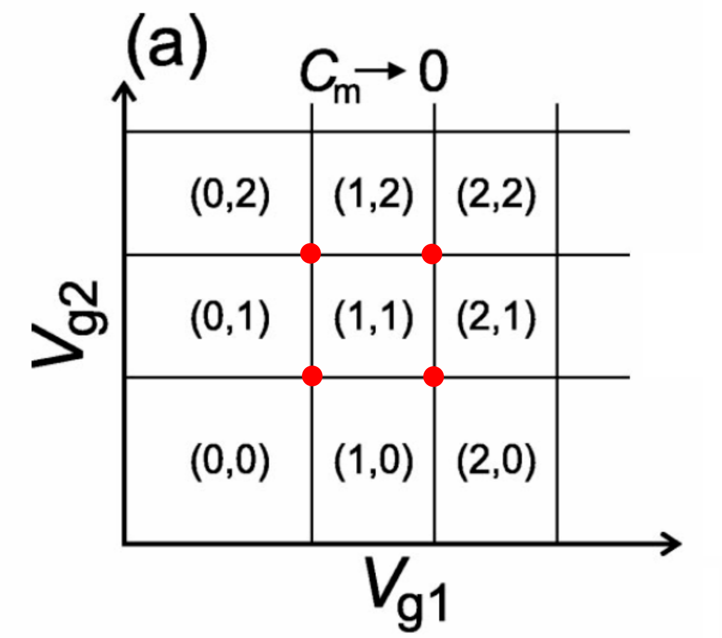


$t < \Delta$ well localized electrons

individual charging energies

$$E_{c1(2)} = \frac{e^2}{C_{1(2)}}\left(1 - \frac{C_m^2}{C_1 C_2}\right)^{-1}$$

Double Quantum Dots: Quadruple Points



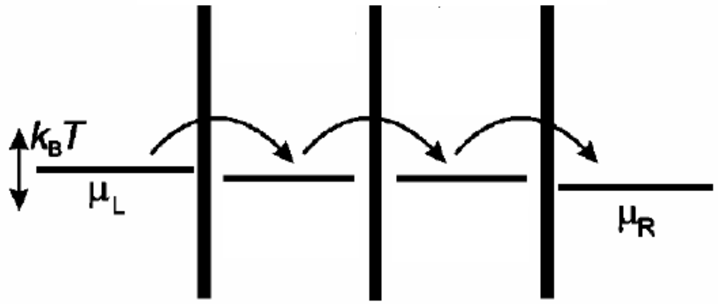
$$E_m = \frac{e^2}{C_m}\left(\frac{C_1 C_2}{C_m^2} - 1\right)^{-1} \longrightarrow 0$$

costs zero energy to add a 2nd electron
to other dot if one electron is already present

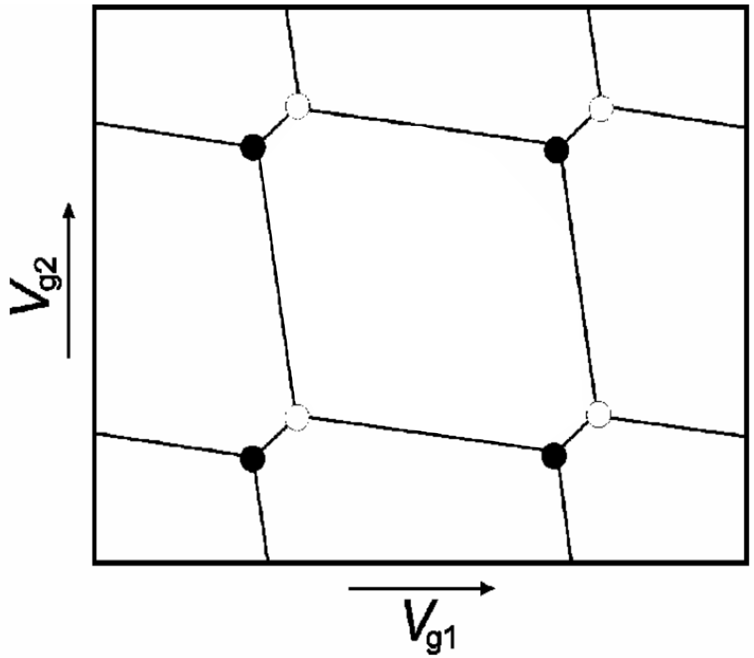
$$E_{C1(2)} = \frac{e^2}{C_{1(2)}}$$ individual charging energies

assume well localized electrons (weak tunneling, but large enough to measure a current)

● quadruple points
degeneracy of four charge states



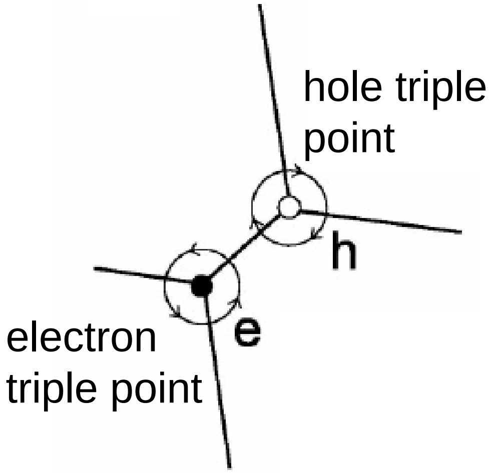
Double Quantum Dots: Triple Points and Honeycombs

