

Conductance fluctuations and partially broken spin symmetries in quantum dots

D. M. Zumbühl, J. B. Miller, and C. M. Marcus

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

D. Goldhaber-Gordon

Department of Physics, Stanford University, Stanford, California 94305, USA

J. S. Harris, Jr.

Department of Electrical Engineering, Stanford University, Stanford, California 94305, USA

K. Campman and A. C. Gossard

Materials Department, University of California, Santa Barbara, California 93106, USA

(Received 20 May 2005; published 16 August 2005)

Conductance fluctuations in GaAs quantum dots with spin-orbit and Zeeman coupling are investigated experimentally and compared to a random matrix theory formulation that defines a number of regimes of spin symmetry depending on experimental parameters. Accounting for orbital coupling of the in-plane magnetic field, which can break time-reversal symmetry, yields excellent overall agreement between experiment and theory.

DOI: [10.1103/PhysRevB.72.081305](https://doi.org/10.1103/PhysRevB.72.081305)

PACS number(s): 73.63.Kv, 73.23.-b, 73.20.Fz, 73.50.Gr

The combination of confinement, spin-orbit (SO) coupling, and Zeeman splitting in semiconductor quantum dots gives rise to rich physics, including experimental access to interesting partially broken spin symmetries¹ and a suppression of SO effects due to confinement¹⁻⁵ that provides long spin lifetimes in small quantum dots.^{5,6} Further consequences of these combined effects are that the confinement-induced suppression of SO effects is lifted by adding a Zeeman field¹⁻³ or by allowing spatial dependence of the SO coupling.⁷ Because of the finite thickness of a two-dimensional electron gas (2DEG), an in-plane magnetic field B_{\parallel} will have an orbital coupling that affects the electron dispersion and can break time-reversal symmetry^{4,8-12} (TRS), adding additional complexity to this system.

This paper presents an experimental study of mesoscopic conductance fluctuations in quantum dots that possess both significant SO and Zeeman coupling. We find that the B_{\parallel} dependence of the variance of conductance fluctuations, $\text{var } g$, with TRS explicitly broken by a perpendicular field [$B_{\perp} \neq 0$, i.e., $B_{\perp} \gg h/(eA)$, where A is the dot area] depends critically on the strength of SO coupling and dot size. This dependence can be understood in terms of spin symmetries partially broken by B_{\parallel} and is in quantitative agreement with an appropriate random matrix theory (RMT) formulation.¹ We also find that $\text{var } g(B_{\perp}, B_{\parallel})$ becomes independent of B_{\perp} at large B_{\parallel} due to B_{\parallel} breaking TRS, consistent with previous results.^{4,10} Taking into account orbital coupling,^{8,9} agreement between theory and experiment is excellent for both broken and unbroken TRS and various regimes of spin symmetry.

In quantum dots, the effects of Rashba and linear Dresselhaus SO coupling are suppressed due to confinement in the absence of Zeeman coupling.^{1,2,5} For large Zeeman splitting or weak confinement, this suppression is lifted and symmetry classes with partially broken spin symmetry appear. A RMT analysis of this system was developed by Aleiner *et al.*¹ and extended to include inhomogeneous SO coupling and interpolation between ensembles.¹ The

RMT formulation identifies three symmetry parameters that govern the amplitude (variance) of conductance fluctuations, $\text{var } g \propto s/(\beta\Sigma)$. Here, $\beta = \{1, 2, 4\}$ is the usual Dyson parameter reflecting TRS, $s = \{1, 2\}$ accounts for Kramers degeneracy, and $\Sigma = \{1, 2\}$ characterizes mixing between Kramers pairs. With these parameters, spin symmetry may be either unbroken ($s=2, \Sigma=1$), partially broken ($s=1, \Sigma=1$), or fully broken ($s=1, \Sigma=2$), causing a reduction of the variance by a factor of 2 each time spin symmetry is incrementally broken. Temperature and decoherence also reduce $\text{var } g$, but ratios such as $\text{var } g(B_{\parallel})/\text{var } g(B_{\parallel}=0)$ are affected only weakly.

