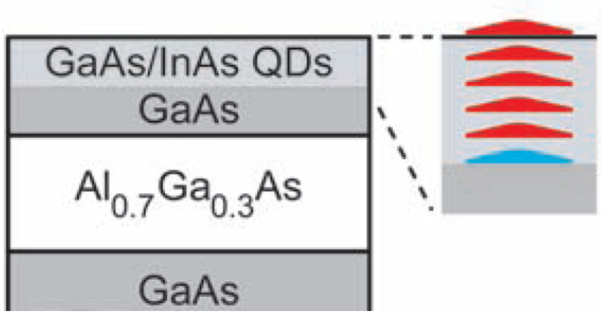
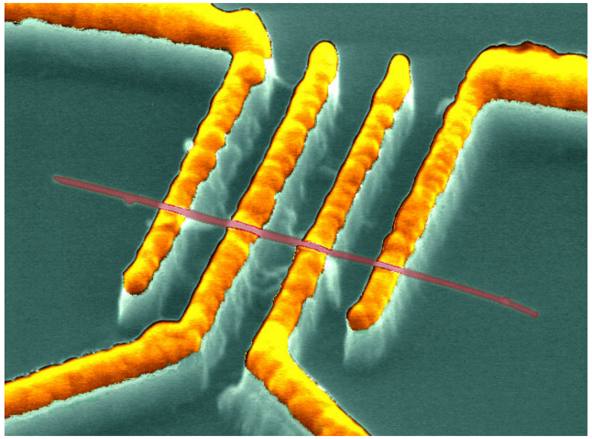
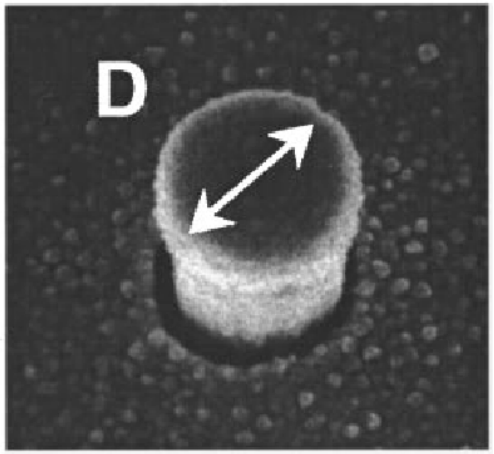
Quantum Dots



