Supplementary information

Ultrafast hole spin qubit with gate-tunable spin–orbit switch functionality

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Supplementary Information for Ultrafast Hole Spin Qubit with Gate-Tunable Spin-Orbit Switch Functionality

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1 Supplementary Note 1 Microwave Power Calibration

The transmission of the high-frequency part of the setup is frequency-dependent in the GHz regime. In addition to a continuously increasing attenuation at higher frequencies, there are also a number of pronounced, sharp, resonances where transmission is strongly attenuated due to impedance mismatch e.g. on the printed circuit board on which the sample is mounted. Changing the microwave frequency can therefore lead to a highly nonlinear change of microwave power, thus affecting the Rabi frequency for these technical reasons (see Fig. 2b of the main text). Therefore, in order to acquire the data shown in Fig. 2d of the main text, we calibrate the microwave power arriving at the sample for each data point. To do so, we measure the linewidth of the zero-detuning line which is broadened due to inelastic tunneling assisted by the microwave drive on the gate [1, 2]. In case of a dot-reservoir tunnel coupling Γ that is large in comparison to the drive frequency $f_{\rm MW}$, applying a microwave tone predominantly leads to broadening and a power-dependent splitting with continuous shift of peak positions with applied microwave power. As shown in Figure S1, this results in a V-shaped dependence when detuning is plotted as a function of power. Here, $f_{\rm MW} = 3.75 \, {\rm GHz}$, which corresponds to the datapoint at $|B_{\text{ext}}| = 264 \,\text{mT}$ in Fig. 2d of the main text. We compare line cuts along the detuning axis to a reference measurement at $f_{\rm MW} = 6 \,\text{GHz}$ and $P_{\rm MW} = 25 \,\text{dBm}$ applied at the signal generator (blue dashed curve in Figure S1 b), and match the linewidth by adjusting $P_{\rm MW}$ for each $f_{\rm MW}$. For the case of $f_{\rm MW} = 3.75 \,\rm GHz$, shown in Figure S1, the reference linewidth is best matched for a power of 19 dBm (green curve). The highest (lowest) power which still gives a good overlap with the reference measurement (red curves in Figure S1 b) then determines the vertical error bars in Figure 2d in positive (negative) direction, where we assume a linear dependence of f_{Rabi} on A_{MW} .



Figure S1: Microwave power calibration. a Linewidth measurement of the zerodetuning line with varying microwave power for calibrating the microwave power arriving at the sample. The drive frequency in this case is $f_{\rm MW} = 3.75$ GHz. b Line cuts along the detuning axis at different powers, as indicated in **a**. The reference measurement (blue dashed curve) corresponds to $f_{\rm MW} = 6$ GHz and $P_{\rm MW} = 25$ dBm. Comparing the linewidth, the best match for $f_{\rm MW} = 3.75$ GHz is found for $P_{\rm MW} = 19$ dBm (green curve) and the upper (lower) limits are $P_{\rm MW} = 20$ dBm (16.5 dBm), resulting in the error bars in Figure 2**d** of the main text.

2 Supplementary Note 2 Electrical Qubit Tunability

2.1 Tuning of Qubit Resonance with $\Delta V_{\rm P}$

For coherent manipulation with EDSR, the qubit is pulsed deeply into Coulomb blockade in order to protect it from unintended effects of the applied microwave burst. We measure EDSR for different pulse depths $\Delta V_{\rm P}$ of the square voltage pulse which pulses the system into Coulomb blockade (see pulsing scheme depicted in Fig. 1 of the main text). Figure S2 shows the result as a function of the magnetic field $|B_{\rm ext}|$ and the pulse depth $\Delta V_{\rm P}$, while $t_{\rm burst}$ and $f_{\rm MW}$ are kept constant. The two vertical features in Figure S2 **a** at $\Delta V_{\rm P} = -0.1$ V and $\Delta V_{\rm P} = 0.2$ V, between which the region of Pauli spin blockade is located (blue shaded area in Figure S2 **b** and **c**), correspond to the T(1, 1)-T(2, 0) transition and the zerodetuning line of the bias triangle, respectively. The two curved features at $|B_{\rm ext}| = 320$ mT and 360 mT correspond to the microwave induced $T_{+}(1, 1) - S(1, 1)$ and $T_{-}(1, 1) - S(1, 1)$ EDSR transitions, respectively.