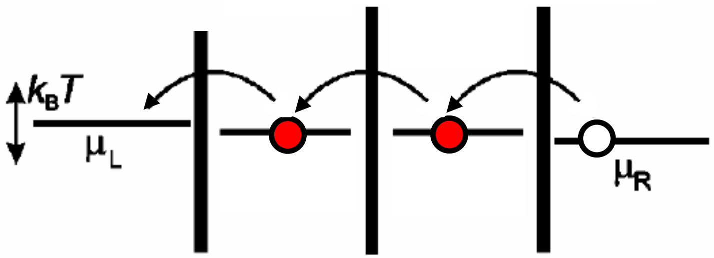


$$0 < C_m < C_{1,2} \quad 0 < E_m < E_{C_1,C_2}$$

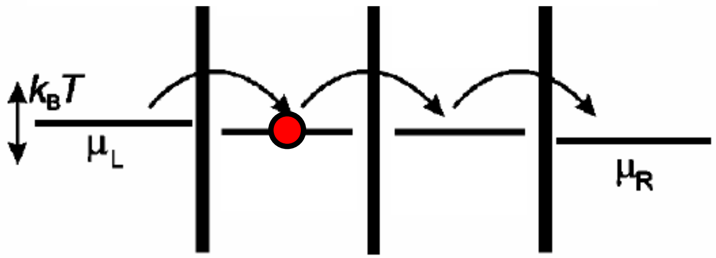
(1,1) – (0,0) degeneracy lifted

quadruple points split into two triple points



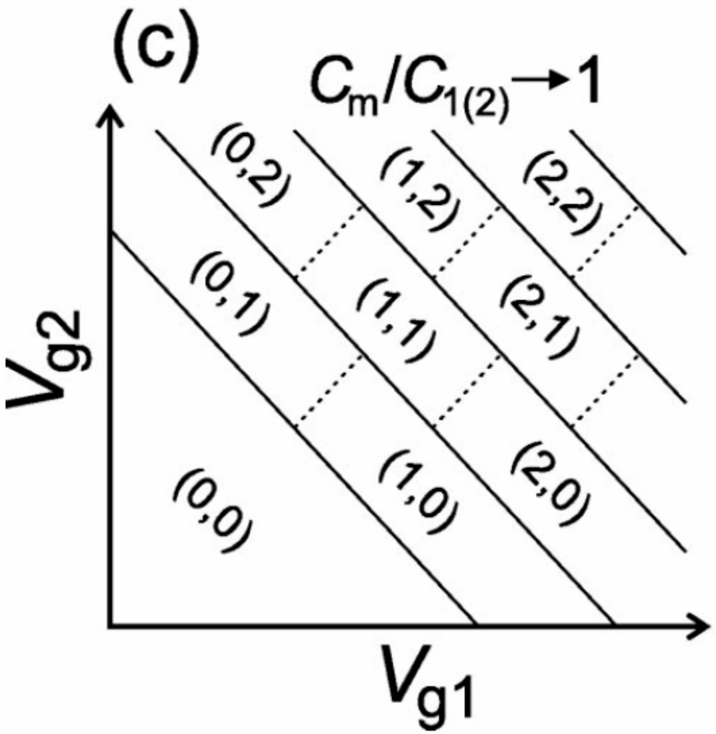


hole like process