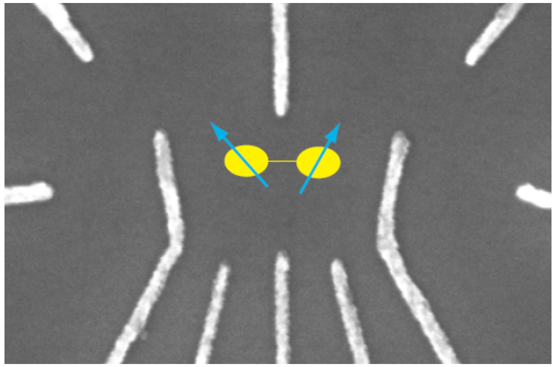
MBE grown



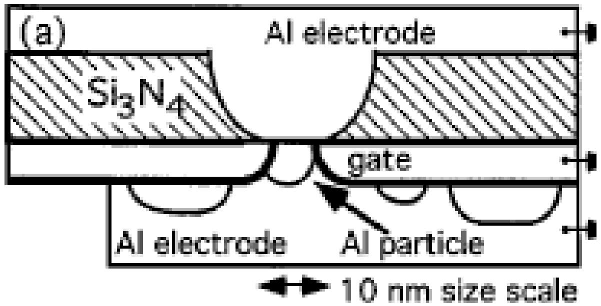
nanotube



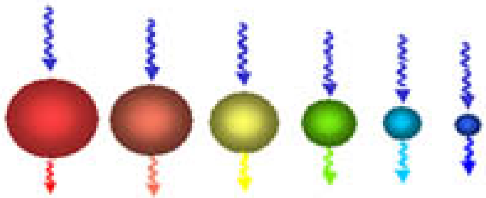
vertical dot

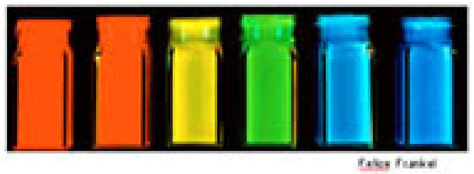


lateral

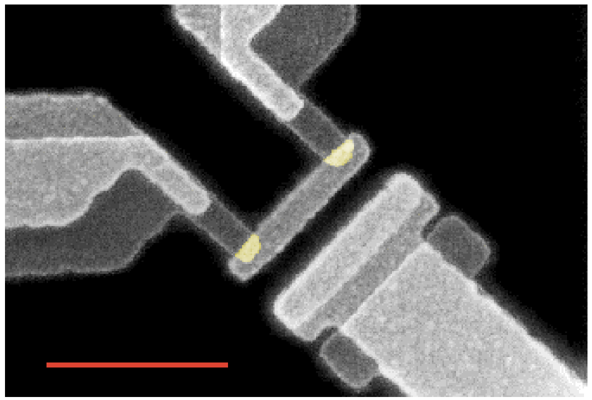


metal grain



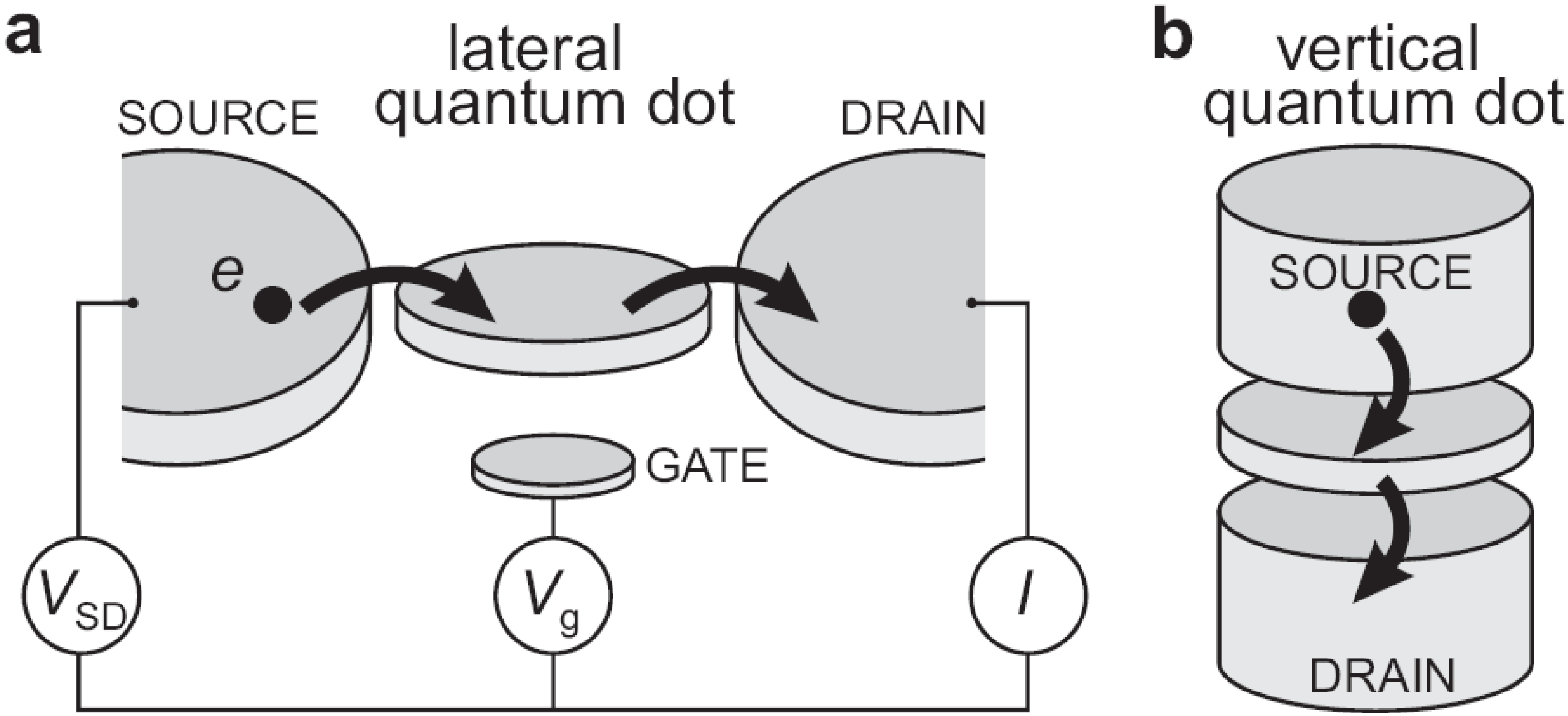


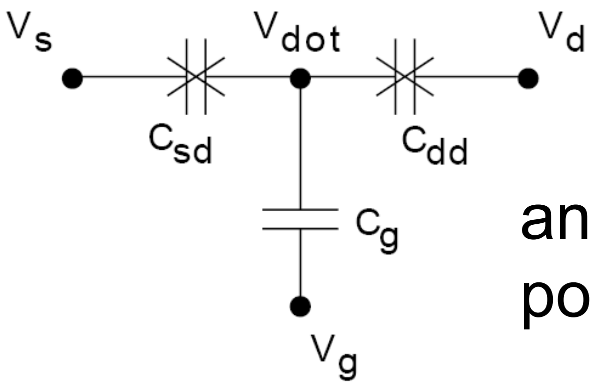
self assembled



metallic SET

lateral vs. vertical

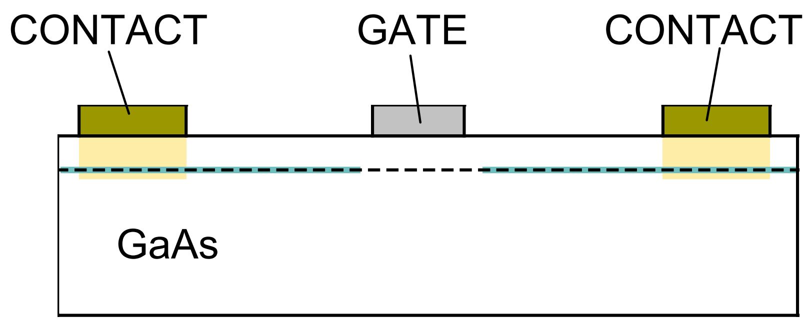


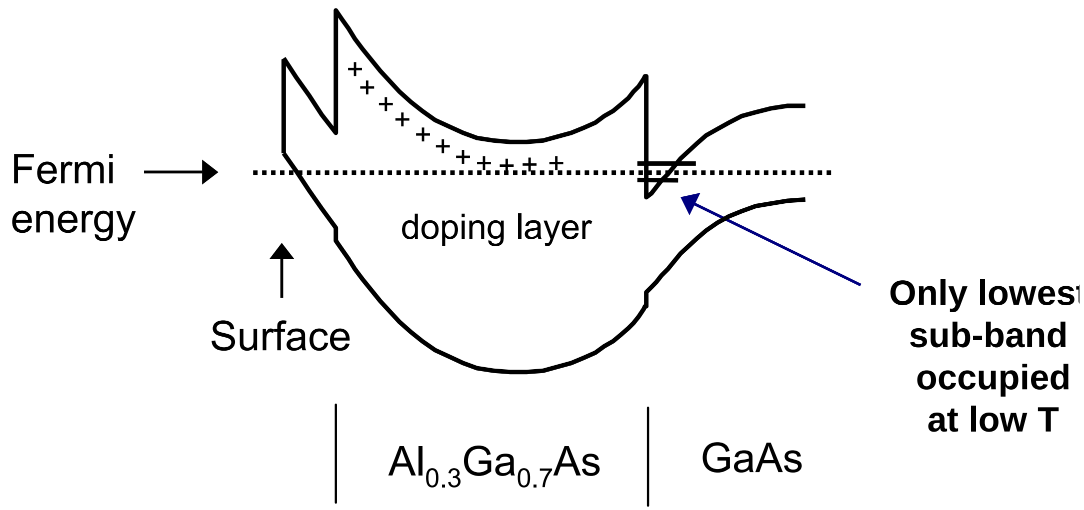


an electrical engineers point of view

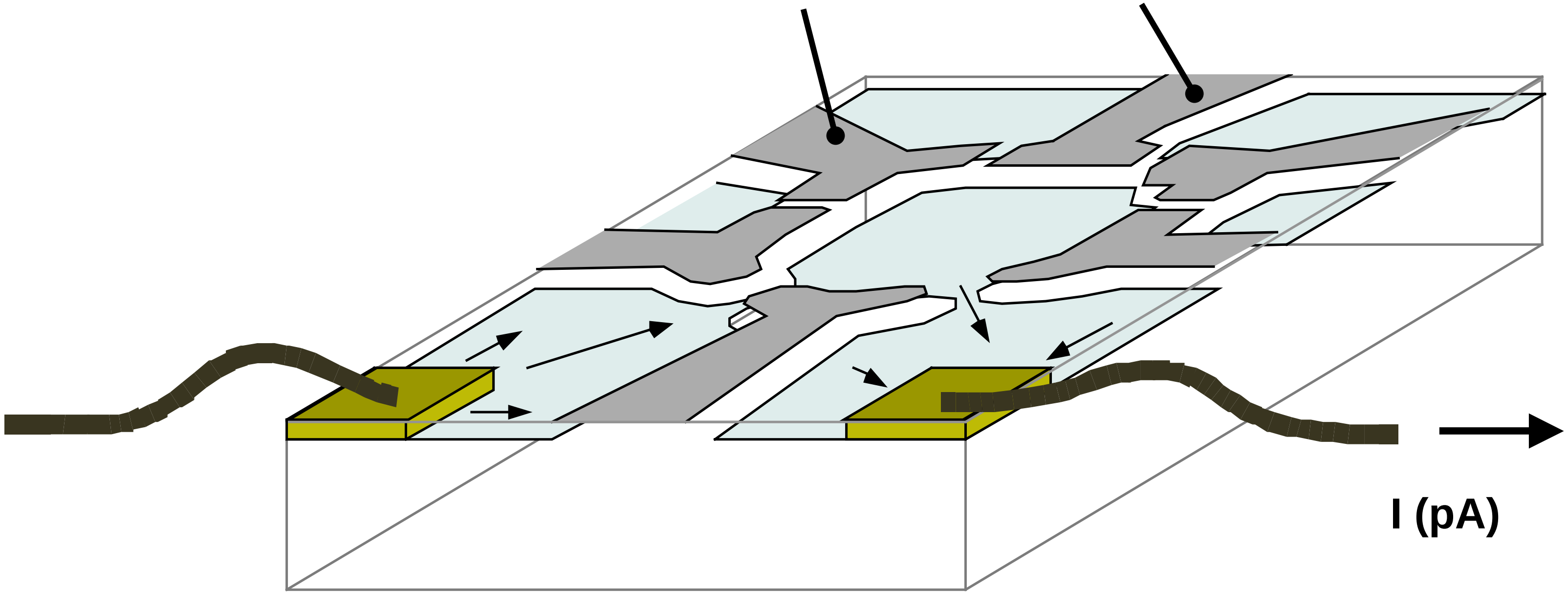
Lateral Dots: Formed in GaAs/AlGaAs 2DEG

Electrons travel in sub-surface layer:

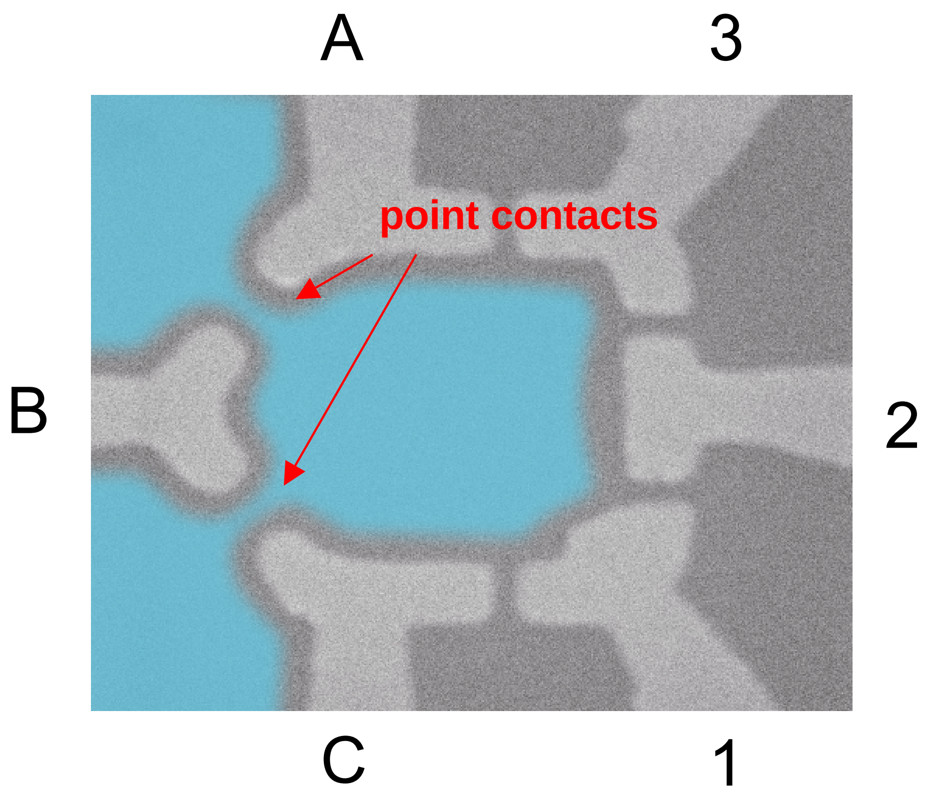


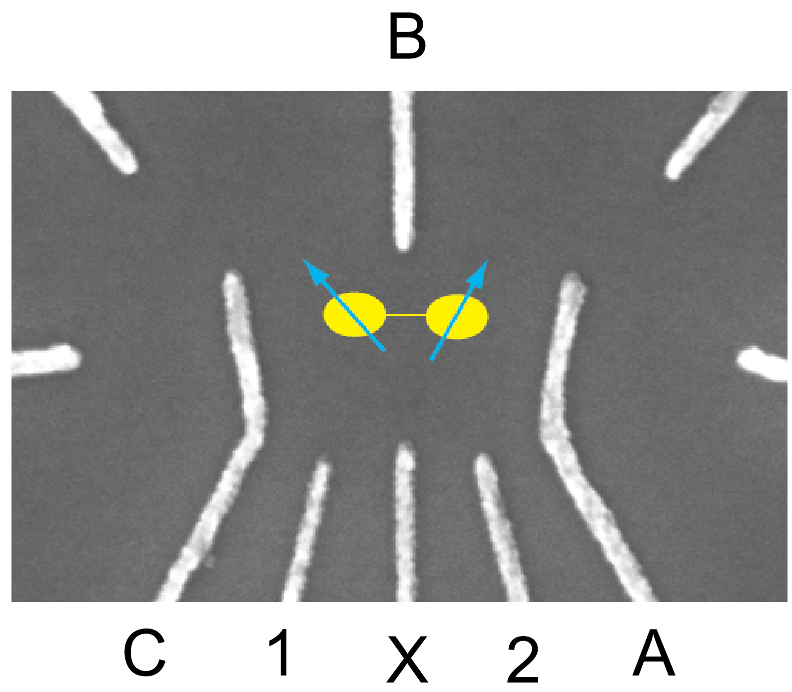


Negative voltage on gates depletes underlying electrons & defines dot cavity



gate defined dots



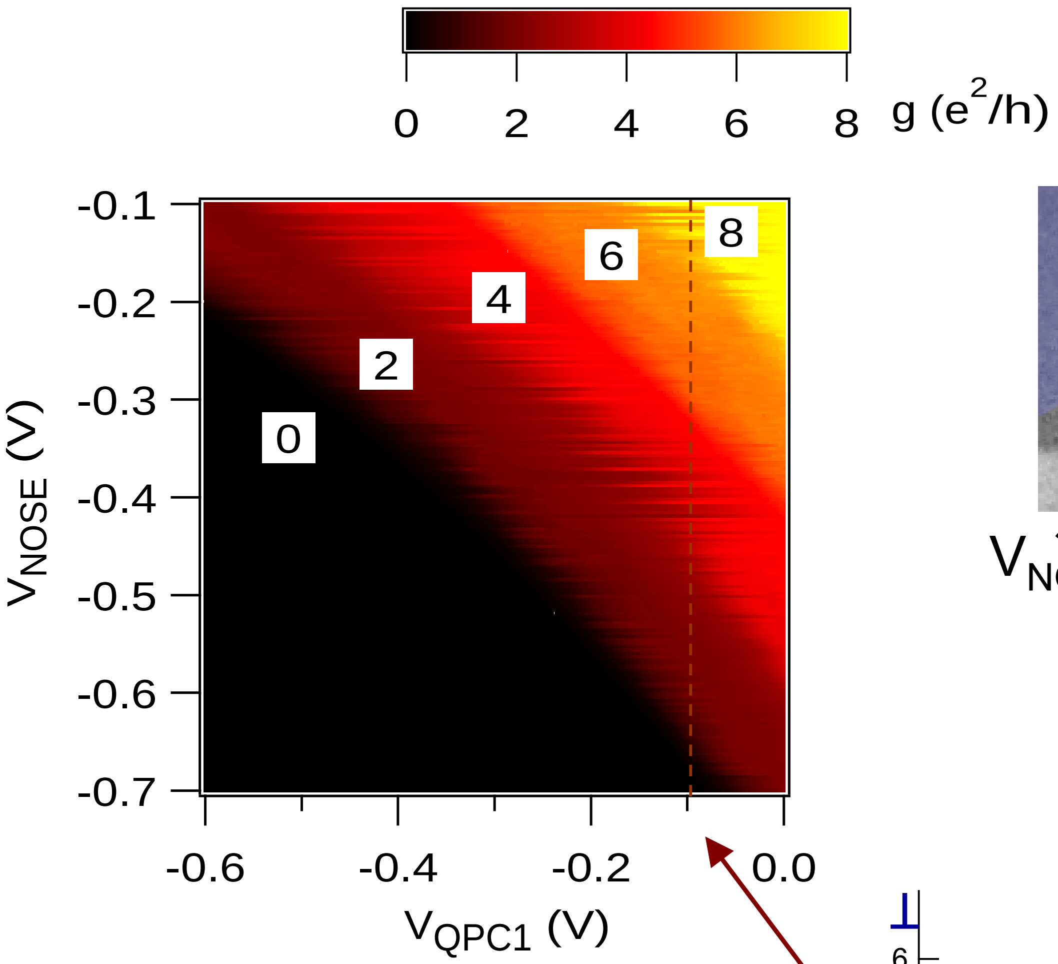


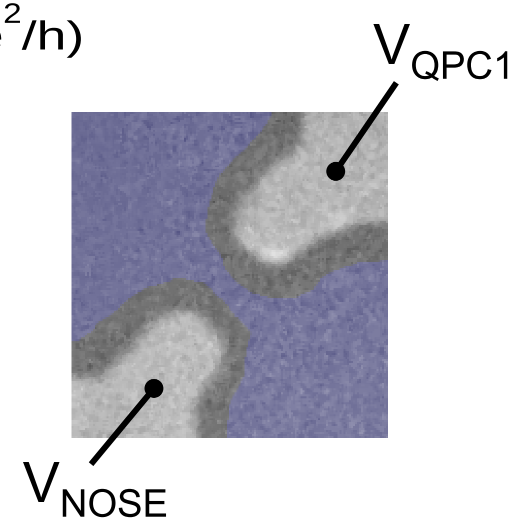
A,B,C : control quantum point contacts
transmission to reservoirs

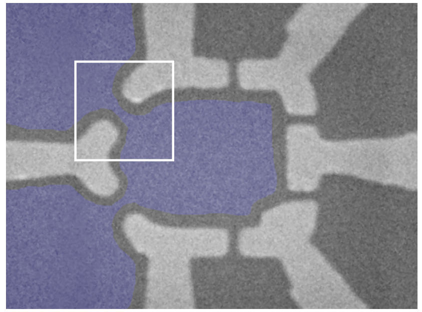
1,2,3: control confinement potential / energy levels only