For $-0.1 \text{ V} \lesssim \Delta V_{\text{P}} \lesssim 0.2 \text{ V}$, spin manipulation takes place inside the bias triangle, leading to comparably bright features because charge transitions are not inhibited by Coulomb blockade, resulting in large interdot and dot-reservoir tunneling rates. The readout point, indicated by the green line in Figure S2 **b** and **c**, is positioned inside this area at $\Delta V_{\text{P}} =$ 0 V. For pulse amplitudes above $\Delta V_{\text{P}} \gtrsim 0.2 \text{ V}$, spin manipulation takes place inside Coulomb blockade, which prevents dot-reservoir tunneling during the microwave burst. This leads to the faint EDSR resonance indicated by the red dashed line in Figure S2 **a**. Note that due to the exchange interaction the resonance splits up for $\Delta V_{\text{P}} \lesssim 0.35 \text{ V}$. Moreover, we observe a pronounced shift of the qubit resonance frequency as a function of the pulse depth ΔV_{P} , corresponding to a decrease of the *g*-factor. Figure S2 **b** shows a relative change of the *g*-factor by up to 20% in a different gate configuration than the data shown in Figure S2 **a**. We emphasize here that when pulsing into Coulomb blockade, one must compensate for the induced change in *g*-factor at the manipulation point M. For a hole spin qubit tuned to be on resonance in the PSB region, the Coulomb pulse



Figure S2: Rabi frequency as a function of the Coulomb pulse amplitude. a Measurement of the EDSR resonances as a function of magnetic field B_{ext} and Coulomb pulse amplitude ΔV_{P} . The microwave burst $t_{\text{burst}} = 100 \text{ ns}$, the frequency $f_{\text{MW}} = 3.8 \text{ GHz}$, and the power $P_{\text{MW}} = 23 \text{ dBm}$ are constant. When the spin manipulation is performed inside the Coulomb-blockaded region, the resonance signal is weak in intensity (red dashed line). **b**, **c** Relative change of *g*-factor and Rabi frequency as a function of the Coulomb pulse depth ΔV_{P} for a different dataset than shown in **a**. The region of Pauli spin blockade (shaded blue) and the readout point at $\Delta V_{\text{P}} = 0 \text{ V}$ (green line) are indicated.

will easily drive it far off-resonance, thus obliviating Rabi oscillations. Next, we measure the Rabi frequency for different values of $\Delta V_{\rm P}$ along the red dotted line in Figure S2 **a** and find the results shown in Figure S2 **c**. The Rabi frequency is highest at manipulation points with small $\Delta V_{\rm P}$ and decreases from 180 MHz to 130 MHz when increasing $\Delta V_{\rm P}$.

2.2 Electrical Tunability with $V_{\rm M}$

The main text discusses the variation of the Rabi frequency due to changes of the voltage $V_{\rm M}$. In addition to the spin-orbit length, also the electric field amplitude $|\vec{E}_{\rm MW}(t)|$, the *g*-factor and the orbital level splitting $\Delta_{\rm orb}$ determine the Rabi frequency, which can be written as (see also equation (1) of the main text):

$$f_{\text{Rabi}} = f_{\text{MW}} \cdot \frac{l_{\text{dot}}}{l_{\text{so}}} \cdot \frac{e |\vec{E}_{\text{MW}}(t)| l_{\text{dot}}}{\Delta_{\text{orb}}}.$$
(1)