Conductance fluctuations are known to be reduced by SO and Zeeman coupling in bulk (disordered) mesoscopic samples, and theories¹³⁻¹⁵ are in good agreement with experiments.¹⁶ Recently, the combined effects of SO and Zeeman coupling on magnetoresistance in bulk samples were investigated,¹¹ and spin-induced breaking of TRS was reported.¹² Partially broken spin symmetry has been theoretically predicted.^{1,14} The results of Ref. 1 were used to explain the existing data on $\text{var } g(B_{\perp} \neq 0, B_{\parallel})$ (Ref. 3) as well as subsequent experiments on average conductance (weak localization and antilocalization) in quantum dots.^{4,17}

Four gate-defined quantum dots of various sizes on two heterostructure wafers were measured (see Table I and Figs. 1 and 2 insets). The lower-density wafer showed weak localization,¹⁰ while the higher-density material has sufficient SO coupling to exhibit antilocalization at $B_{\parallel}=0$ (Ref. 4). Further details of these wafers are given in Refs. 4 and 10. Measurements were made in a ³He cryostat at 0.3 K using a current bias of 1 nA at 338 Hz. In order to apply tesla scale B_{\parallel} while maintaining subgauss control of B_{\perp} , we mount the sample with the 2DEG aligned to the axis of the primary solenoid (accurate to $\sim 1^{\circ}$) and use an independent split-coil magnet attached to the cryostat to provide B_{\perp} .³ The Hall voltage measured in a comounted Hall bar sample as well as the symmetry of transport through the dot itself

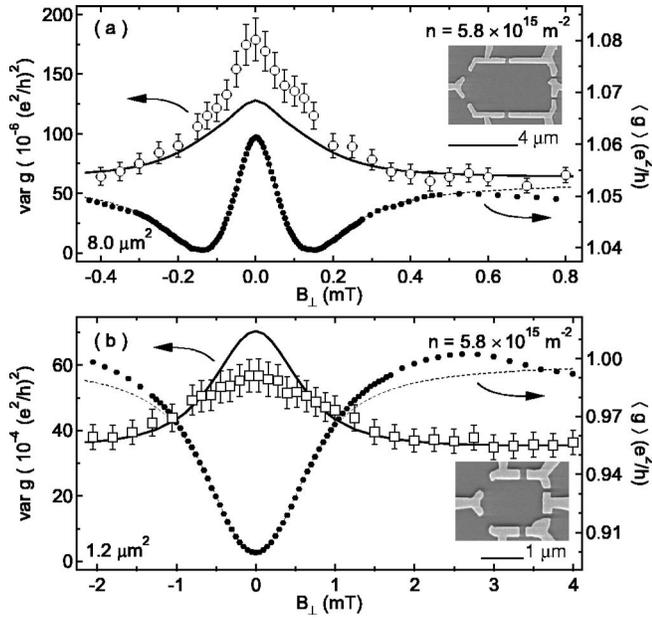


FIG. 1. Conductance average $\langle g(B_{\perp}) \rangle$ (solid dots) and variance $\text{var } g(B_{\perp})$ (open circles and squares) as a function of perpendicular magnetic field B_{\perp} with $B_{\parallel}=0$, for large and small dots on the high-density material, at $T=300$ mK. (a) Antilocalization in $\langle g(B_{\perp}) \rangle$ for the $8\text{-}\mu\text{m}^2$ dot. (b) Weak localization in $\langle g(B_{\perp}) \rangle$ for the $1.2\text{-}\mu\text{m}^2$ dot, demonstrating confinement suppression of SO effects. Both dots show an enhancement of $\text{var } g$ at $B_{\perp}=0$. Fits of RMT (Ref. 1) to $\langle g(B_{\perp}) \rangle$ (dashed curves) and $\text{var } g(B_{\perp})$ (solid curves) using fit parameters determined from fits to $\langle g \rangle$ plus an overall scale factor for $\text{var } g$ (see text). The insets show device micrographs.