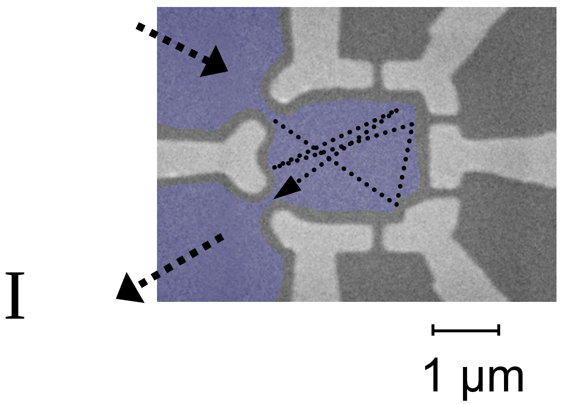
X control dot-internal tunneling rate

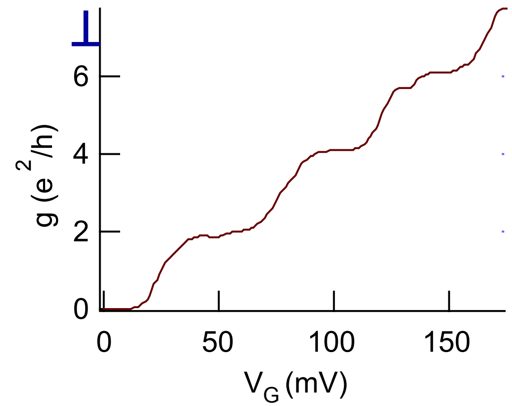
Quantum Point Contact Leads



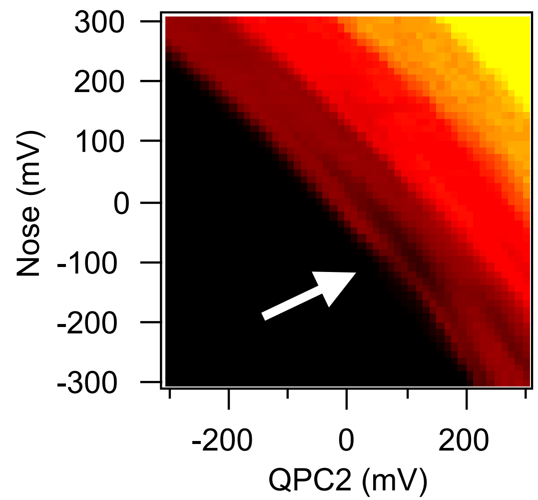








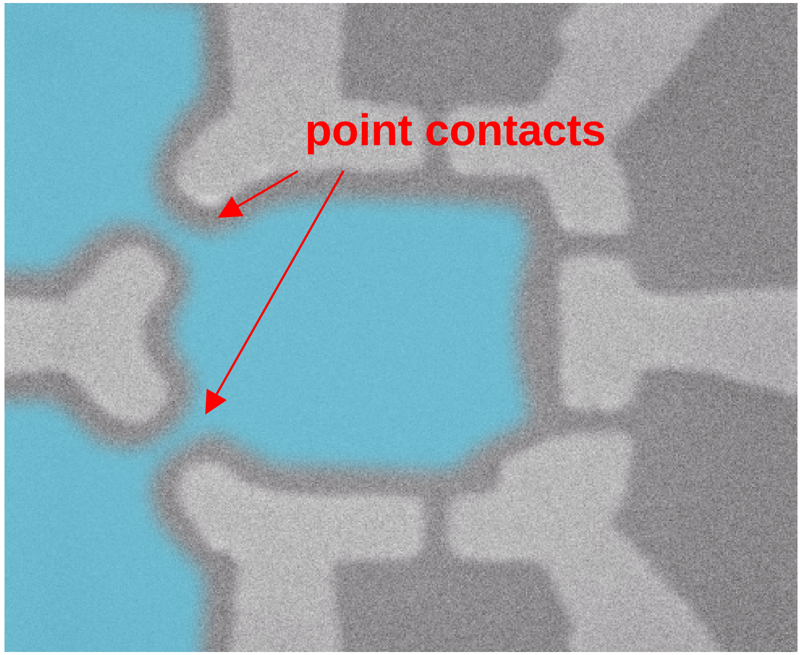
Not all QPCs are beautiful:



slide from A. Huibers, Thesis (1999)

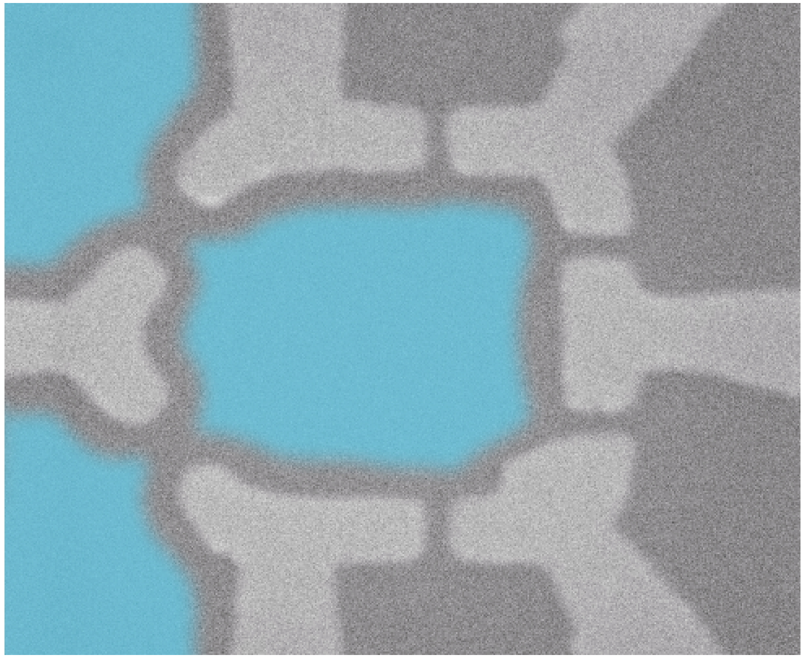
Open vs. Closed

Open Dot



- V_{gate} set to allow ≥ $2e^2/h$ conductance through each point contact
- Dot is well-connected to reservoirs
- Transport measurements exhibit CF and Weak Localization

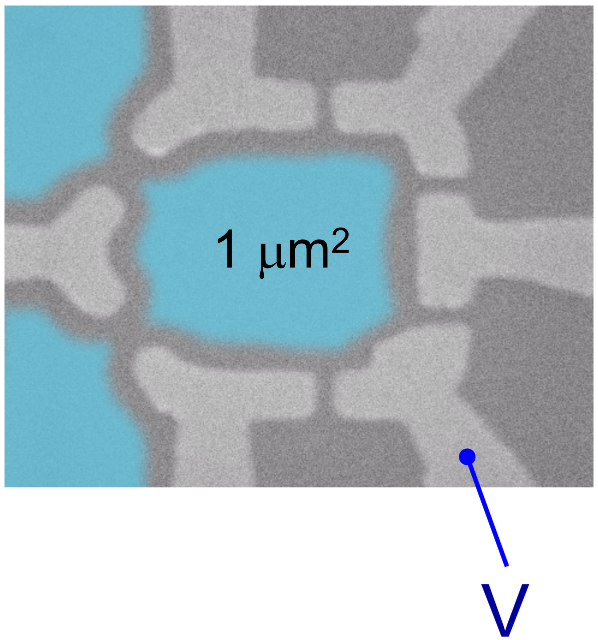
Closed Dot



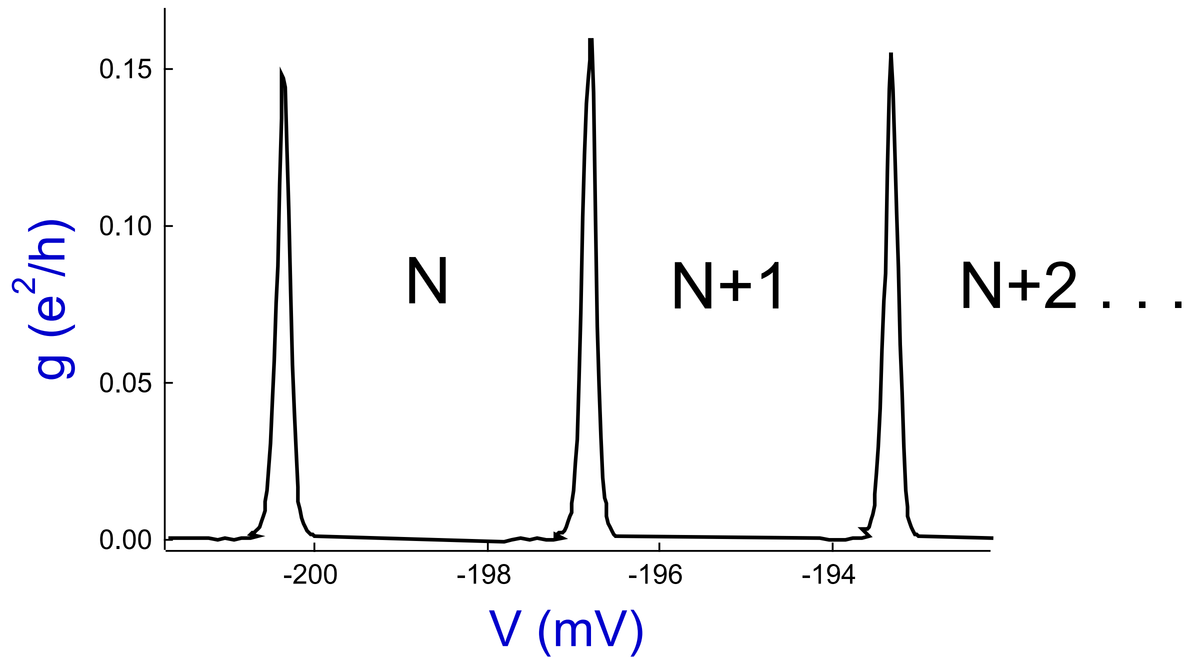
- V_{gate} set to require tunnelling across point contacts
- Dot is isolated from reservoirs, contains discrete energy levels
- Transport measurements exhibit Coulomb Blockade

Coulomb Blockade in Closed Dots

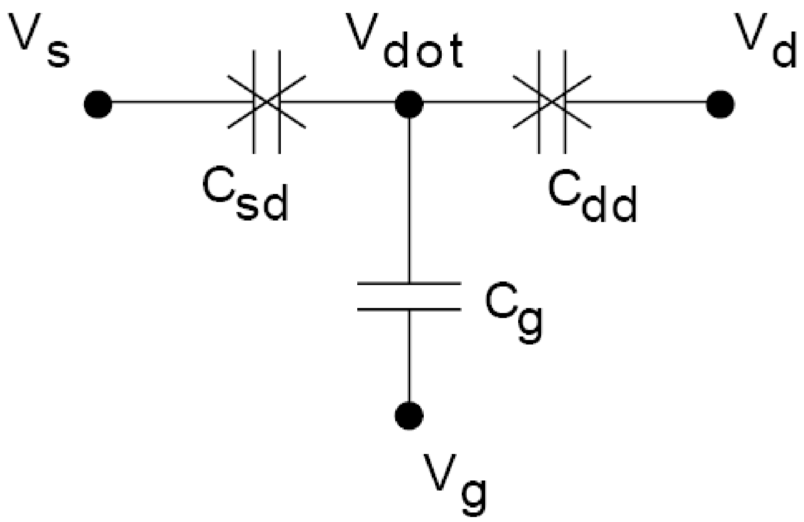
Finite energy $E_c=e^2/C_{dot}$ is needed to add an additional electron to the dot. When $kT<<E_c$ charging blocks conduction in valleys.



Coulomb blockade peaks:
resonant transport through dot levels



Electrostatic Energy



apply voltages

what is potential on dot?

voltage divider...

$$C_\Sigma = C_{sd} + C_{dd} + C_{g1} + C_{g2} + \ldots$$

$$V_{dot} = \sum_i \alpha_i V_i$$

$$\alpha_i = \frac{C_i}{C_\Sigma}$$

can use V_g to shift dot energy!!