electron like process

Double Quantum Dots: Single Dot Limit

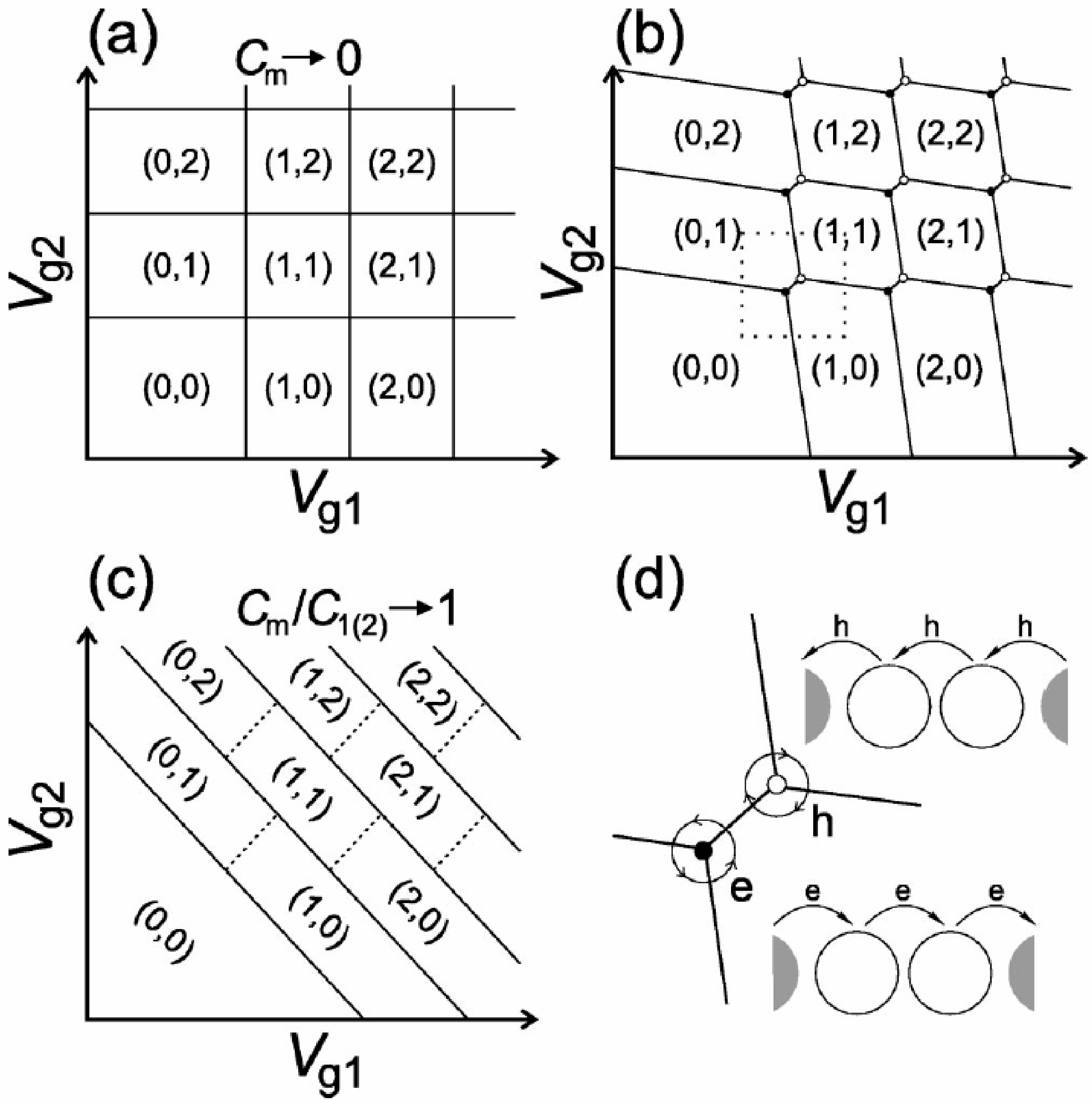


$$0 < C_m \sim C_{1,2}$$

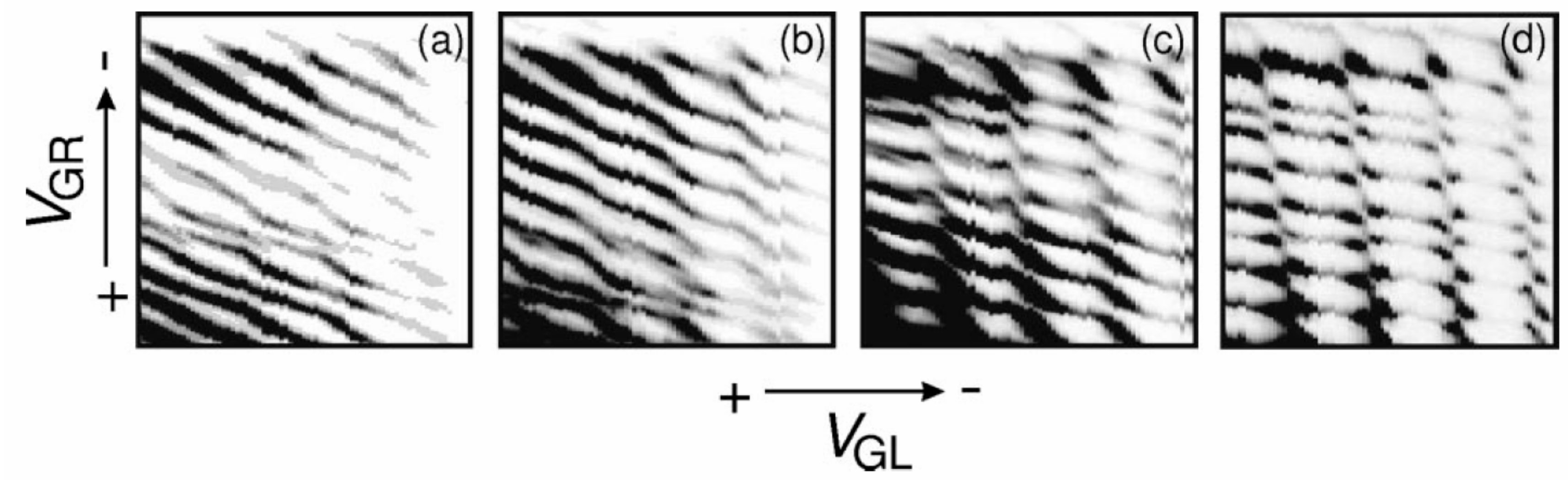
$$E_m \sim E_{C_1,C_2}$$

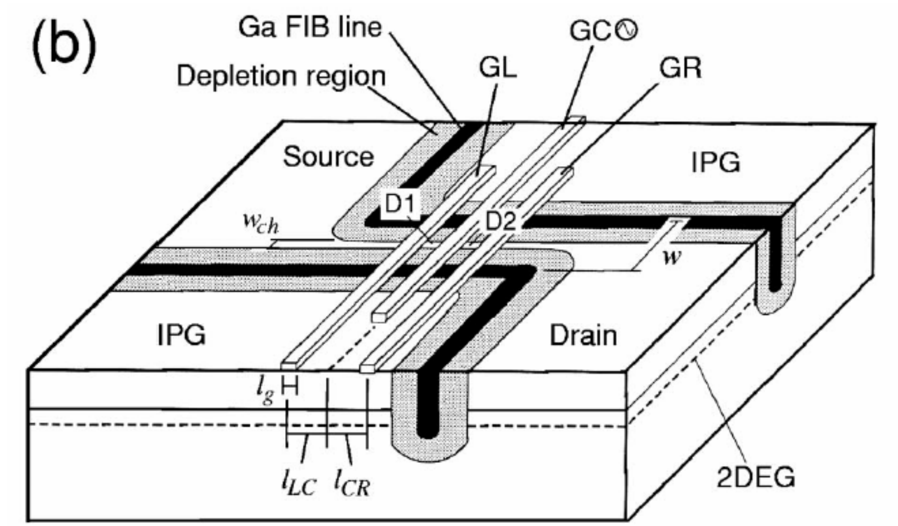
double dot behaves like a
single dot with two plunger gates