(visible for $B_{\parallel} \lesssim 2T$) was used to locate $B_{\perp}=0$ as it changed with B_{\parallel} .

Statistics of conductance fluctuations were gathered using two shape-distorting gates¹⁸ while the point contacts were actively held at one fully transmitting mode each. At each value of B_{\perp} and B_{\parallel} , mean and variance were estimated based on ~ 400 (~ 200) statistically independent samples for the low-density (high-density) dots. For $\text{var } g(B_{\parallel}, B_{\perp} \neq 0)$ data with TRS explicitly broken, B_{\perp} was used to gather additional samples to reduce the statistical error.

Fitting the RMT results to $\langle g(B_{\perp}) \rangle$ yields values for the average SO length $\lambda_{so} = \sqrt{|\lambda_1 \lambda_2|}$, where $\lambda_{1,2}$ are the SO lengths along the main crystal axes, as well as the phase coherence time τ_{φ} and a geometrical parameter κ_{\perp} . The SO inhomogeneity $\nu_{so} = \sqrt{|\lambda_1 / \lambda_2|}$ can be extracted from $\langle g(B_{\parallel}) \rangle$ in the presence of antilocalization (AL), and is taken as ν_{so}

TABLE I. Carrier density n , dot area $A=L^2$, coherence time τ_{φ} , spin-orbit parameters λ_{so} and ν_{so} , RMT parameters κ_{\perp} , f_{var} , and ξ , and FJ parameters a and b (see text).

n m^{-2}	A μm^2	τ_{φ} ns	λ_{so} μm	ν_{so}	κ_{\perp}	f_{var}	ξ	a $\text{ns}^{-1} \text{T}^{-2}$	b $\text{ns}^{-1} \text{T}^{-6}$
2.0	3.0	0.18	8.5	1.0	0.15	1.0	2.8	0.5 ± 0.1	0.028
2.0	8.0	0.21	8.5	1.0	0.25	0.6	3.0	0.37 ± 0.07	0.028
5.8	1.2	0.10	3.2	1.4	0.33	1.9	1.0	6.6 ± 1	0.14
5.8	8.0	0.39	4.4	1.4	0.23	0.7	0.45	1.4 ± 0.4	0.14

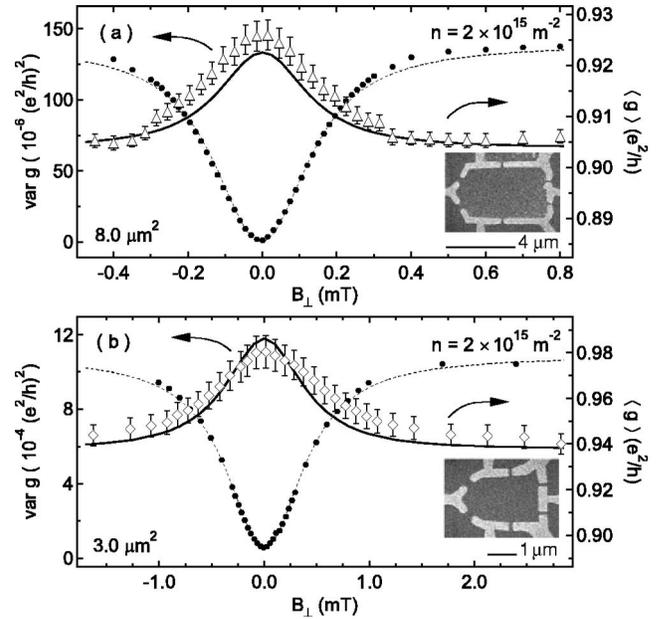


FIG. 2. Conductance average $\langle g(B_{\perp}) \rangle$ (solid dots) and variance $\text{var } g(B_{\perp})$ (open triangles and diamonds) as a function of perpendicular field B_{\perp} with $B_{\parallel}=0$, for (a) large and (b) small dots, both fabricated on low-density material. Both devices display weak localization in $\langle g(B_{\perp}) \rangle$. Fits to RMT are shown as dashed and solid curves, as described in the caption of Fig. 1. Insets show device micrographs; geometry of large device is identical to large dot in high density material.