Charging Energy

capacitance of dot to world = C

$$C = \epsilon_0 \epsilon \frac{A}{d}$$

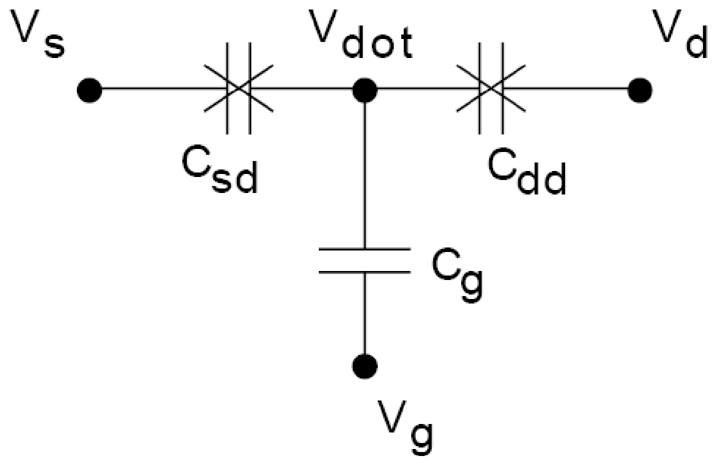
energy stored in capacitor

$$U = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C}$$

charging energy

$$E_C = \frac{e^2}{C_\Sigma}$$

can range from
~0 to many meV



$$C_\Sigma \gtrsim 10\,\mathrm{aF}$$

Classical Effect, NOT quantum

Constant Interaction Model

$$E_i = \sum_{k=1}^{N} q_k \phi_k$$

$$q_k = -e$$

ϕ_k : interaction of electron k with rest

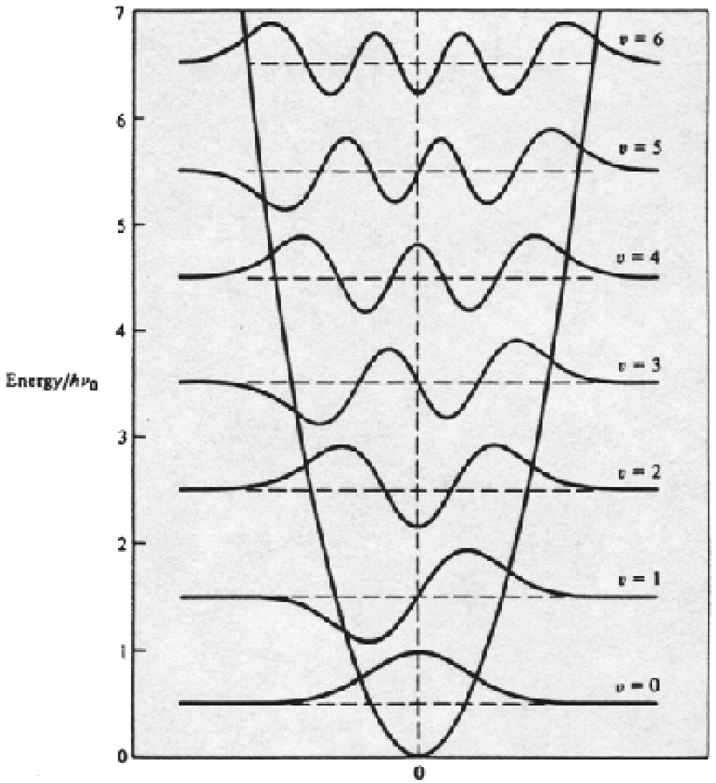
constant inteaction: model ϕ_k with C_Σ

$$\phi_k = -(k-1)e/C_\Sigma$$

$$E_i = \frac{e^2}{C_\Sigma} \sum_{k=1}^{N} (k-1) = \frac{N(N-1)e^2}{2C_\Sigma}$$

Confinement Energy

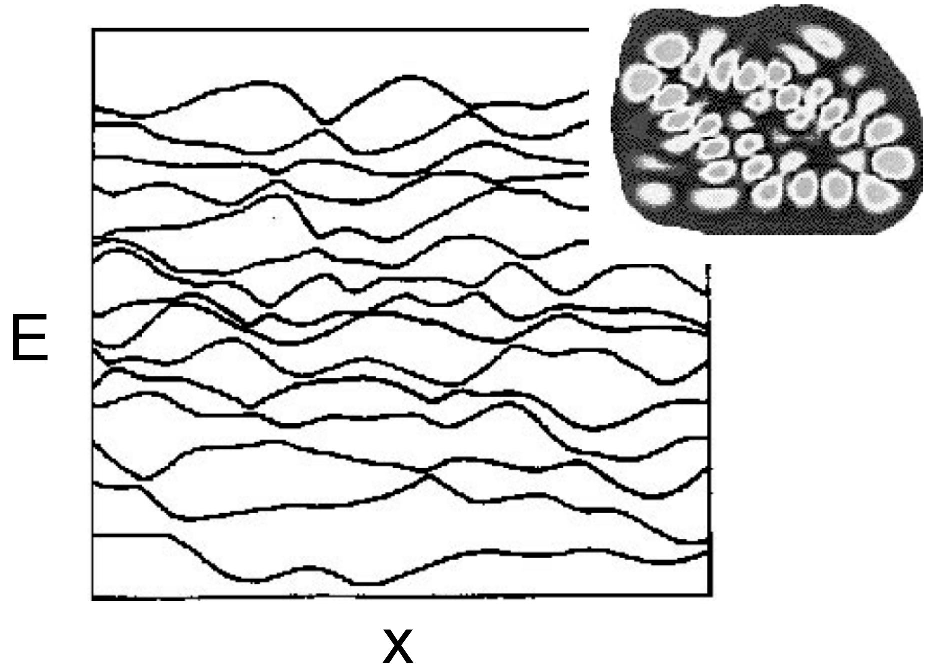
harmonic potential



$$E_n = \left[n + \frac{1}{2}\right] \hbar\omega$$

μeV to meV

complicated potential



average level spacing

$$\Delta = \frac{2\pi\hbar^2}{m^* A}$$

quantum mechanical effect!!

Constant Interaction Model

$$E(N) = E_{\text{QM}} + E_i + E_e$$

total dot energy

$$= \sum_{n=1}^{N} \epsilon_n + \frac{N(N-1)e^2}{2C_\Sigma} - Ne\sum_{i=1}^{6} \alpha_i \text{V}_i$$

$$\mu_{\text{dot}}(N) \equiv E(N) - E(N-1)$$

$$\mu_{dot}(N) = \epsilon_N + (N-1)\frac{e^2}{C} - e\sum_i \alpha_i V_i$$

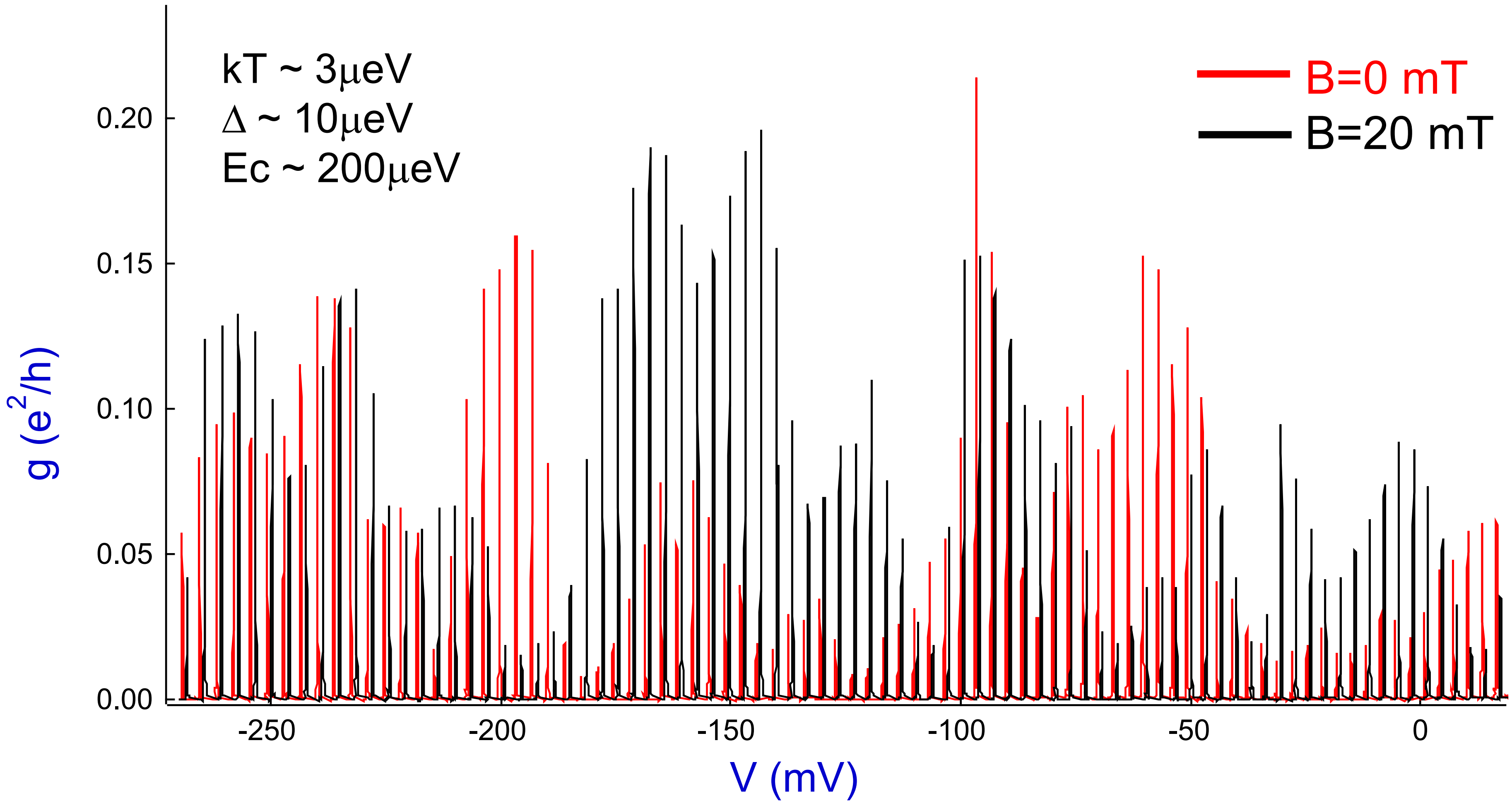
el.chemical potential
μ=0: change N
current flows