Double Quantum Dots



Double Dot Experiment





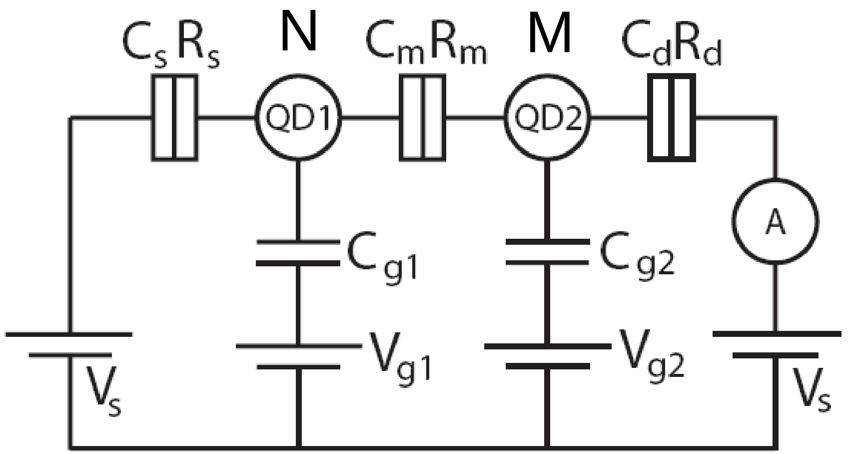
van der Wiel et al., RMP75, 1 (2003)

Double Dot Hamltonian

individual charging — electrostatic — quantum confinement

$$
\begin{aligned}
H_{DQD} &= \frac{E_{c1}}{2}N(N-1) - \frac{NE_{c1}+ME_m}{e}(C_{g1}V_{g1}+C_sV_s) + \sum_{i,\sigma} N_{i\sigma}\epsilon_{i\sigma} \\
&+ \frac{E_{c2}}{2}M(M-1) - \frac{ME_{c2}+NE_m}{e}(C_{g2}V_{g2}+C_dV_d) + \sum_{j,\sigma} M_{j\sigma}\epsilon_{j\sigma} \\
&+ E_mNM + \sum_{i,j,\sigma} t_{ij\sigma}(c^{\dagger}_{i\sigma}c_{j\sigma}+h.c.). \quad \text{+ lead tunneling} \qquad (3.11)
\end{aligned}
$$

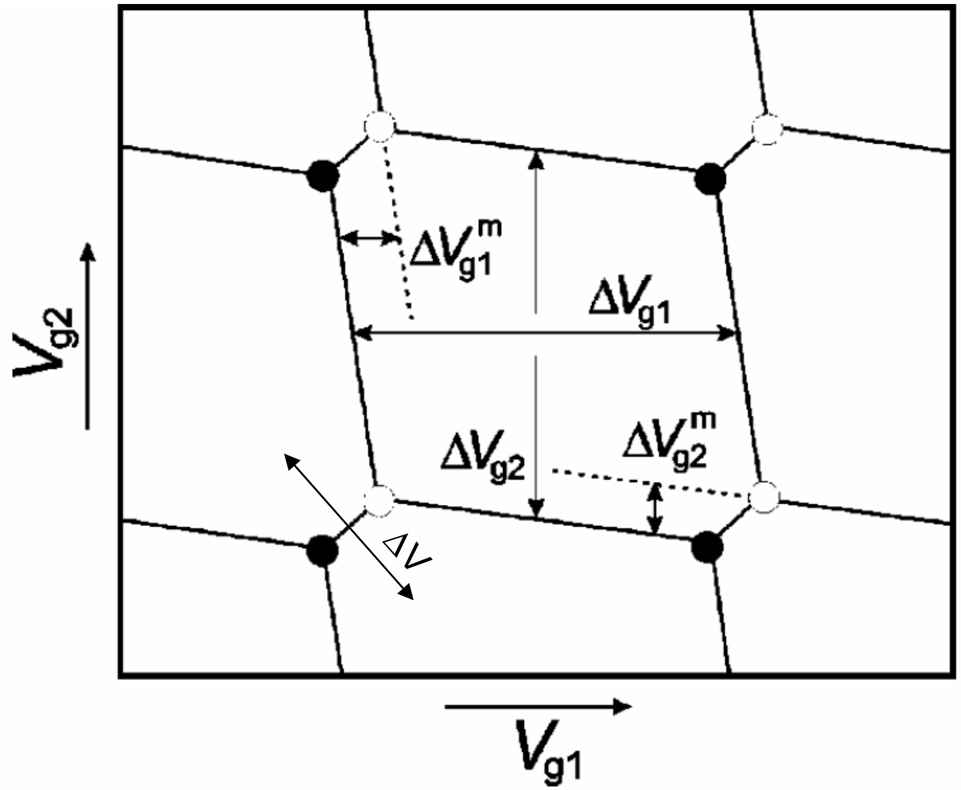
mutual charging — inter-dot tunneling



electrons well localized

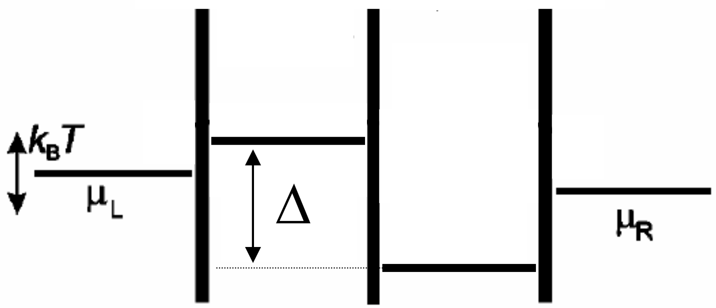
$G_m < e^2/h$

Double Dot Capacitances in the Honeycombs



$$\Delta V_{g1} = \frac{|e|}{C_{g1}}$$

$$\Delta V_{g2} = \frac{|e|}{C_{g2}}$$



$$\Delta V_{g1}^{m} = \frac{|e| C_m}{C_{g1} C_2} = \Delta V_{g1} \frac{C_m}{C_2}$$