$=1.4$ (1.0) for the high- (low-) density devices.⁴ An additional order-one geometrical parameter κ' , relevant for the strong SO regime and not readily extracted from transport measurements, is set to $\kappa'=1$ for all devices. Further details of fits to the average conductance are given in Ref. 4. We note that in the absence of AL λ_{so} can only be bounded from below. Values for τ_{φ} are similar for all devices and consistent with previous measurements.¹⁹

As seen in Figs. 1 and 2, all devices except the large high-density dot show weak localization (WL), indicating that these dots are in the regime $\lambda_{so} \gg L$, where SO effects are strongly suppressed by confinement. The observation of WL down to ~ 40 mK in previous measurements on identical devices¹⁹ bounds the SO rate to $\lambda_{so} \gtrsim 8.5 \mu\text{m}$. We note that SO coupling of this order will noticeably reduce the low-temperature WL feature and therefore contribute to the saturation of WL observed in Ref. 19.

For both the high- and low-density material (respectively,

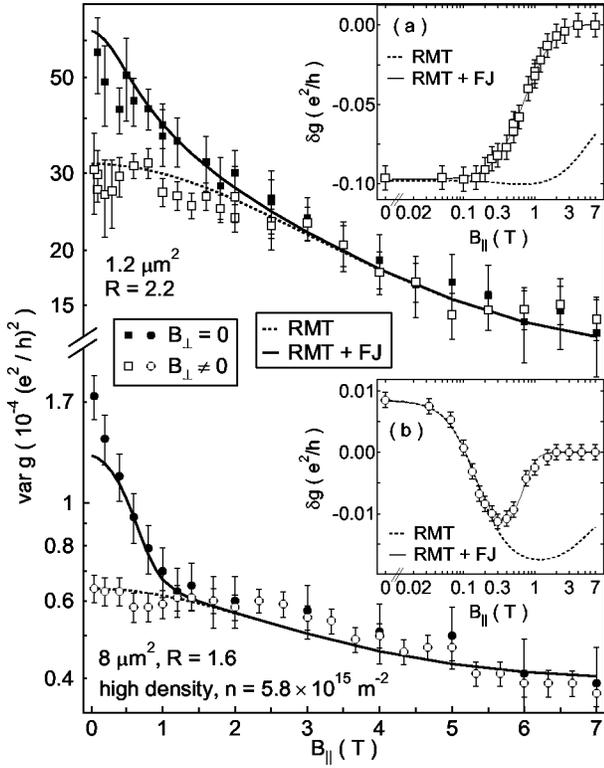


FIG. 3. Variance of conductance fluctuations, $\text{var } g$, in high-density dots, as a function of in-plane field B_{\parallel} with $B_{\perp} \neq 0$ sufficient to break TRS (open symbols) and $B_{\perp} = 0$ (solid symbols). The big dot shows less reduction in $\text{var } g(B_{\perp} \neq 0)$ with B_{\parallel} than the small dot, consistent with RMT (see text). The insets show quantum correction to average conductance, $\delta g(B_{\parallel}) = \langle g(B_{\perp} = 0, B_{\parallel}) \rangle - \langle g(B_{\perp} \neq 0, B_{\parallel}) \rangle$. In both main figure and insets, dashed curves are fits to RMT, solid curves (labeled RMT+FJ) are fits to RMT including orbital coupling of B_{\parallel} (see text).