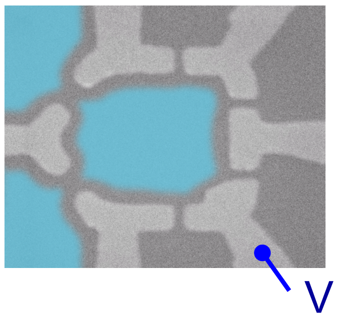
addition energy

$$(\mu_{\text{dot}}(N+1) - \mu_{\text{dot}}(N))|_{\text{fixed V}_i} = \epsilon_{N+1} - \epsilon_N + e^2/C_\Sigma$$

$$\equiv \Delta\epsilon_{N\to N+1} + U$$

Quantum Coulomb Blockade

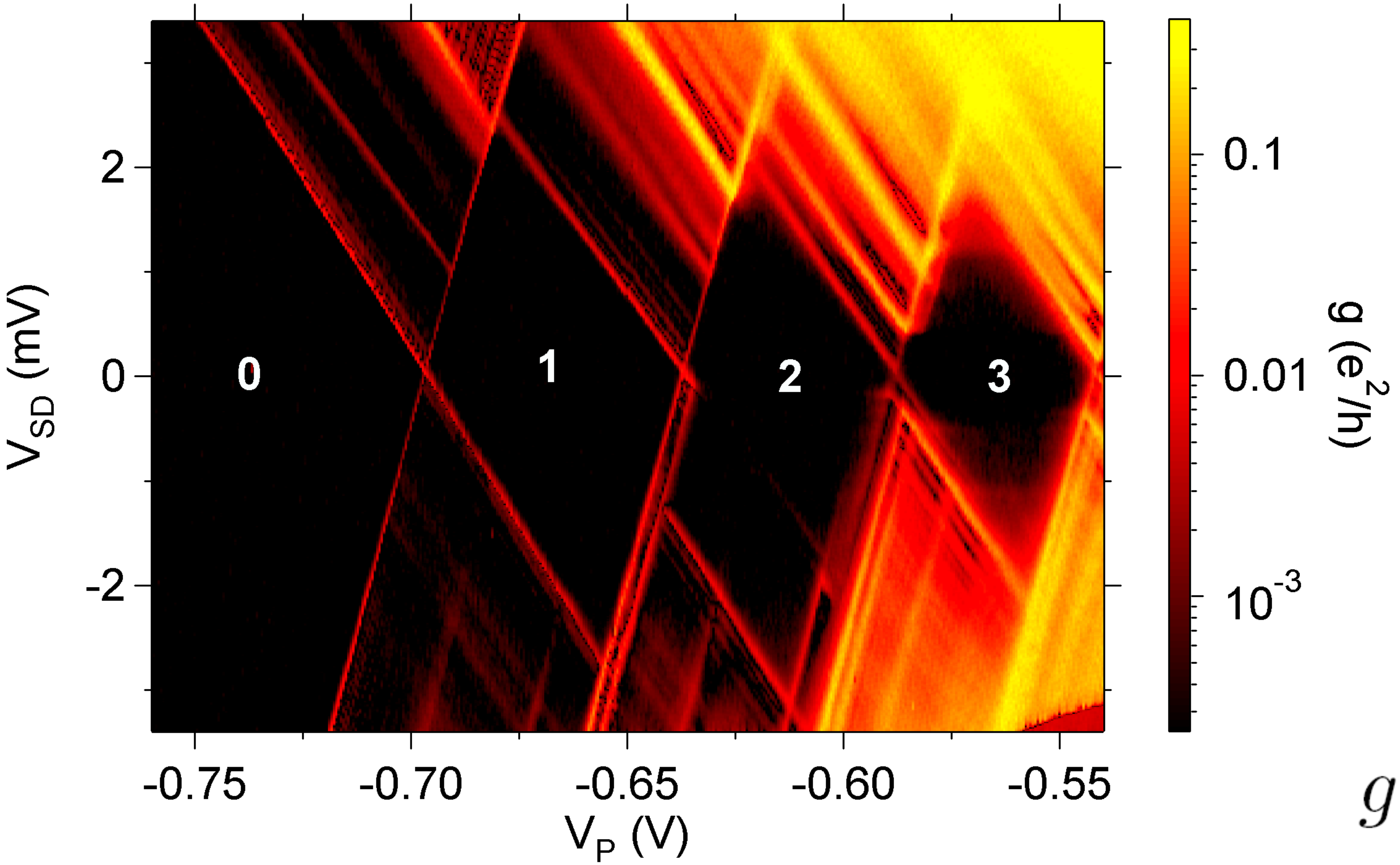




For kT < Δ, each peak describes tunnelling into a single eigenstate. Wavefunction amplitude fluctuations lead to peak height fluctuations.

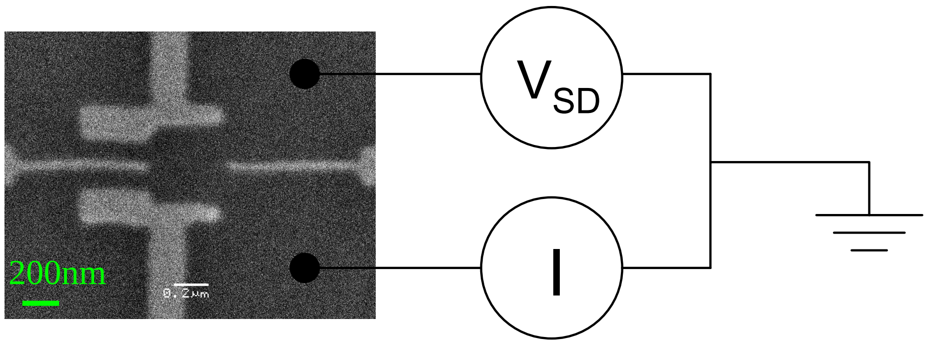
slide from J. Folk (2002)

Coulomb Diamonds



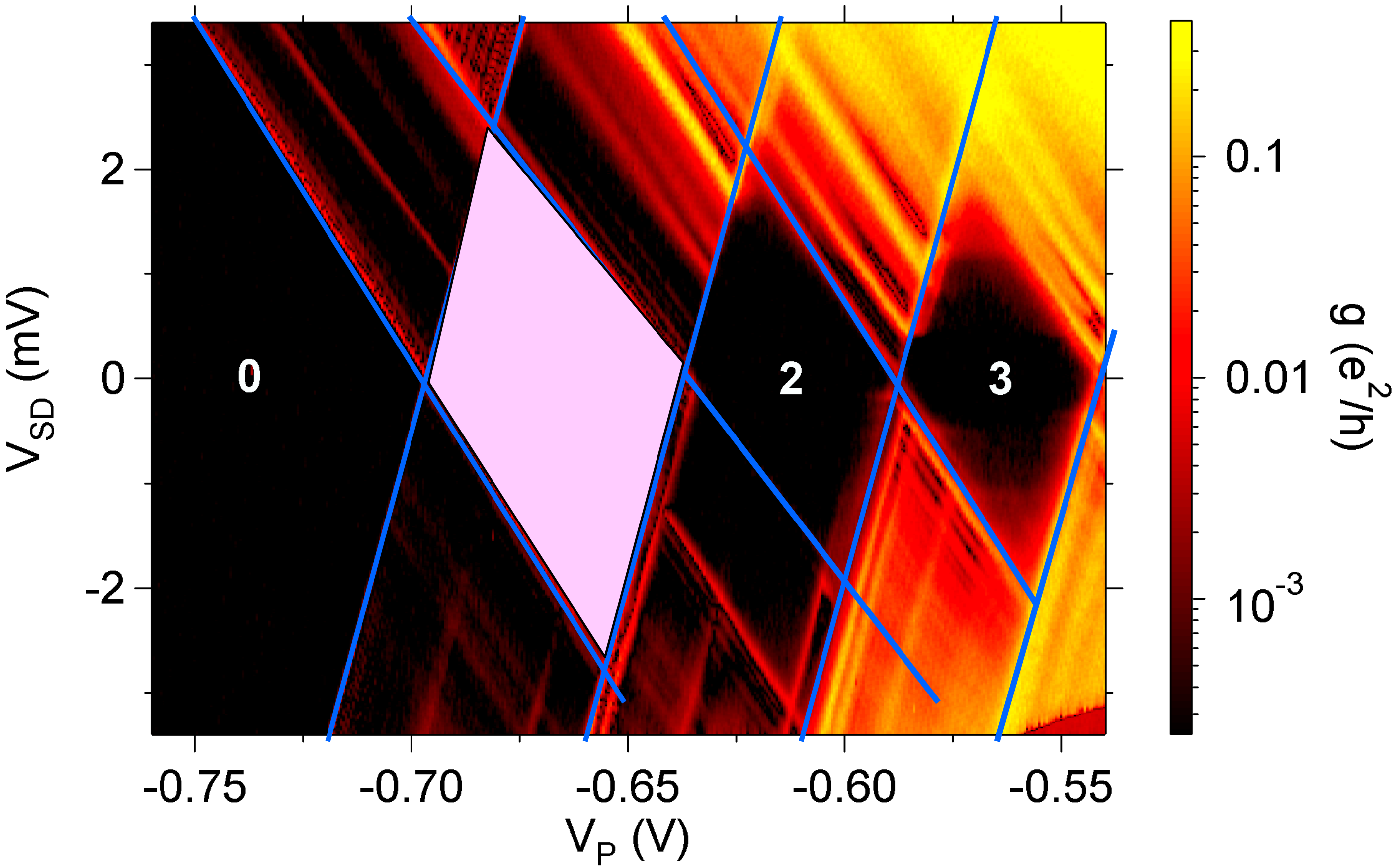
kT ~ 1.5μeV
Δ ~ 1000μeV
Ec ~ 2000μeV

$$g = \frac{dI}{dV}$$

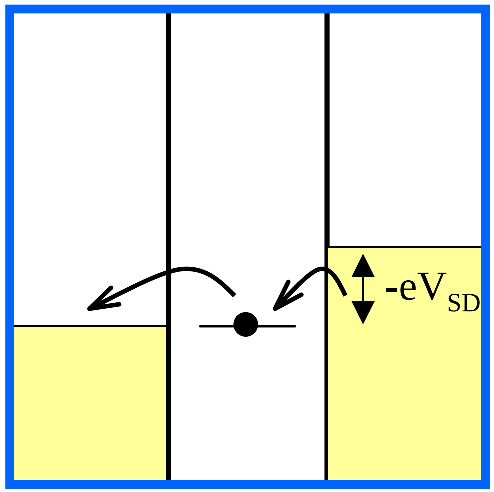


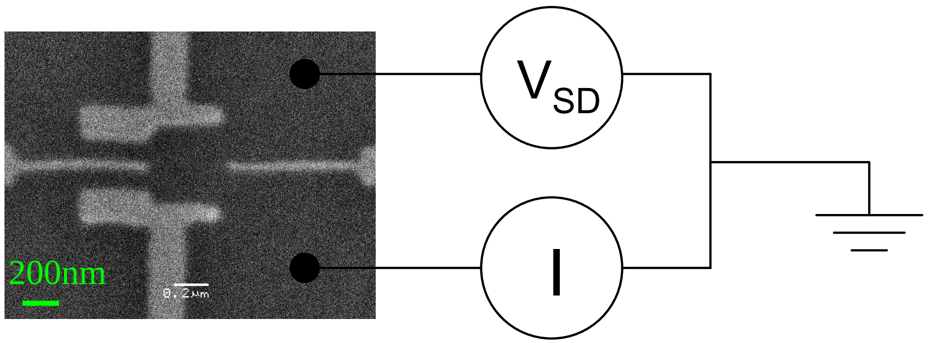
differential conductance:
peaks when current through dot is changing

Coulomb Diamonds

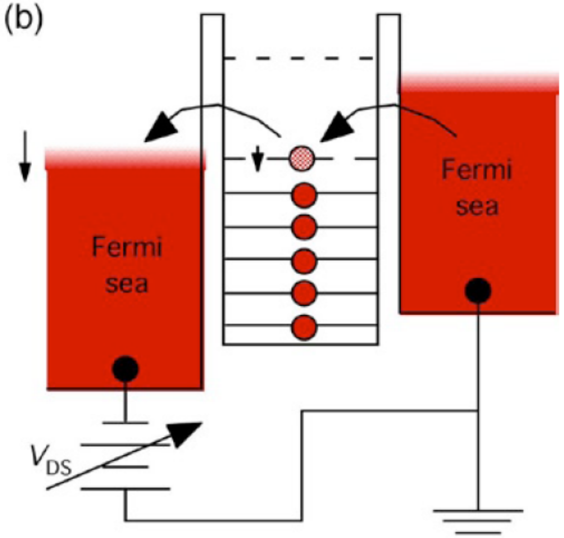


peaks in g appear when dot level aligned with either source or drain chemical potential