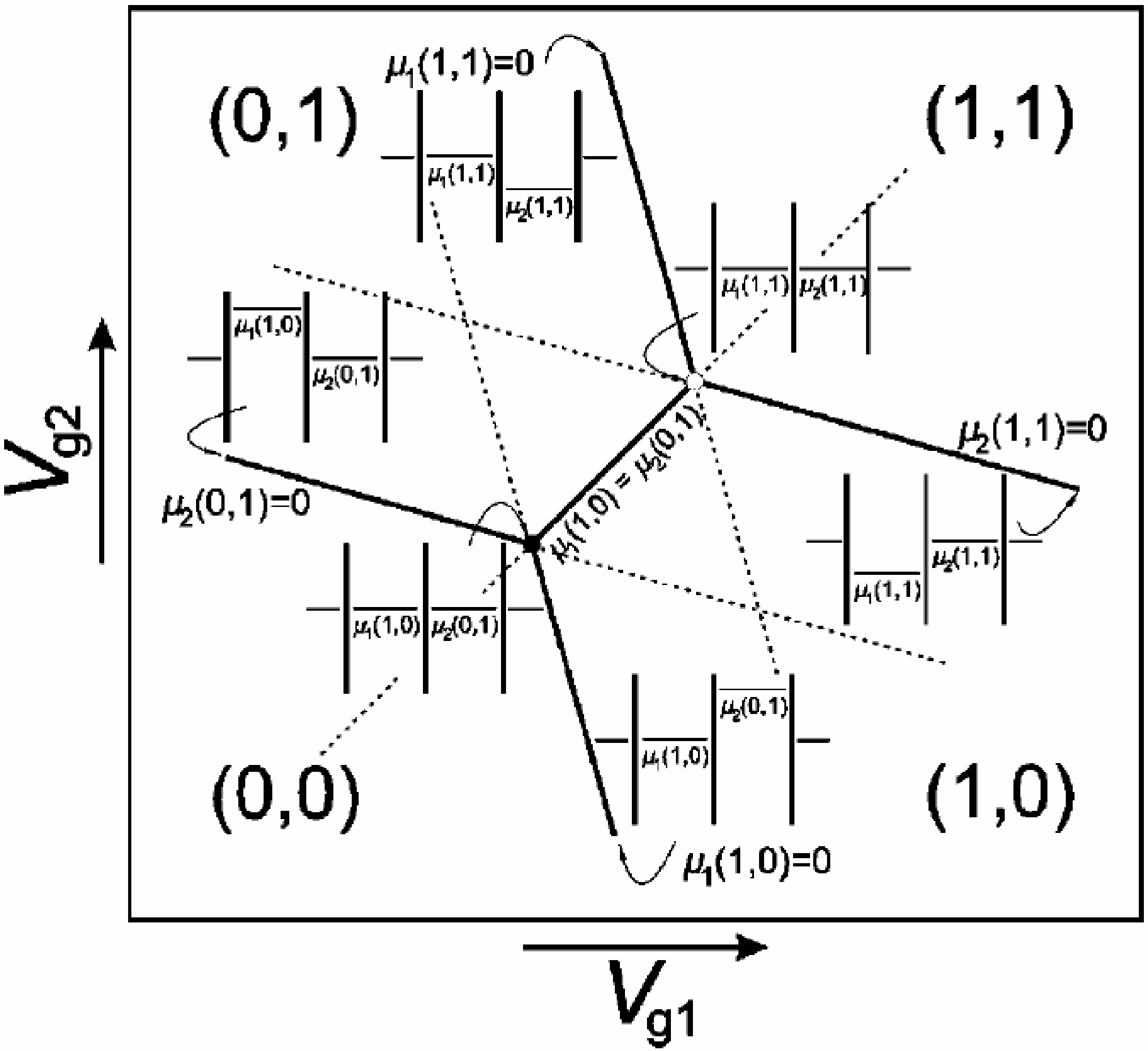
$$\Delta V_{g2}^{m} = \frac{|e| C_m}{C_{g2} C_1} = \Delta V_{g2} \frac{C_m}{C_1}$$

ΔV : detuning
controls energy difference Δ
between the dot levels
keeping constant the
total dot occupation N + M

