

quantenmechanische Motivationen der Dipolübergangsmatrixelemente A

$H\psi = E\psi$ stat. Schr. gl. / Eigenwert Problem

$H\psi = -\frac{\hbar}{i} \frac{\partial \psi}{\partial t}$ zeitabh. Schr. gl.

Nehme $H = H_0 + V'(x, t)$

↑ Störpotential z.B. $V' = eAx \cos(\omega t)$

$H_0 = -\frac{\hbar^2}{2m} \Delta + V(x)$ ungest. Problem $F_x = -\frac{\partial V'}{\partial x}$

$H_0 u_\alpha = E_\alpha u_\alpha$

$H_0 u_\beta = E_\beta u_\beta$

$\psi_{\alpha, \beta} = u_{\alpha, \beta}(x, y, z) \cdot e^{-\frac{i}{\hbar} E_{\alpha, \beta} t}$ Lsg. zeitabh. Schr.

Ansatz: $\Psi = \underbrace{c_\alpha}_{c_\alpha(t)} \psi_\alpha + c_\beta \psi_\beta$

andere Lsg. von $H_0 \Psi = -\frac{\hbar}{i} \frac{\partial \Psi}{\partial t}$
aber nicht von H

finde Näherungslösung \rightarrow Übung \Rightarrow

* unter Annahme:

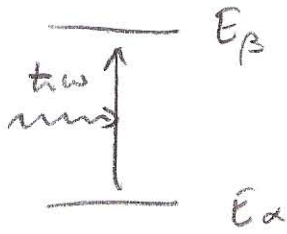
- ebene Welle
- nur \vec{E} -feld (wird B)
- \vec{E} feld homogen ($\lambda \gg a_0$)
- Übergang von $\psi_\alpha \rightarrow \psi_\beta \leftarrow$ stat. Endzustand
 \uparrow
 stat. Anfangsz.
 $\omega_{\beta\alpha} := \frac{E_\beta - E_\alpha}{\hbar}$

$\frac{dc_\beta}{dt} \approx \frac{i}{\hbar} e^{-\frac{i}{\hbar} (E_\beta - E_\alpha) t} E_0 \cos(\omega t) M_{\beta\alpha}$ $M_{\beta\alpha} = \int d^3r u_\beta^* x u_\alpha$

$= \frac{i}{2\hbar} E_0 M_{\beta\alpha} \left(e^{i(\omega_{\beta\alpha} + \omega)t} + e^{i(\omega_{\beta\alpha} - \omega)t} \right)$

$c_\beta(t) = \int_0^t \frac{dc_\beta}{dt} dt = \frac{E_0}{2\hbar} M_{\beta\alpha} \left\{ \frac{e^{i(\omega_{\beta\alpha} + \omega)t} - 1}{(\omega_{\beta\alpha} + \omega)} + \frac{e^{i(\omega_{\beta\alpha} - \omega)t} - 1}{(\omega_{\beta\alpha} - \omega)} \right\}$

Absorption



$$\omega_{\beta\alpha} = \frac{E_\beta - E_\alpha}{\hbar} > 0$$

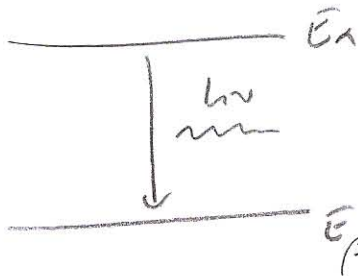
$$\rightarrow |\omega_{\beta\alpha} + \omega| \gg |\omega_{\beta\alpha} - \omega| = \Delta\omega$$

$$c_\beta(t) \approx \frac{E_0}{2\hbar} M_{\beta\alpha} \frac{e^{i(\omega_{\beta\alpha} - \omega)t} - 1}{\omega_{\beta\alpha} - \omega}$$

calculate $c_\beta^* c_\beta$, use $(e^{i\alpha} - 1)(e^{-i\alpha} - 1)$
 $= 2(1 - \cos(\alpha)) = 4 \sin^2\left(\frac{\alpha}{2}\right)$

$$|c_\beta(t)|^2 \sim \frac{E_0^2}{\hbar^2} |M_{\beta\alpha}|^2 \frac{\sin^2\left(\frac{1}{2}\Delta\omega t\right)}{(\Delta\omega)^2}$$