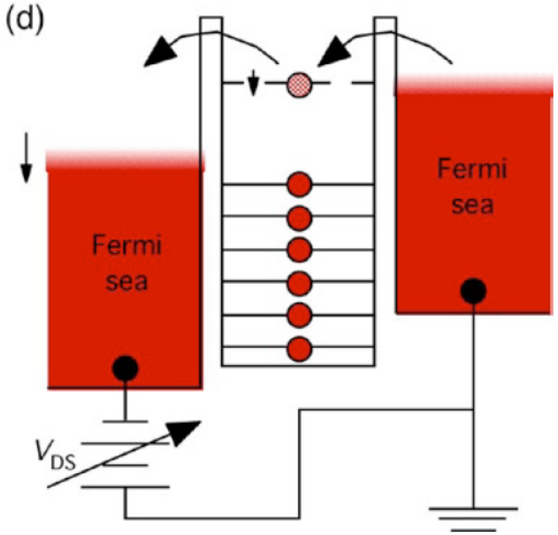


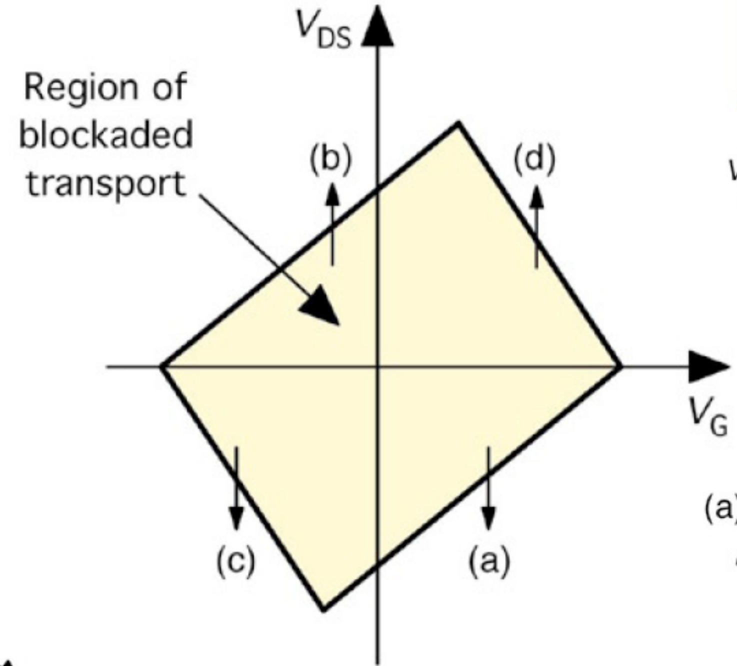


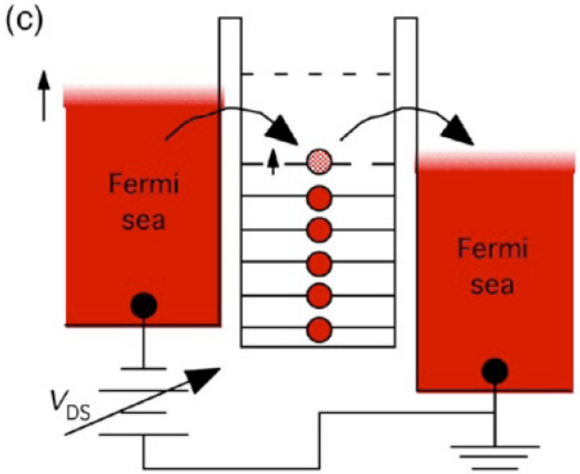
Coulomb Diamonds, Sequential Tunneling Transport



electrons tunnel on / off dot
one by one
(charging energy)

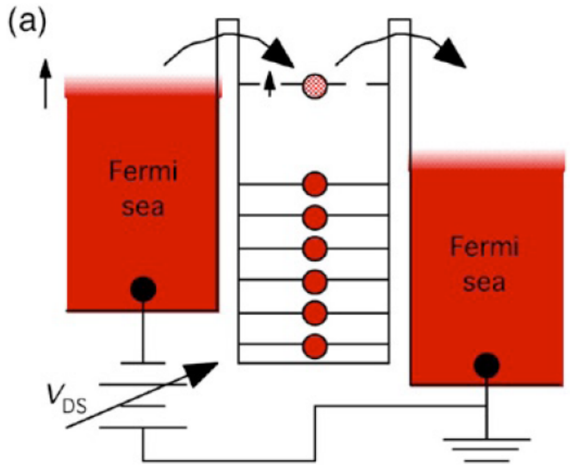




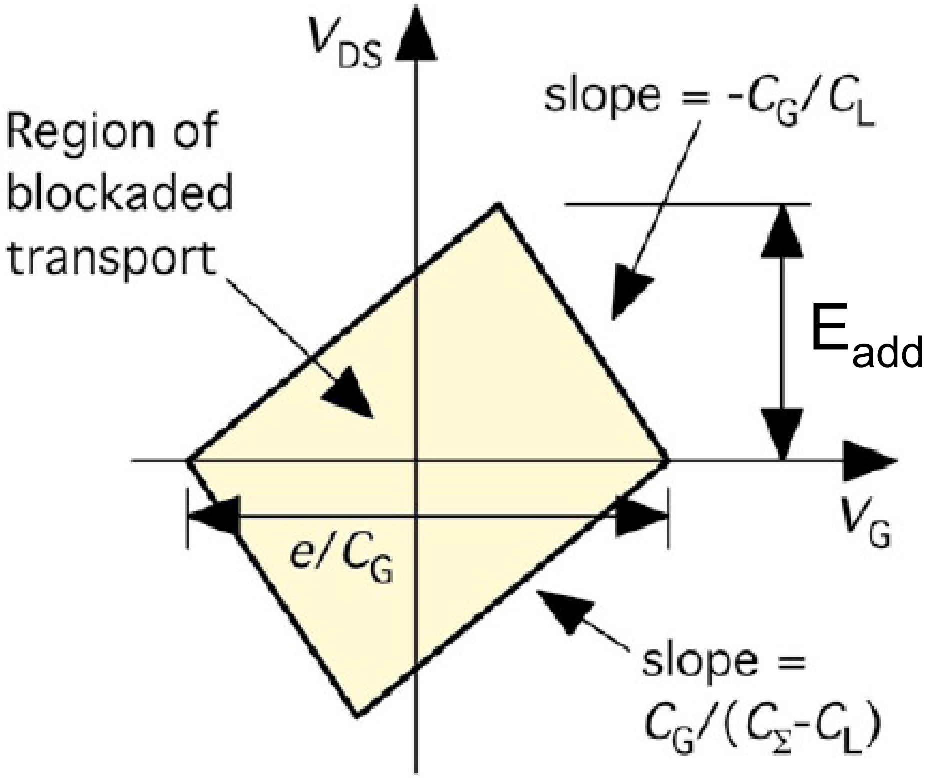


electrons do not change
energy when tunneling

dot energy unchanged

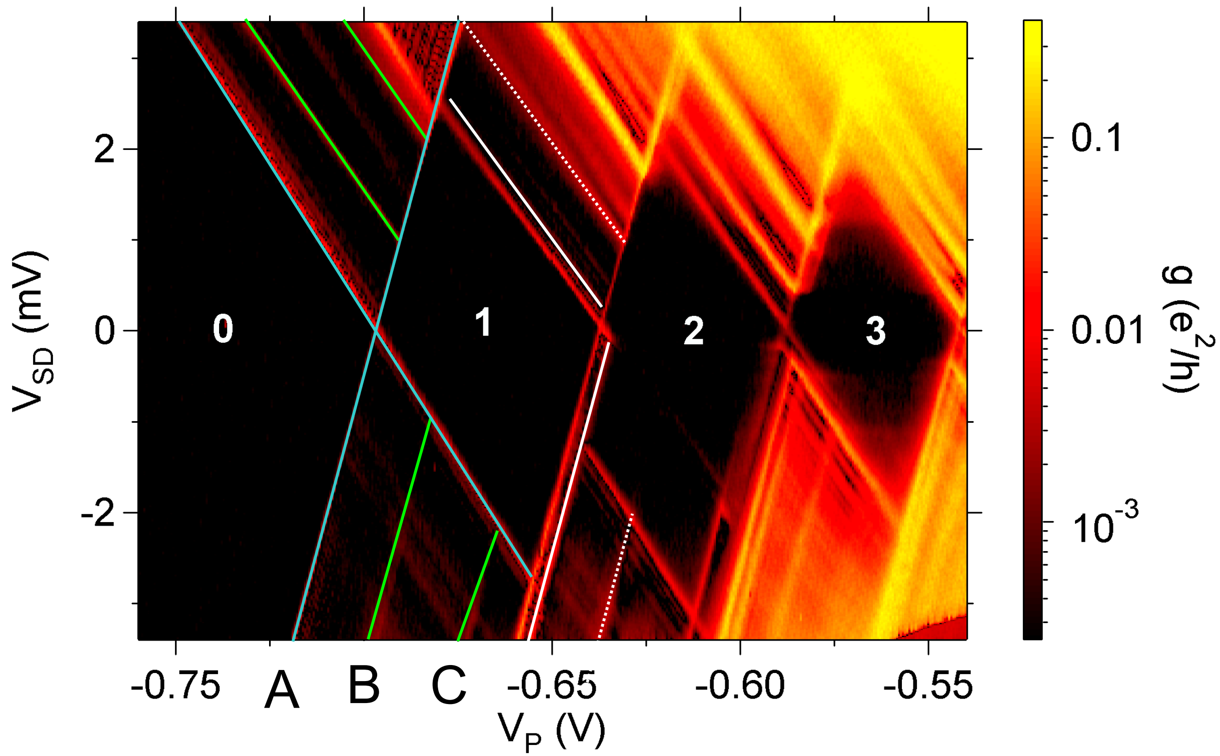


Coulomb Diamonds



two slopes, each associated with its respective dot-lead capacitance

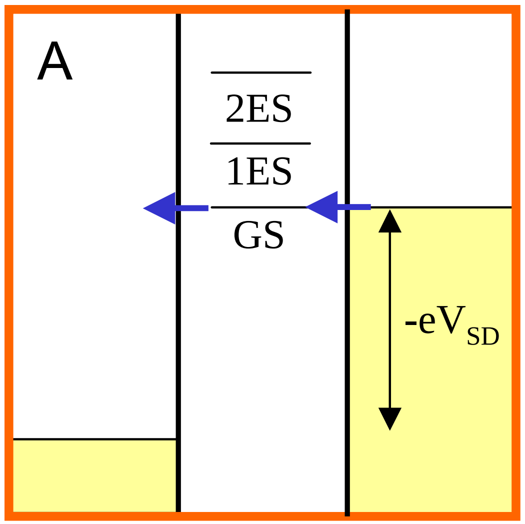
Excited State Spectroscopy

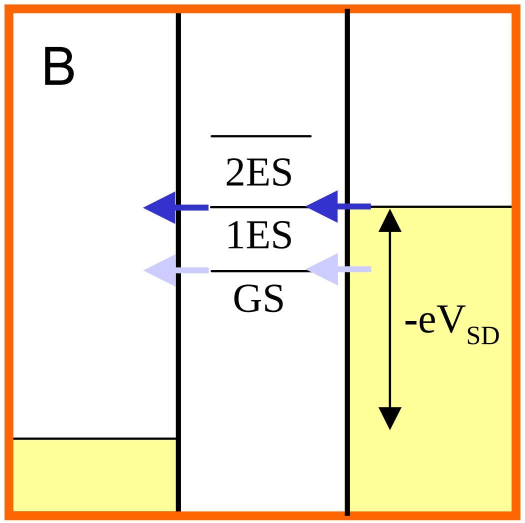


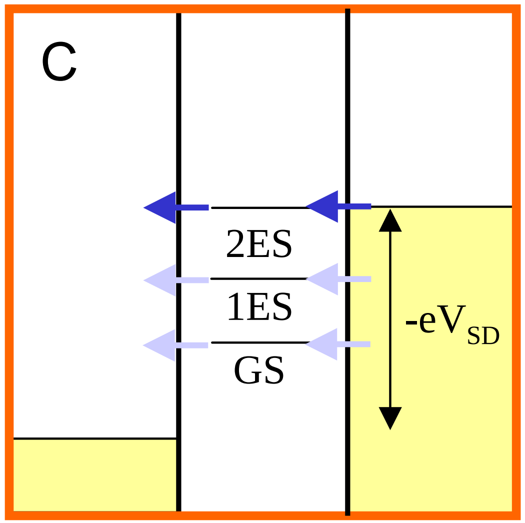
lab to investigate quantum levels in device!!

quantum confinement energies

internal excitations (spin)