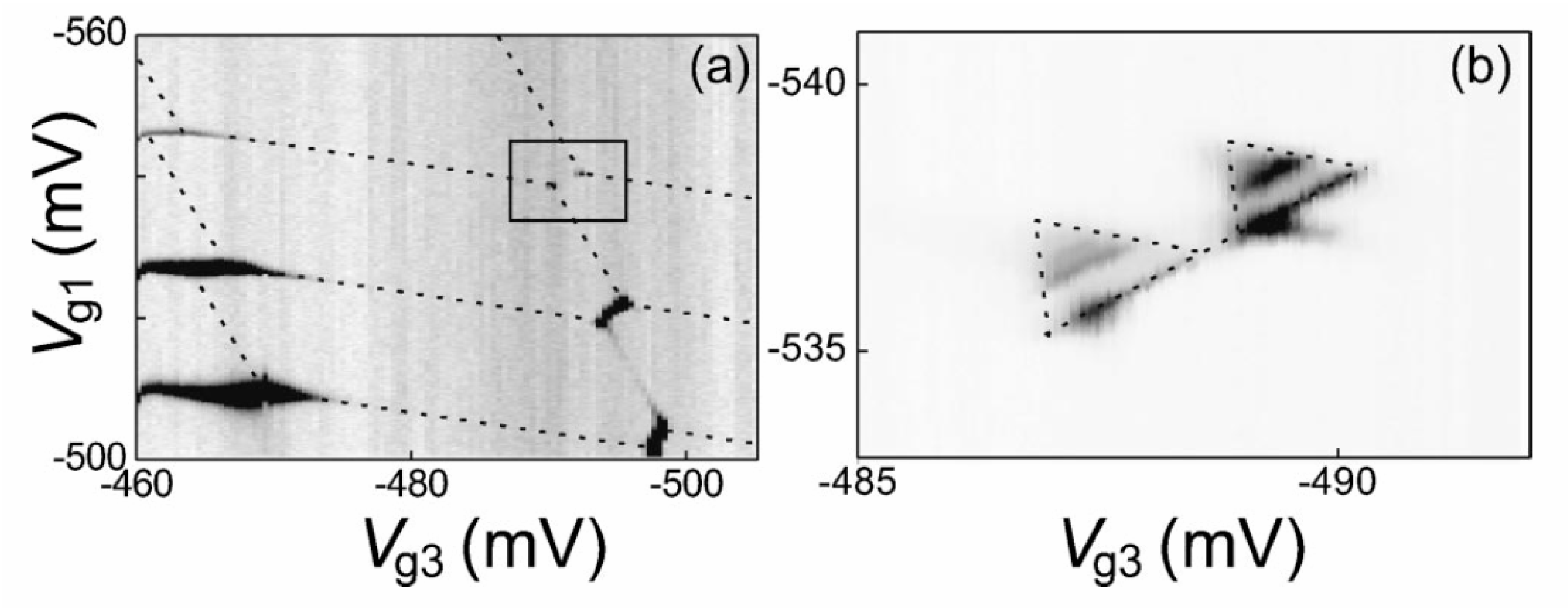
Double Dot Transport

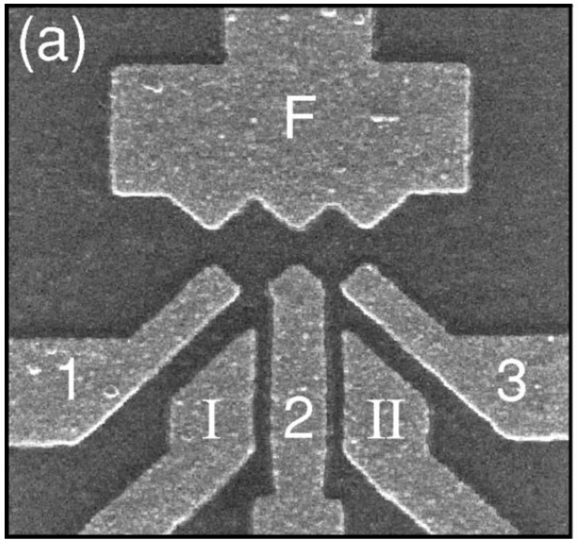
triple points:
sequential tunneling

honey comb lines:
cotunneling



Double Dot Experiment

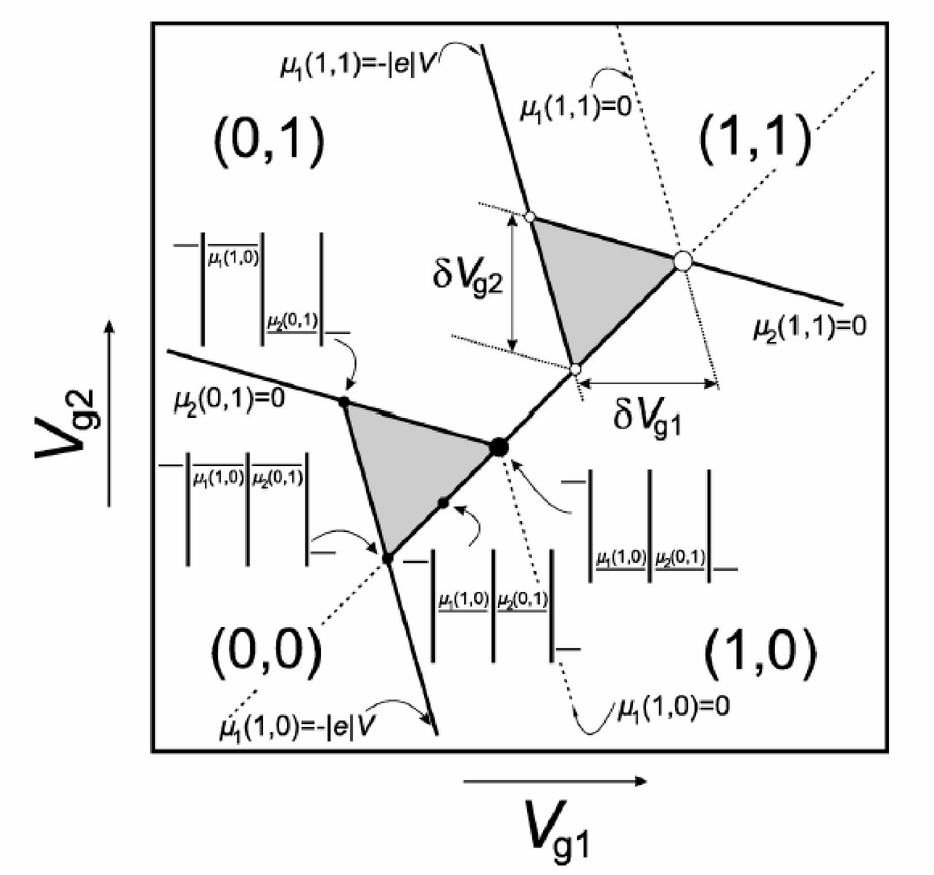




finite bias: nonlinear transport