Emission

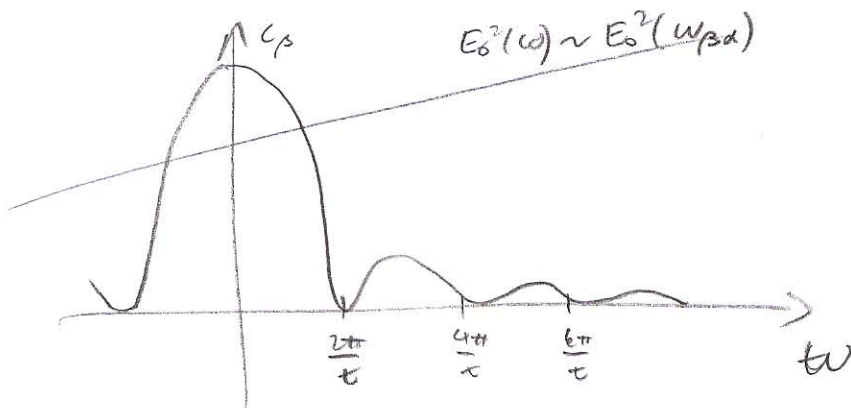


$$\omega_{\beta\alpha} = \frac{E_\beta - E_\alpha}{\hbar} < 0$$

$$|\omega_{\beta\alpha} - \omega| \gg |\omega_{\beta\alpha} + \omega| = \Delta\omega$$

ditto Absorption, i.e. same $|c_\beta(t)|^2$

$$\rightarrow \boxed{\text{Absorption} = \text{Emission } W_i}$$



$$\int_{-\infty}^{\infty} d\omega \frac{E_0^2(\omega_{\beta\alpha}) |M_{\beta\alpha}|^2}{\hbar^2} \frac{\sin^2\left(\frac{1}{2}\Delta\omega\right)}{\Delta\omega^2}$$

$$= \frac{E_0^2(\omega_{\beta\alpha}) |M_{\beta\alpha}|^2}{t^2} \frac{t^2}{4} \int_{-\infty}^{\infty} \frac{\sin^2\left(\frac{1}{2}\Delta\omega t\right)}{\left(\frac{1}{2}\Delta\omega t\right)^2} d(\Delta\omega)$$

$$= \frac{E_0^2(\omega_{\beta\alpha}) |M_{\beta\alpha}|^2}{t^2} 2t \underbrace{\int_{-\infty}^{\infty} \frac{\sin^2 \frac{\omega}{2}}{\frac{\omega}{2}} d\frac{\omega}{2}}_{\pi} \quad \begin{aligned} \frac{1}{2}\Delta\omega t &= \frac{\omega}{2} \\ d\Delta\omega &= \frac{2}{t} d\frac{\omega}{2} \end{aligned}$$

$$= \frac{\pi E_0^2(\omega_{\beta\alpha}) |M_{\beta\alpha}|^2}{2t^2} t$$

↑

Übergangsw. wächst mit t

$$W_{\alpha\beta} = \frac{|c_{\beta}(t)|^2}{t} = \frac{\pi E_0^2(\omega_{\beta\alpha}) |M_{\beta\alpha}|^2}{2t^2}$$

↓
 $W(\omega_{\beta\alpha})$

Hermitesches Ggw., isotrop, additiv x, y, z Komp.

$$B_{\beta\alpha} = \frac{\pi}{3\epsilon_0 t^2} |M_{\beta\alpha}|^2$$

Abs. / stim. Ein