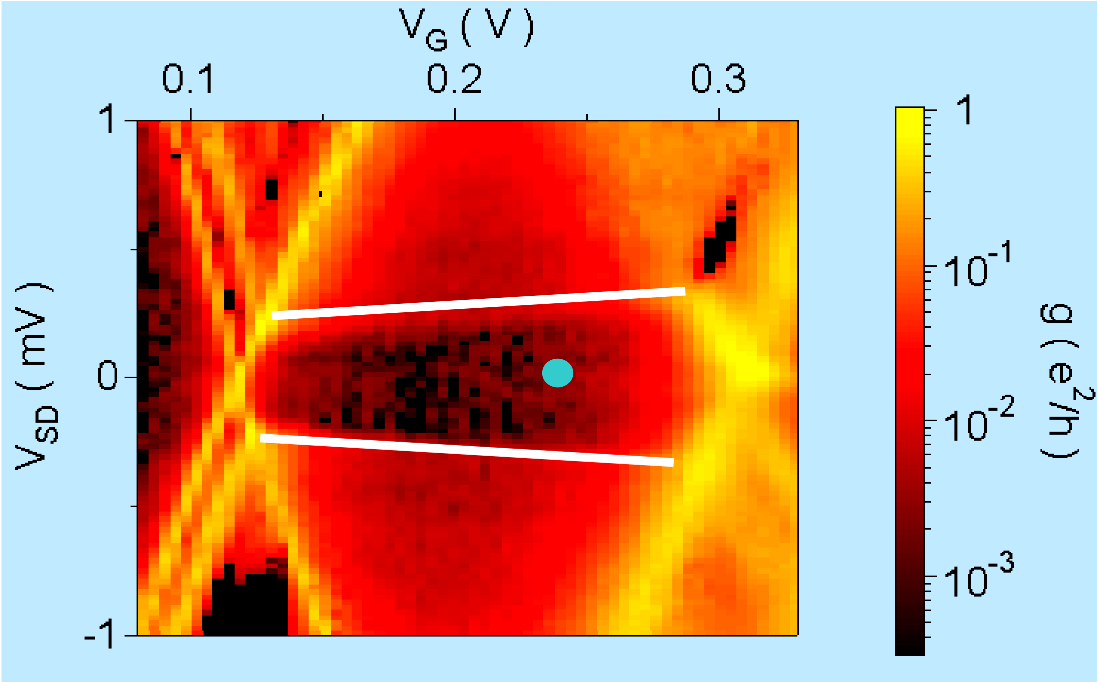






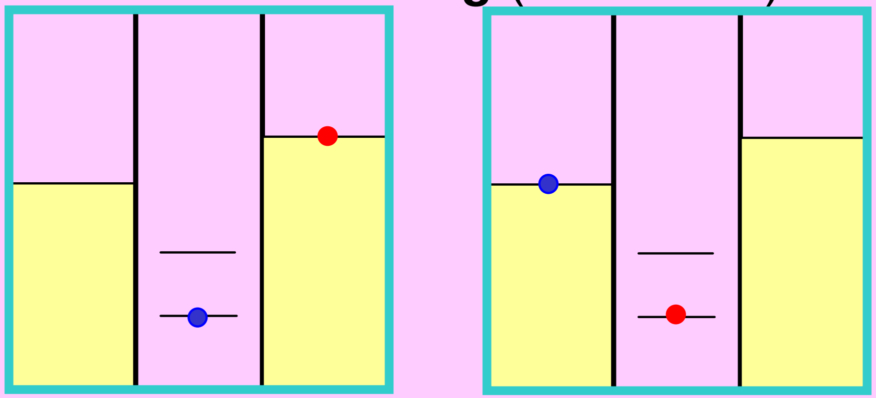
only one electron excess electron can be on dot (charging energy)

Cotunneling Transport



higher order process:
two electrons tunnel and change energy

elastic cotunneling (blue circle)

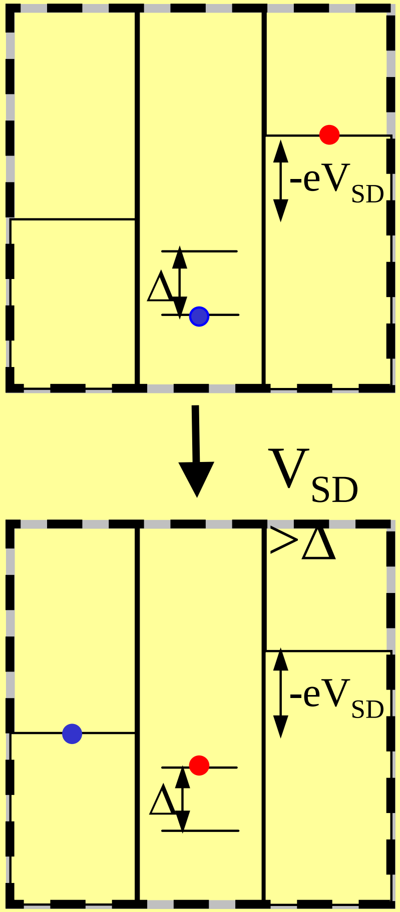


before tunneling

after

dot energy unchanged

white lines: inelastic cotunneling
dot energy changes
only possible for $V_{SD} > \Delta$



before tunneling

after

dot now in excited state

Temperature Regimes

$$\Delta, \frac{e^2}{C} \ll kT$$

no charging effects, no Coulomb blockade

$$g_\infty = \left(\frac{1}{g_L} + \frac{1}{g_R}\right)^{-1}$$

$$\Gamma, \Delta \ll kT \ll \frac{e^2}{C}$$

classical Coulomb blockade (metallic CB)
temperature broadened
transport through several quantum dot energy levels

$$g \sim \frac{g_\infty}{2} \cosh^{-2}\left(\frac{\epsilon}{2.5kT}\right)$$

peak conductance idependent of T
FWHM ~ 4.35kT

$\Gamma = \Gamma_L + \Gamma_R$

escape broadening (tunneling rates)

Temperature Regimes

$$\Gamma \ll kT \ll \Delta \ll \frac{e^2}{C}$$

quantum Coulomb blockade
temperature broadended regime
resonant tunneling
transport through only one dot level

$$g \sim \frac{e^2}{h}\frac{\gamma}{4kT}\cosh^{-2}\left(\frac{\epsilon}{2kT}\right)$$

peak conductance 1/T
FWHM ~ 3.5kT

$$\gamma = \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R}$$

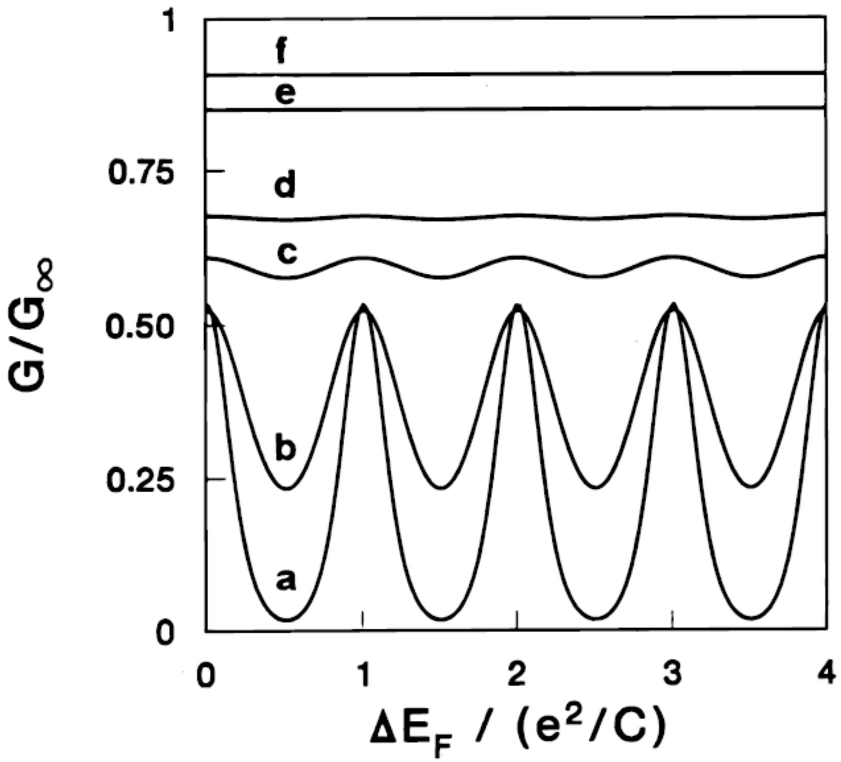
$$kT \ll \Gamma, \Delta \ll \frac{e^2}{C}$$

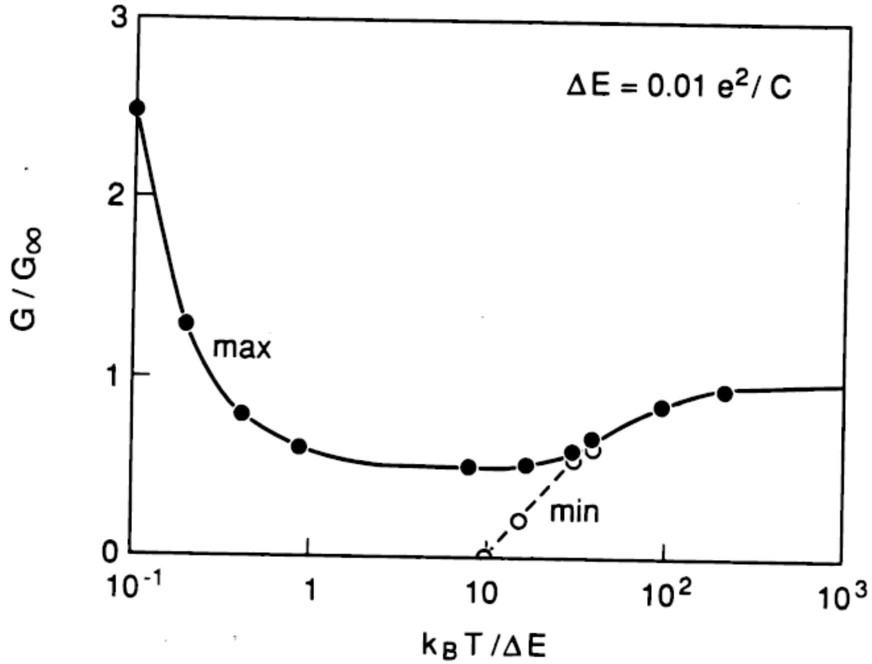
quantum Coulomb blockade
lifetime broadended regime
transport through only one dot level

$$g_{BW} \sim \frac{e^2}{h}\frac{\gamma\Gamma}{(\epsilon/\hbar)^2 + (\Gamma/2)^2}$$