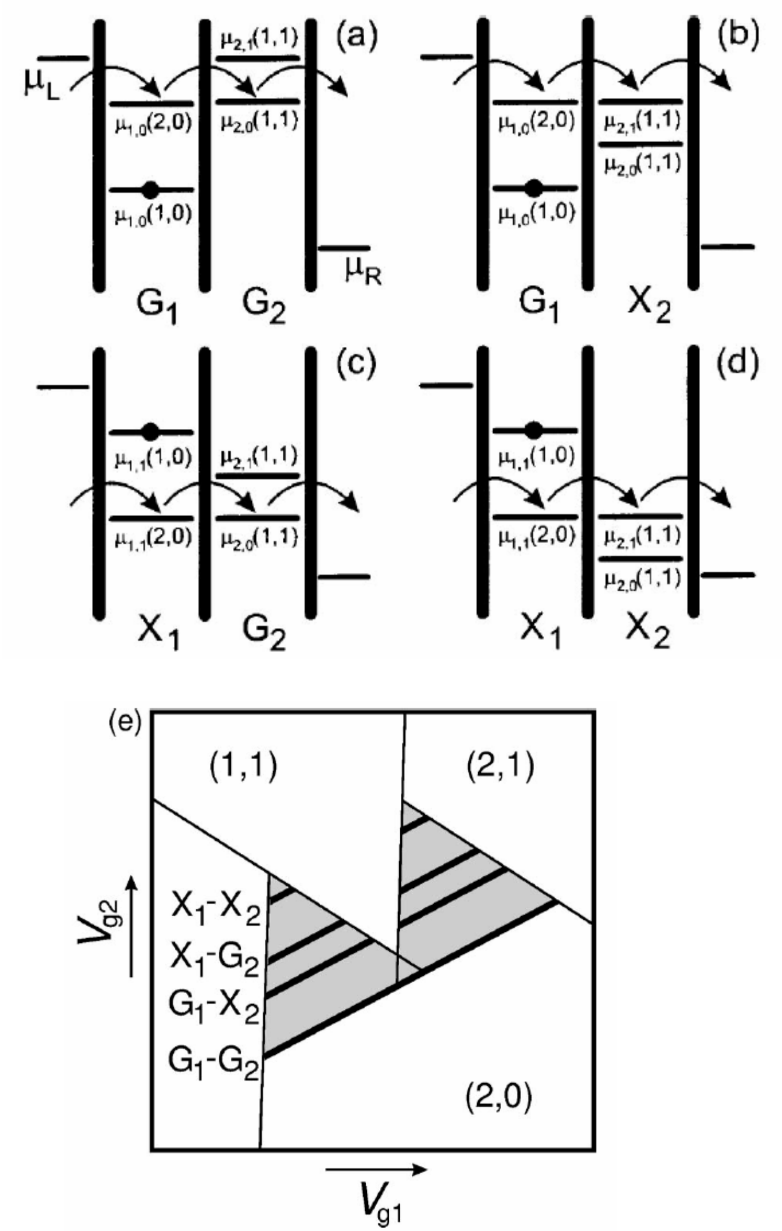
van der Wiel et al., RMP75, 1 (2003)

Double Dot at finite bias: Excited State Spectroscopy

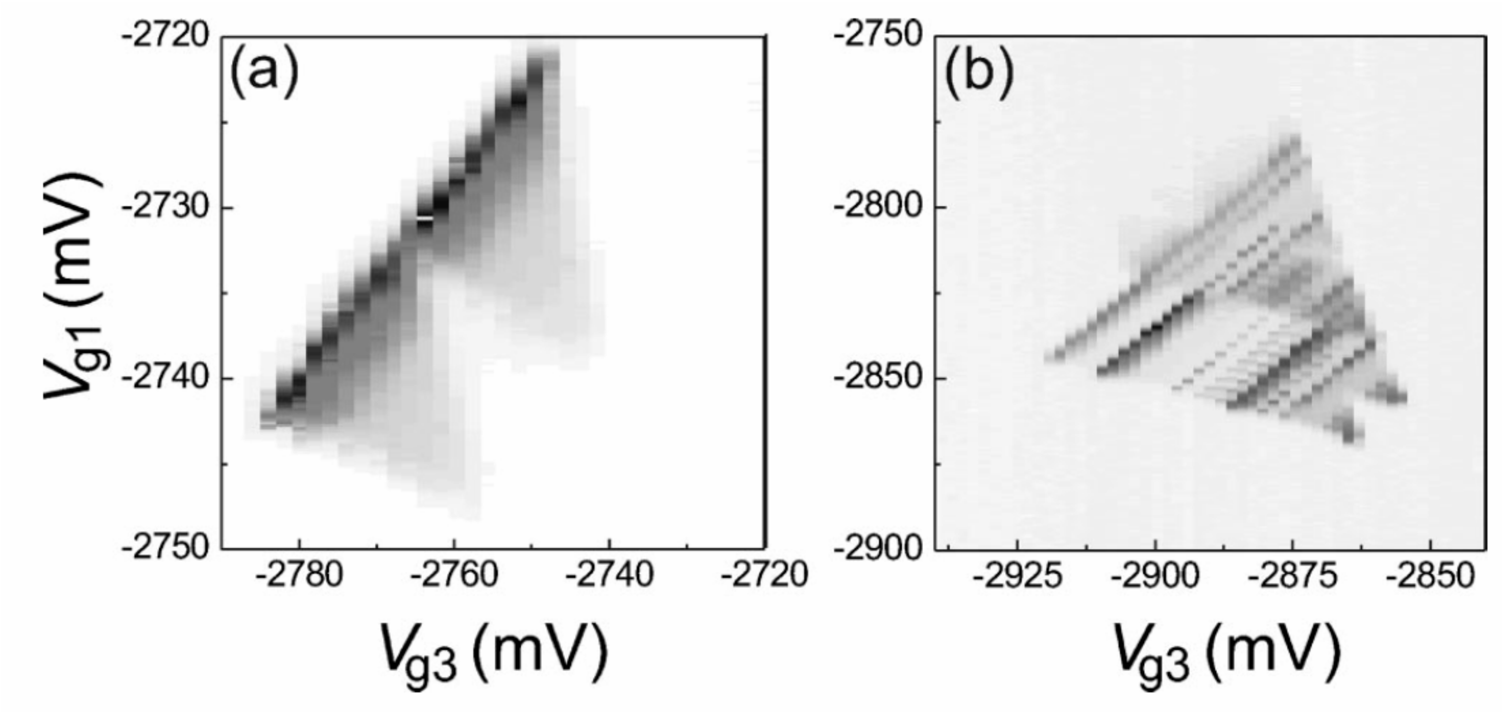


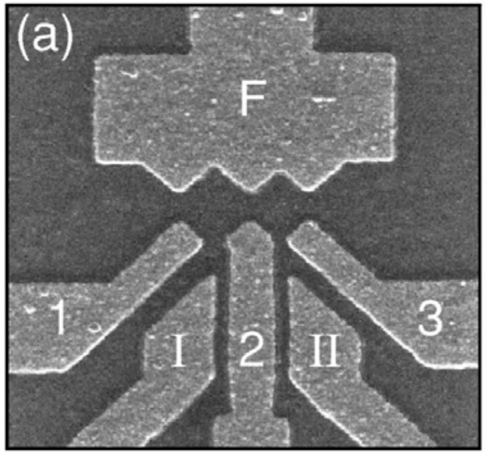
triple points expans into triangles obeying