with AL and WL), $\text{var } g$ at $B_{\parallel} = 0$ is reduced upon application of a TRS-breaking perpendicular field, as seen in Figs. 1 and 2. This effect has been investigated previously for the weak SO (WL) case,^{18,20} and has been observed but not analyzed for the strong SO (AL) case.⁴ The solid theory curves in Figs. 1 and 2 include thermal smearing and decoherence effects and use parameters obtained from fits of RMT to $\langle g(B_{\perp}) \rangle$, plus one additional parameter, f_{var} (Table I) to normalize the $\text{var } g(B_{\perp} \neq 0)$ the RMT value. This factor compensates the assumption of multimode leads, $N \gg 1$, in the RMT,¹ whereas the experiment has $N = 2$. RMT for $\text{var } g$ in the $N = 2$ case has been given, but does not include SO or Zeeman terms.²¹

We next investigate the effect of an in-plane magnetic field on $\text{var } g$, focusing first on the case where TRS is broken by a small perpendicular field, $B_{\perp} \neq 0$. As seen in Figs. 3 and 4, $\text{var } g(B_{\perp} \neq 0, B_{\parallel})$ decreases with increasing B_{\parallel} and saturates at large B_{\parallel} , giving reduction factors $R = \text{var } g(B_{\perp} \neq 0, B_{\parallel} = 0) / \text{var } g(B_{\perp} \neq 0, B_{\parallel} \gg 0)$ between $R \sim 1.6$ for the large high-density dots (which show AL at $B_{\parallel} = 0$) and $R \sim 4$ for large low-density dots (which show WL at $B_{\parallel} = 0$). This range of values for R can be readily interpreted within RMT: For the large high-density dot (relatively strong SO coupling, not suppressed by confinement), Kramers degeneracy

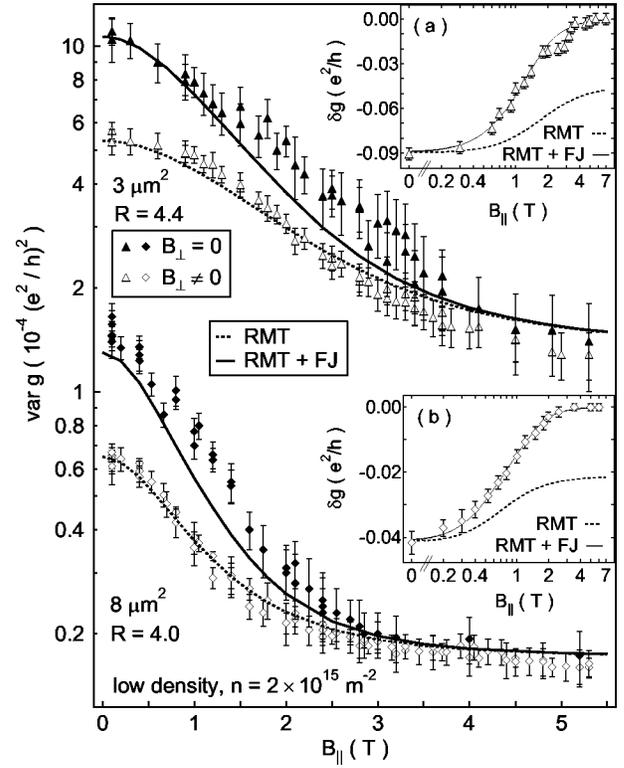


FIG. 4. As in Fig. 3, but for low-density dots. In this case, both dots have a reduction factor $R \sim 4$ in parallel field, consistent with RMT (curves). Orbital coupling of B_{\parallel} breaks TRS, making $\text{var } g$ independent of B_{\perp} and quenching the quantum correction to average conductance, $\delta g \rightarrow 0$ (insets) on the same (dot-size-dependent) scale of B_{\parallel} .

is broken whenever TRS is broken, and there is weak spin mixing ($\beta = 2, s = 1, 1 < \Sigma < 2$) at $B_{\parallel} = 0$. The effect of B_{\parallel} is to fully mix the Kramers pair ($\beta = 2, s = 1, \Sigma = 1$), thus the reduction $1 \leq R \leq 2$. On the other hand, dots showing WL at $B_{\parallel} = 0$ retain spin degeneracy ($\beta = 2, s = 2, \Sigma = 2$) at $B_{\parallel} = 0$, which is then lifted by Zeeman coupling ($\beta = 2, s = 1, \Sigma = 2$) and mixed at larger B_{\parallel} ($\beta = 2, s = 1, \Sigma = 1$) due to SO coupling revived by B_{\parallel} , giving $R \sim 4$.