peak conductance e^2/h indep. of T
FWHM ~ Γ

Temperature Dependence: Theory





Δ= 0.01 e^2/C

kT / e^2C

a 0.075

b 0.15

c 0.3

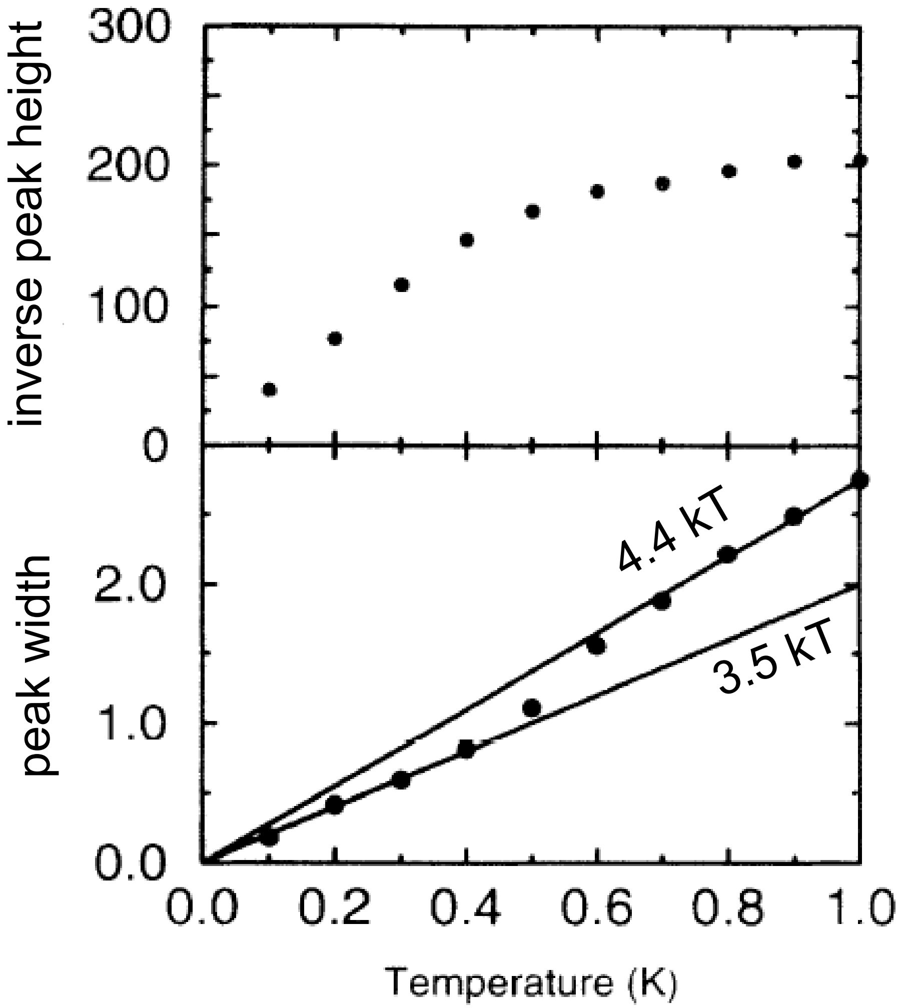
d 0.4

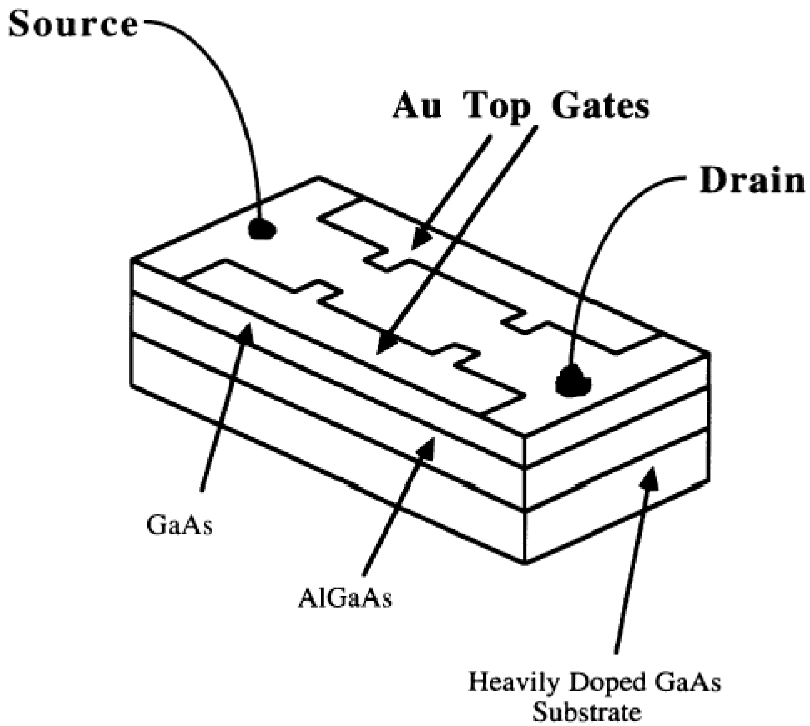
e 1

f 2

van Houten, Beenakker & Staring, NATO ASI Series

Temperature Dependence: Experiment



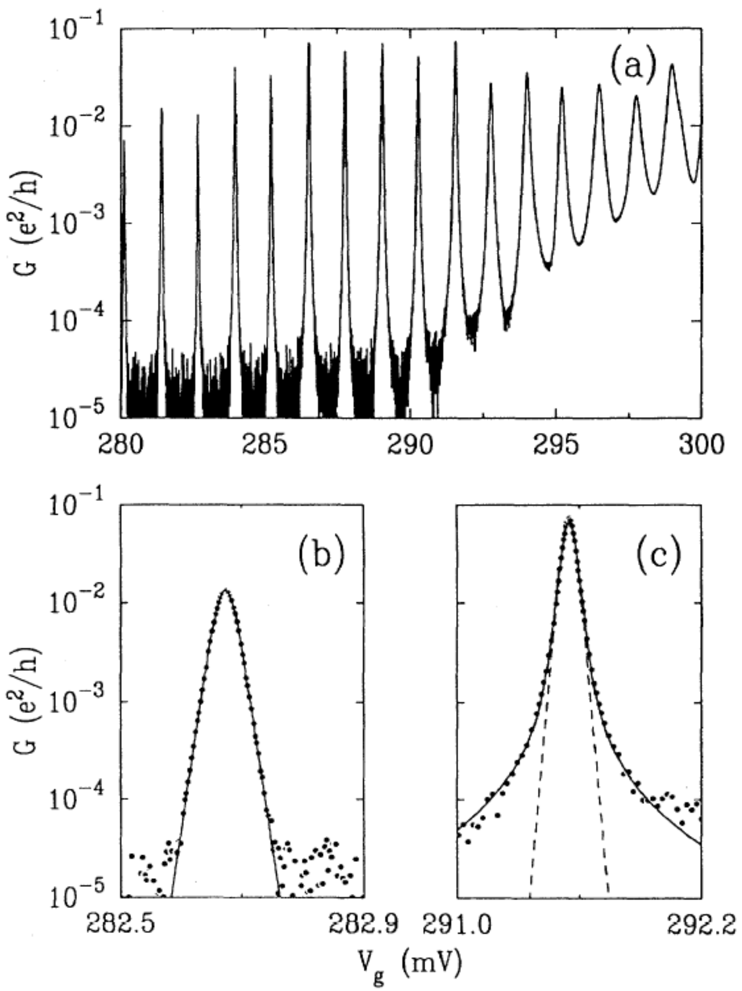


crossover 3.5 to 4.3kT peak width

peak g
1/T dependence: quantum regime
T independent: cassical regime

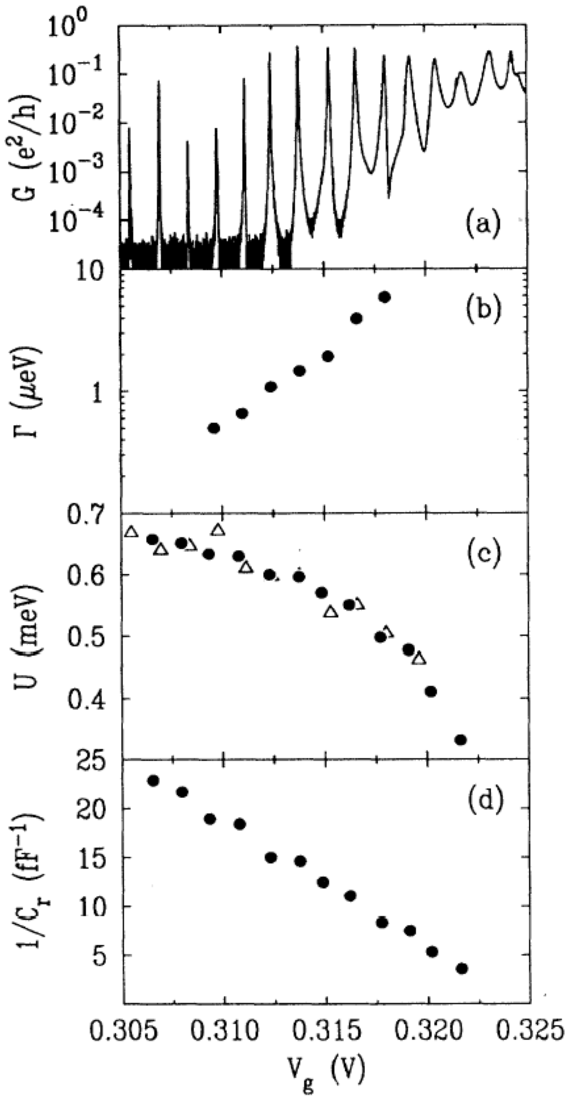
Foxman et al., PRB50, 14193 (1994)

Line Shapes: Experiments



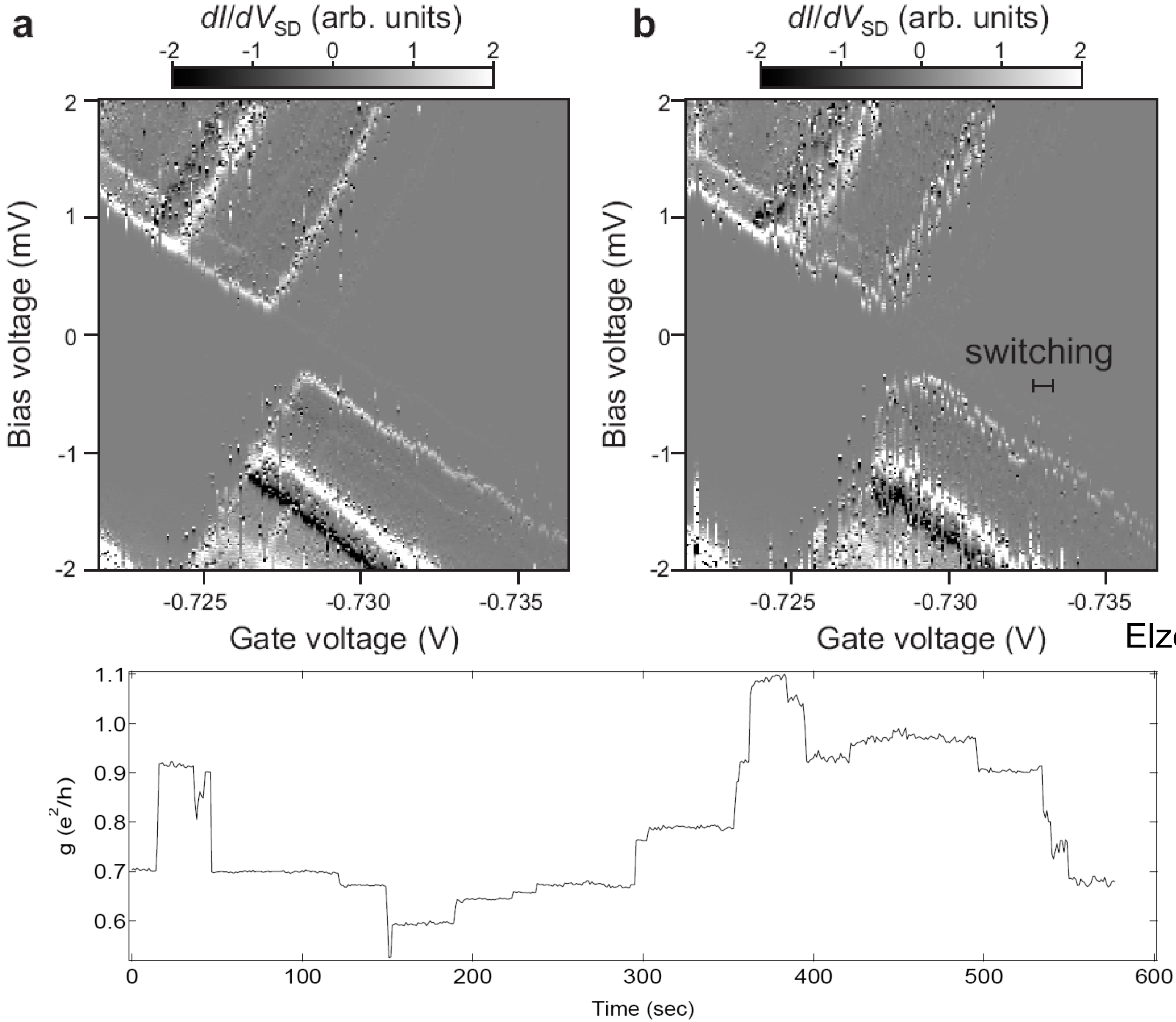
T-broadened

lifetime broadened



Foxman et al., PRB47, 10020 (1993)

Charge Switching / Telegraph Noise



Elzermann, 2003