Here, we used $f_{\text{Rabi}} = g_{\parallel} \mu_{\text{B}} |\vec{B}_{\text{eff}}(t)|/(2h)$ with $|\vec{B}_{\text{eff}}(t)|$ as defined in Eq. 1 of the main text, and $|\vec{B}_{\text{ext}}(t)|$ is converted to f_{MW} via the resonance condition $g_{\parallel} \mu_{\text{B}} |\vec{B}_{\text{ext}}(t)| = h f_{\text{MW}}$, since f_{MW} is held fixed. In order to stay on resonance, we compensate the variation of g_{\parallel} (shown in Figure 3 **c** of the main text) with a proportional change of $|\vec{B}_{\text{ext}}(t)|$ in the opposite direction, effectively making f_{Rabi} independent of g_{\parallel} . We note that $l_{\text{dot}} = \hbar/\sqrt{\Delta_{\text{orb}}m_{\text{eff}}}$ with the effective hole mass m_{eff} .

Because several quantities in equation (1) could depend on the gate voltages, we discuss the relative contributions of these individual terms to the 7-fold change of the Rabi frequency as observed in Figure 3b of the main text. We treat the contribution of the change in spin-orbit length l_{so} separately in section 2.2.1.

First, we investigate the contribution of the electric field amplitude, which can change as a function of $V_{\rm M}$ even at constant applied microwave power, due to changes of the quantum dot shape, size, or position, which may alter the electric dipole coupling. To roughly estimate the microwave electric field amplitude at the quantum dot for different configurations of $V_{\rm M}$, we convert the voltage drop over gate $V_{\rm L}$ into an electric field across the quantum dot. The voltage drop, in turn, we determine from linewidth measurements of the modified zero-detuning line [2], as discussed in section 1. We assume that the microwave voltage drops over 50 nm, corresponding to the distance between two neighboring gates. Figure S3 a shows the microwave electric field amplitude in the dot as a function



Figure S3: Additional data supporting the qubit tunability with electric fields. a Electric field strength of the microwave drive for different values of $V_{\rm M}$ in the same range as in Figure 3 of the main text. The values are extracted from the shift of the zero detuning line (see section 1) at the used microwave power P = 11 dBm. b Singlettriplet splitting $E_{\rm ST}$ as a function of $V_{\rm M}$ extracted from line cuts in the bias triangle along the detuning line. c g-factor in perpendicular directions compared to the magnetic field orientation in Figure 3 of the main text. d Anisotropy measurements of the g-factor in the $\hat{x}\hat{z}$ - (left) and $\hat{x}\hat{y}$ - (right) plane for $V_{\rm M} = 1422.5$ mV and $B_{\rm ext} = 260$ mT. The anisotropy is highlighted by the red dashed line. e From measurements as shown in d, the g-factor anisotropy in the $\hat{x}\hat{z}$ - (left) and $\hat{x}\hat{y}$ - (right) plane is extracted for several values of $V_{\rm M}$. In order to compare differences of the g-factor anisotropy due to a change in $V_{\rm M}$, each curve is normalized to its maximum.

of gate voltage $V_{\rm M}$ in the same range as used in the main text. Overall, the microwave electric field amplitude does not change significantly, which rules out an unintentional variation of the microwave power while changing the value of $V_{\rm M}$.

Second, the Rabi frequency scales with the orbital level splitting Δ_{orb} roughly as $\sim 1/\Delta_{\text{orb}}^2$ (see equation (1)). In order to estimate this contribution, we extract the single-dot singlettriplet splitting E_{ST} for all values of V_{M} used in the main text and plot it in Figure S3 b. Note that $E_{\text{ST}} = \Delta_{\text{orb}} - E_{\text{exchange}}$ includes contributions from both Δ_{orb} and the single-dot exchange energy E_{exchange} , which is not easily determined independently. In a realistic case, a more positive V_{M} increases both Δ_{orb} and E_{exchange} as the holes become more confined. Here, however, we disregard any changes in E_{exchange} as a function of V_{M} and assign the observed decrease of E_{ST} in Figure S3 b solely to a change of Δ_{orb} . Note that this approach leads to an overestimation of the role of a change in Δ_{orb} with V_{M} . If the exchange energy indeed varies with V_{M} , the decrease of Δ_{orb} would be even smaller and therefore contribute less to the change in f_{Rabi} . If Δ_{orb} increases with V_{M} , it would even lead to a downward trend of f_{Rabi} with V_{M} .