Spin mixing induced by B_{\parallel} , marking the $\Sigma = 1$ to 2 crossover, occurs when a field-dependent energy scale ϵ_{\perp}^Z exceeds the level broadening $\tilde{\gamma} = \hbar(\tau_{\text{esc}}^{-1} + \tau_{\phi}^{-1})^{-1}$ ($\tau_{\text{esc}}^{-1} = N\Delta/h$ is the escape rate from the dot). This new energy scale depends on both Zeeman and SO coupling, $\epsilon_{\perp}^Z = \xi^2 \epsilon_Z^2 / (2E_T)(A/\lambda_{\text{so}}^2)$,¹ where $\epsilon_Z = g\mu_B B$ is the Zeeman splitting, E_T is the Thouless energy (for ballistic dots $E_T \sim \hbar v_F / \sqrt{A}$, where v_F is the Fermi velocity), and ξ is a parameter of order one that depends on dot geometry as well as the direction of B_{\parallel} .^{1,2} Note that $\epsilon_{\perp}^Z / \tilde{\gamma} \propto \epsilon_Z^2 A^{5/2}$ (when $\tau_{\text{esc}} \ll \tau_{\phi}$) so that the $\Sigma = 1$ to 2 crossover field will depend on dot size. For the smallest dot, the crossover is inaccessible, $\Sigma = 1$ for all measured fields and $R \sim 2$ due to breaking of Kramers degeneracy only ($s = 2$ to 1). In the low-density dots, the $\Sigma = 1$ to 2 crossover is accessible, occurring around $B_{\parallel} \sim 1(3)$ T for the larger (smaller) dot. The large high-density dot has $1 < \Sigma < 2$ already at $B_{\parallel} = 0$ due to SO coupling, and the crossover to $\Sigma = 2$ occurs around $B_{\parallel} \sim 3$ T. Because of the undetermined coefficient ξ , the SO

length λ_{so} cannot be independently extracted from $\text{var } g(B_{\parallel})$. Taking ξ as a single fit parameter, the dashed curves in Figs. 3 and 4 give the RMT results, which are in good agreement with experiment for all devices.

Finally, we investigate orbital effects of B_{\parallel} on $\text{var } g$, measured when TRS is not explicitly broken by a perpendicular field ($B_{\perp}=0$). Figures 3 and 4 show that as B_{\parallel} is increased, $\text{var } g(B_{\perp}=0, B_{\parallel})$ decreases sharply, approaching $\text{var } g(B_{\perp} \neq 0, B_{\parallel})$. At large B_{\parallel} , $\text{var } g$ becomes independent of B_{\perp} , whereas RMT gives $\text{var } g_{RMT}(B_{\perp}=0)/\text{var } g_{RMT}(B_{\perp} \neq 0)=2$ for all B_{\parallel} . On a similar scale of B_{\parallel} , WL corrections $\delta g(B_{\parallel}) = \langle g(B_{\perp}=0, B_{\parallel}) \rangle - \langle g(B_{\perp} \neq 0, B_{\parallel}) \rangle$ are also vanishing in all devices whereas RMT predicts a finite δg . As discussed previously,^{3,4,10} these effects result from the breaking of TRS by B_{\parallel} .^{8,9}