$$0 \leq \mu_1 \leq \mu_2 \leq eV$$



Double Dot Experiment: Finite Bias





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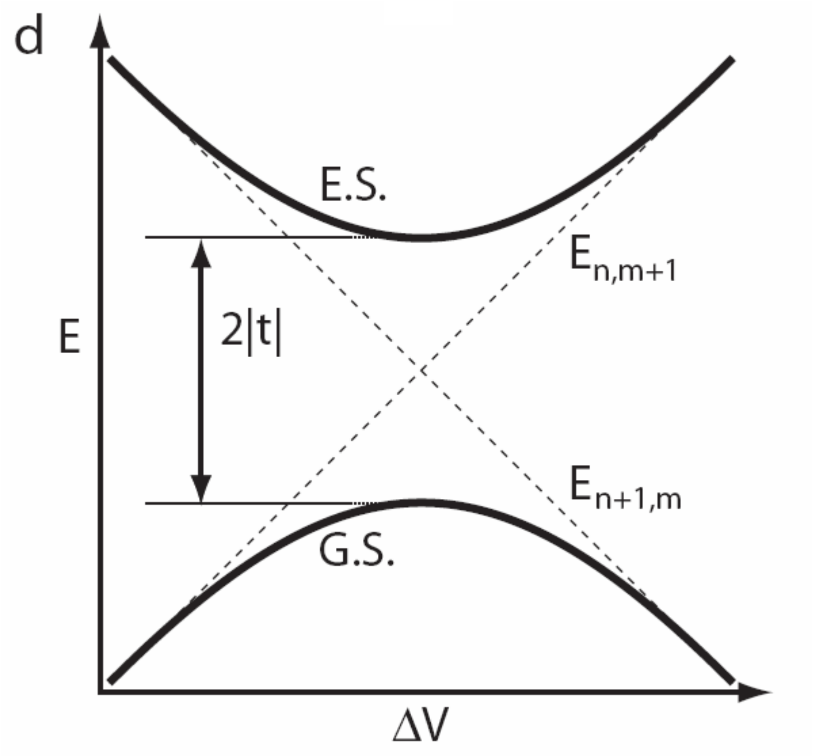
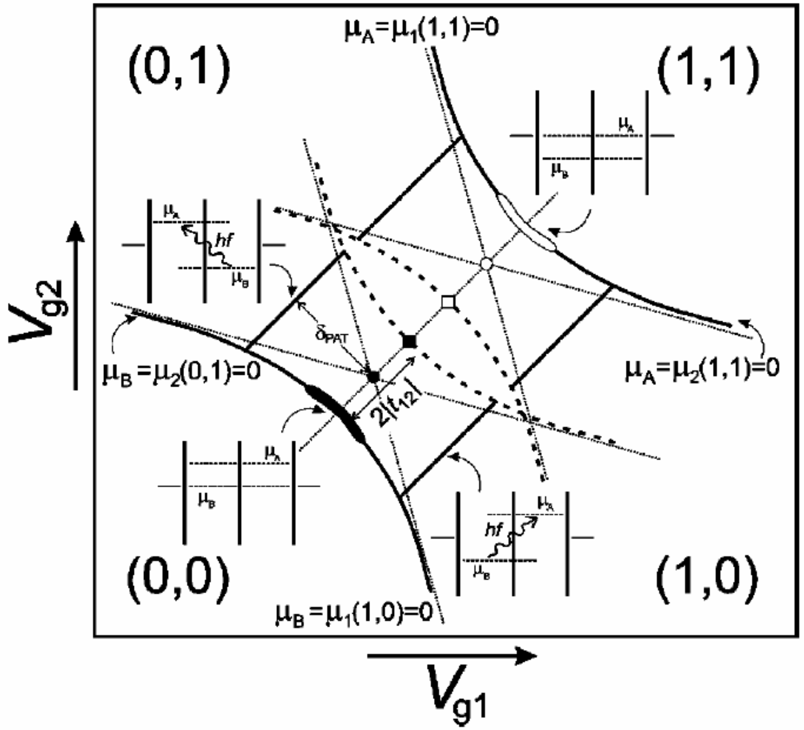
Interdot Tunneling: Anticrossing





$$\mathbf{H}_0|\phi_1\rangle = E_1|\phi_1\rangle$$

$$\mathbf{H}_0|\phi_2\rangle = E_2|\phi_2\rangle$$

$$\mathbf{T} = \begin{pmatrix} 0 & t_{12} \\ t_{21} & 0 \end{pmatrix}, \quad t_{12} = t_{21}^*, \quad t_{21} = |t_{21}|e^{i\varphi}$$

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{T}$$

$$\mathbf{H}|\psi_B\rangle = E_B|\psi_B\rangle$$

$$\mathbf{H}|\psi_A\rangle = E_A|\psi_A\rangle$$

$$E_B = E_M - \sqrt{\frac{1}{4}(\Delta E)^2 + |t_{12}|^2}$$

$$E_A = E_M + \sqrt{\frac{1}{4}(\Delta E)^2 + |t_{12}|^2}$$