We conclude that the reduction of the orbital level splitting Δ_{orb} from 4.1 meV to 3.3 meV can therefore maximally account for only a factor 1.5 out of the 7-fold increase of the Rabi frequency observed in the main text.

In conclusion, we estimate the contributions from changes in electric field amplitude (no change) and orbital level splitting to account for a total of not more than a 1.5-fold change of the Rabi frequency. The very large change of the Rabi frequency by a factor of 7 can therefore be attributed mainly to an electrically tunable spin-orbit interaction, as expected from a direct Rashba type of spin-orbit interaction.

We also study the evolution of the g-factor in other directions as a function of $V_{\rm M}$. Figure S3 c shows these g-factors in two directions that are perpendicular to the orientation of $\vec{B}_{\rm ext}$ as function of $V_{\rm M}$, extracted from measurements of the Larmor frequency. We find that the g-factor in the $\hat{x}\hat{z}$ -plane (red dots in Figure S3 c) does not change significantly as a function of $V_{\rm M}$. In the $\hat{x}\hat{y}$ -plane (blue dots in Figure S3 c), we find that the g-factor increases as a function of $V_{\rm M}$ by a factor of 1.3.

This increase is somewhat less but comparable to the change of g_{\parallel} in Figure 3c of the main text, which increases by a factor of 1.5 in the same range of $V_{\rm M}$, indicating that the changes in *g*-factor are rather isotropic.

Although indicative of an isotropic change of g-factors, a full proof would entail de-

termination of the complete g-matrix. Our measurements do not allow for such a full characterization, but we can give more indications for the g-matrix variation by providing measurements of the g-factor anisotropy in various planes, and as a function of $V_{\rm M}$.

We measure the complete g-factor anisotropy in the $\hat{x}\hat{y}$ and in the $\hat{x}\hat{z}$ planes for a relevant range of $V_{\rm M}$. Figure S3 d shows examples of such anisotropy measurements, for $V_{\rm M} = 1422.5 \,\mathrm{mV}$. We extract the anisotropic g-factor, indicated by the red dashed curves, for different values of $V_{\rm M}$ as shown in Figure S3 e.

Each curve in Figure S3 e has been normalized to the maximum in order to better compare the change of the anisotropy. Overall, the g-factor anisotropy in the $\hat{x}\hat{y}$ -plane decreases by about 10% with increasing voltage $V_{\rm M}$. Conversely, in the $\hat{x}\hat{z}$ -plane the anisotropy increases by about 30% with increasing voltage $V_{\rm M}$ and furthermore the position of the minimum g-factor shifts.

After estimating the impact of $|\vec{E}(t)|$, g-factor and Δ_{orb} , we now estimate the value of l_{so} .

2.2.1 Estimation of spin-orbit length

We estimate the spin-orbit length $l_{\rm so}$ using equation (1) directly from the measured Rabi frequency $f_{\rm Rabi}$. Here, we use an effective value of $l_{\rm so}$ defined by $\pi l_{\rm so}/2$ being the length along the nanowire which a hole has to travel in order to have its spin flipped. This definition of $l_{\rm so}$ includes the possibility that the direction of the spin-orbit field $\vec{B}_{\rm so}$ with respect to the external magnetic field $\vec{B}_{\rm ext}$ can be non-optimal for EDSR, i.e. $\angle(\vec{B}_{\rm so}, \vec{B}_{\rm ext}) \neq 90^{\circ}$. In this case, $|\vec{B}_{\rm eff}|$ assumes a smaller value, leading to lower Rabi frequencies than achievable with $\vec{B}_{\rm so} \perp \vec{B}_{\rm ext}$.