Following Fal'ko and Jungwirth⁸ (FJ) we account for the suppression of $\delta g(B_{\parallel})$ and $\text{var } g(B_{\parallel})$ by introducing a field-dependent factor $f_{FJ}(B_{\parallel}) = (1 + \tau_{B_{\parallel}}^{-1}/\tau_{esc}^{-1})^{-1}$, where $\tau_{B_{\parallel}}^{-1} \sim aB_{\parallel}^2 + bB_{\parallel}^6$. The B_{\parallel}^2 term reflects interface roughness and dopant inhomogeneities. The B_{\parallel}^6 term is due to the asymmetry of the well. The RMT results are then modified as $\delta g(B_{\parallel}) = \delta g_{RMT}(B_{\parallel})f_{FJ}(B_{\parallel})$ and $\text{var } g(B_{\perp}=0, B_{\parallel}) = \text{var } g_{RMT}(B_{\perp} \neq 0, B_{\parallel}) [1 + f_{FJ}(B_{\parallel})]$ to account for flux effects of the parallel field.^{9,10,22} The coefficient a is obtained from a fit to the

experimental $\delta g(B_{\parallel})$ while b is estimated from device simulations²² (Table I). The resulting theory curves for both $\delta g(B_{\parallel})$ (solid curves, insets in Figs. 3 and 4) and $\text{var } g(B_{\perp}=0, B_{\parallel})$ (solid curves, main panels) are in good agreement with experiment. We emphasize that the theoretical variance curves $\text{var } g(B_{\perp}=0, B_{\parallel})$ are not fitted to the data. Estimates of a, b based on correlation functions of parallel-field conductance fluctuations¹⁰ are consistent with the values obtained here based on $\delta g(B_{\parallel})$.

In summary, mesoscopic conductance fluctuations in open quantum dots in the presence of SO coupling and in-plane fields can be understood in terms of fundamental symmetries in the system, including observed partially broken spin rotation symmetries as well as time-reversal symmetry, which can be broken by both perpendicular and in-plane fields.

We thank I. Aleiner, B. Altshuler, P. Brouwer, J. Cremer, V. Fal'ko, J. Folk, B. Halperin, T. Jungwirth, and Y. Lyanda-Geller. This work was supported in part by DARPA-QuIST, DARPA-SpinS, ARO-MURI, and NSF-NSEC. We also acknowledge support from NDSEG (J.B.M.) and the Harvard Society of Fellows (D.G.-G). Work at UCSB was supported by QUEST, a NSF Science and Technology Center.

- ¹I. L. Aleiner and V. I. Fal'ko, Phys. Rev. Lett. **87**, 256801 (2001); J.-H. Cremer, P. W. Brouwer, and V. I. Fal'ko, Phys. Rev. B **68**, 125329 (2003).
- ²B. I. Halperin, A. Stern, Y. Oreg, J. N. H. J. Cremer, J. A. Folk, and C. M. Marcus, Phys. Rev. Lett. **86**, 2106 (2001).
- ³J. A. Folk, S. R. Patel, K. M. Birnbaum, C. M. Marcus, C. I. Duruöz, and J. S. Harris, Jr., Phys. Rev. Lett. **86**, 2102 (2001).
- ⁴D. M. Zumbühl, J. B. Miller, C. M. Marcus, K. Campman, and A. C. Gossard, Phys. Rev. Lett. **89**, 276803 (2002).
- ⁵A. V. Khaetskii and Y. V. Nazarov, Phys. Rev. B **61**, 12639 (2000); **64**, 125316 (2001).
- ⁶T. Fujisawa, D. G. Austing, Y. Tokura, Y. Hirayama, and S. Tarucha, Nature (London) **419**, 278 (2002); J. M. Elzerman, R. Hanson, L. H. W. van Beveren, B. Witkamp, L. M. K. Vandersypen, and L. P. Kouwenhoven, *ibid.* **430**, 431 (2004); V. N. Golovach, A. Khaetskii, and D. Loss, Phys. Rev. Lett. **93**, 016601 (2004).
- ⁷P. W. Brouwer, J. N. H. J. Cremer, and B. I. Halperin, Phys. Rev. B **65**, 081302(R) (2002).
- ⁸V. I. Fal'ko and T. Jungwirth, Phys. Rev. B **65**, 081306(R) (2002).
- ⁹J. S. Meyer, A. Altland, and B. L. Altshuler, Phys. Rev. Lett. **89**, 206601 (2002).
- ¹⁰D. M. Zumbühl, J. B. Miller, C. M. Marcus, V. I. Fal'ko, T. Jungwirth, and J. S. Harris, Jr., Phys. Rev. B **69**, 121305(R) (2004).
- ¹¹A. G. Mal'shukov, K. A. Chao, and M. Willander, Phys. Rev. B **56**, 6436 (1997); G. M. Minkov, A. V. Germanenko, O. E. Rut, A. A. Sherstobitov, L. E. Golub, B. N. Zvonkov, and M. Willander, *ibid.* **70**, 155323 (2004).
- ¹²F. E. Meijer, A. F. Morpurgo, T. M. Klapwijk, T. Koga, and J. Nitta, Phys. Rev. B **70**, 201307(R) (2004); F. E. Meijer, A. F. Morpurgo, T. M. Klapwijk, and J. Nitta, Phys. Rev. Lett. **94**, 186805 (2005).
- ¹³Y. Meir, Y. Gefen, and O. Entin-Wohlman, Phys. Rev. Lett. **63**, 798 (1989).
- ¹⁴Y. B. Lyanda-Geller and A. D. Mirlin, Phys. Rev. Lett. **72**, 1894 (1994).
- ¹⁵B. L. Al'tshuler, JETP Lett. **41**, 648 (1985); B. L. Al'tshuler and B. I. Shklovskii, Sov. Phys. JETP **64**, 127 (1986); S. Feng, Phys. Rev. B **39**, 8722 (1989); A. D. Stone, Phys. Rev. B **39**, 10736 (1989); C. W. J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).
- ¹⁶N. O. Birge, B. Golding, and W. H. Haemmerle, Phys. Rev. Lett. **62**, 195 (1989); P. Debray, J.-L. Pichard, J. Vicente, and P. N. Tung, *ibid.* **63**, 2264 (1989); O. Millo, S. J. Klepper, M. W. Keller, D. E. Prober, S. Xiong, A. D. Stone, and R. N. Sacks, *ibid.* **65**, 1494 (1990); Y. K. Fukai, H. Nakano, S. Nakata, S. Tarucha, and K. Arai, Solid State Commun. **94**, 757 (1995); J. S. Moon, N. O. Birge, and B. Golding, Phys. Rev. B **56**, 15124 (1997).
- ¹⁷B. Hackens, C. Gustin, V. Bayot, and M. Shayegan, Physica E (Amsterdam) **12**, 833 (2002); C. Gustin, S. Faniel, B. Hackens, E. P. DePoortere, M. Shayegan, and V. Bayot, *ibid.* **17**, 154 (2003).
- ¹⁸I. H. Chan, R. M. Clarke, C. M. Marcus, K. Campman, and A. C. Gossard, Phys. Rev. Lett. **74**, 3876 (1995).
- ¹⁹A. G. Huibers, M. Switkes, C. M. Marcus, K. Campman, and A. C. Gossard, Phys. Rev. Lett. **81**, 200 (1998); A. G. Huibers, J. A. Folk, S. R. Patel, C. M. Marcus, C. I. Duruöz, and J. S. Harris, Jr., *ibid.* **83**, 5090 (1999).
- ²⁰A. G. Huibers, S. R. Patel, C. M. Marcus, P. W. Brouwer, C. I. Duruöz, and J. S. Harris, Jr., Phys. Rev. Lett. **81**, 1917 (1998).
- ²¹E. R. P. Alves and C. H. Lewenkopf, Phys. Rev. Lett. **88**, 256805 (2002).
- ²²V. Fal'ko and T. Jungwirth (private communication).