In our experiment, we do not determine the direction of \vec{B}_{so} . However, values extracted for l_{so} from Eq. (1) of the main text using this effective definition correspond to upper bounds of the spin-orbit length defined in terms of the Rashba coefficient α as $\hbar/(m_{eff}\alpha)$ as obtained by taking the optimal orientation $\vec{B}_{so} \perp \vec{B}_{ext}$. We estimate such upper bounds for l_{so} for four different values of $V_{\rm M}$. The corresponding values of $|\vec{B}_{ext}|$ are given by matching the Larmor frequency to $f_{\rm MW}$ and we determine the values of the remaining parameters |E(t)|, Δ_{orb} , and l_{dot} from other measurements. Note that these four values of V_{M} form a subset of those used in the main text.

The electric field amplitude |E(t)| is estimated as described in section 2.2 and shown in Figure S3 **a**. Given the assumption that the entire voltage of the applied microwaves drops over the distance of 50 nm between neighbouring gates the value of $|E(t)| \approx 13 \text{ kV m}^{-1}$ (see Figure S3 **a**) will result in conservative values for l_{so} , as taking a larger distance over which the voltage drops would result in a weaker electric field and hence larger l_{so} . For Δ_{orb} we take the values shown in Figure S3 **b**, which were obtained from measurements of bias triangles. Note again that for determining Δ_{orb} , we neglect the exchange energy and a possible variation of it with $V_{\rm M}$. Therefore, Δ_{orb} could be larger than we estimate here, which would make the extracted values of l_{so} even smaller than those we find.

Lastly, to estimate the longitudinal dot length l_{dot} , we use $l_{dot} = \hbar/\sqrt{\Delta_{orb}m_{eff}}$, given an effective hole mass m_{eff} . For holes in Ge/Si nanowires, the effective mass depends strongly on the number of holes in the quantum dot [3, 4]. Since we did not independently measure m_{eff} , we estimate l_{dot} and l_{so} both for m_{eff} being equal to the heavy-hole (HH) mass $(0.28 m_0, \text{ with } m_0$ the electron mass) and for m_{eff} being equal to the light-hole (LH) mass $(0.044 m_0)$. Table S1 and Fig. S4 present the calculated upper bounds for l_{dot} and l_{so} for the four different values of V_{M} . For the measurement with $V_{M} = 1422.5 \text{ mV}$, corresponding to the highest Rabi frequency, we find extemely short spin-orbit lengths of maximally 4 nm (26 nm) assuming a HH (LH) effective mass. Note that the fact that the current running through the double dot does not get significantly quenched even for high values of a magnetic field transverse to the nanowire (up to 6 T, not shown) points towards m_{eff} being rather closer to the HH mass than to the LH mass [5]. Further, note that for the relatively low microwave powers used here, equation (1) of the main text is valid even in the case that $l_{so} < l_{dot}$ [6].

$V_{\rm M}~({\rm mV})$	$l_{\rm dot}$ HH (nm)	$l_{\rm dot}$ LH (nm)	l_{so} HH (nm)	$l_{\rm so}$ LH (nm)
1395	8	20	23	146
1400	8	20	15	98
1410	8	20	7	44
1422.5	9	23	4	26

Table S1: Extracted values of l_{dot} and upper bounds of l_{so} for different values of V_{M} . Here LH (HH) corresponds to the case of a light-hole (heavy-hole) effective mass. The errors on all extracted values of l_{dot} and l_{so} are smaller than 15 %.



Figure S4: Estimation of spin-orbit length and dot length. Plot of $l_{\rm so}$ and $l_{\rm dot}$ as a function of $V_{\rm M}$, estimated using measured values of $\Delta_{\rm orb}$ and $|E_{\rm MW}|$ for each value of $V_{\rm M}$ as described in text. Here $m_{\rm eff}$ corresponds to the heavy-hole mass. Note that assuming another mass will change the absolute values of $l_{\rm so}$ and $l_{\rm dot}$, but will leave the tunability with $V_{\rm M}$ unaffected.

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