

Figure 4.27: Essential components of a ${ }^{3} \mathrm{He} /{ }^{4} \mathrm{He}$ dilution refrigerator.

### 4.3 Electronic measurements on nanostructures

Measuring the resistances and conductances of a sample requires the application of currents and/or voltages, as well as the detection of voltage drops and/or currents, respectively. Conceptually, these measurements are very simple. The greatest efforts in practice are usually related to the reduction of the electronic noise level. This is done by avoiding ground loops, filtering, and choosing the right cables, among other important issues. As in the previous section, we are not that much interested in these technical details. This topic has been dealt with in great detail in excellent books (see the reference section at the end of this chapter). Our goal here is to present in brief some basic setups, just enough for the reader to know what type of setup has been used in the experiments to be discussed. In the previous section, we have seen that the cryostats available set some limitations to the temperature range. Likewise, the measurement setup limits the physical quantities, as well as their ranges, that can be measured. The present section, together with the previous one, should put us in a position to judge why a particular experimental setup has been used, and how it affects the parameter ranges. We begin by showing how the samples are actually mounted in the low-temperature environment, before we discuss the most important electronic measurement setups.

### 4.3.1 Sample holders



Figure 4.28: Sketch of a sample holder used to stick a specimen in the sample space of a cryostat.

A sample holder contains the sample in an appropriate way, and is mounted in the sample space of the cryostat. Its basic components are sketched in Fig. 4.28. The sample is mounted in some kind of carrier, which is placed inside the cryostat, in the center of the magnetic field. Cables are brought into the sample space via a vacuum feedthrough at the top end. Typically, the wires run in twisted pairs, which reduces the currents induced by the magnetic field due to vibrations, since the magnetic flux through adjacent loops points in opposite directions. Furthermore, the sample holder contains baffles, i.e., polished metal plates which reflect the thermal radiation from the top. Some sample holder are equipped with a rotator, which permits the sample to be tilted with respect to the magnetic field (which points in the vertical direction in most cryostats).

### 4.3.2 Application and detection of electronic signals

## General considerations

For many experiments, measuring in a low temperature environment only makes sense when the electric signals are kept sufficiently small. Suppose, for example, we plan to investigate the transmission properties of a tunnel barrier. The low temperature reduces the thermal smearing of the Fermi function, which corresponds to an energy scale of $\delta E=3.52 k_{B} \Theta \approx$ $300 \mu \mathrm{eV} / \mathrm{K} \cdot \Theta$. Therefore, the voltage drop across the barrier at, e.g., $\Theta=1 \mathrm{~K}$, should be small compared to $300 \mu V$. For larger voltage drops, the temperature does no longer determine the energy resolution.

Measurements can be performed AC or DC . AC measurements have the advantage that a lock-in amplifier can be used, a device that selectively detects signals with the source frequency, within a narrow band width. In addition, phase-sensitive measurements are possible, such that, e.g., capacitance measurements can be performed by measuring the voltage drop with a phase shift of $\pi / 2$ with respect to the source signal. Although the frequency selection greatly reduces the noise, it is not always best to use an AC signal. For example, imagine the sample has a very large resistance, such that the capacitances, which are always present in the leads, cause significant phase shifts. This makes it hard to determine the resistive part of the impedance. Also, theoretical results are often obtained for DC transport.

Furthermore, it has to be clearly distinguished between the resistance and the resistivity (conductance and conductivity, respectively). The plain result of, say, applying a current $I$ and
measuring the voltage drop $\Delta V$ is the resistance $R=\Delta V / I$. If the sample is macroscopic, we can assume that the voltage drops homogeneously in between the voltage probes, and we can translate the resistance into a resistivity, an intrinsic property of the sample, by taking the sample geometry into account. This is no longer true in mesoscopic samples. Here, the measurement does not average over a large volume of randomly distributed scatterers, and the sample simply does not have a resistivity.

## Voltage and current sources



Figure 4.29: A voltage divider (left) and a voltage to current conversion (right).
High quality commercial voltage sources typically provide voltages in the regime of Volts, with an accuracy of, say $10^{-6}$. Hence, some conversion to smaller voltages, or to a small current, is often necessary. This is done by a voltage divider, or a voltage-to-current conversion, respectively, see Fig. 4.29. The voltage divider simply consists of two resistors in series connected to the output voltage $V_{S}$ of the commercial voltage source. The potential in between the two resistors is applied to the sample with respect to ground. This voltage is given by

$$
V_{\text {out }}=V_{S} \frac{R_{2}}{R_{1}+R_{2}}
$$

In order to divide $V_{S}$ by a few orders of magnitude, $R_{1}$ must be much larger than $R_{2}$. This immediately implies an experimental limitation, since $R_{1}$ adds to the effective internal resistance of the voltage source. Connecting a sample with a resistance of $R_{s}$, causes the applied voltage to drop to

$$
V_{\text {out }}=\frac{R_{2}}{R_{1}+R_{2}+\frac{R_{1} R_{2}}{R_{3}}}
$$

The circuit to the left in Fig. 4.29 is only a good voltage source for $R_{s} \gg R_{1} . R_{1}$ should be chosen as small as possible. The required output voltage then determines $R_{2}$. These resistors cannot be arbitrarily small, however, since a minimum current of $I=V_{S} /\left(R_{1}+R_{2}\right)$ must be provided by the voltage source. Hence, only samples of high resistances should be voltagebiased with such a setup.

On the other hand,

$$
V_{\text {out }}(t)=a_{0}\left(V_{\text {in }}^{+}-V_{\text {in }}^{-}\right)=-\frac{a_{0}}{C_{-}} \int_{t^{\prime}=0}^{t} I_{-}\left(t^{\prime}\right) d t^{\prime}
$$

We differentiate this expression with respect to $t$ and substitute $I_{-}(t)$ with the previous equation. This gives

$$
-\frac{C_{-}}{a_{0}} \frac{d V_{\text {out }}}{d t}=I_{\text {in }}(t)+\frac{1}{R} V_{\text {out }}(t)
$$

The left hand side is approximately zero, due to the large open loop gain. Consequently,

$$
V_{\text {out }}(t)=-R I_{i n}(t)
$$

The input current is converted into a voltage with a conversion ratio determined by the resistor. Its resistance can be very high, like $R \approx 1 \mathrm{G} \Omega$, since the condition is that R must be small compared to the input impedance. Thus, the output voltage adjusts in such a way that there is no charge buildup at the input. The current is effectively drained at the "-" - input. For this reason, the "-" input is sometimes referred to as virtual ground. In cryogenic experiments, the conversion resistor $R$ is sometimes mounted inside the cryostat, in order to reduce the thermal noise.

## Some important measurement setups



Figure 4.32: (a): two-point and four point resistance measurements. (b): setup for a conductance measurement.

As we have just seen, low-resistance samples should be investigated by applying a current and detecting the voltage drop (Fig. 4.32). This can be done in a two-probe configuration, where the voltage drop is measured at the connections used to apply the current and ground the sample. This has the disadvantage that the not only the sample is measured, but also leads and the contacts, which, in case of a 2DEG, are the Ohmic contacts between the sample surface and the electron gas. In a quasi four-probe setup, two wires are connected to both contacts used. Applying a current $I_{\text {in }}$ in Fig: 4.32(a) and measuring the voltage between
$4 a$ and $4 b$ eliminates the wire resistance, but not the contact resistances. Therefore, a true four-probe configuration is preferable, where the contacts used for measuring the voltage are different from those used to pass the current through the sample. In Fig. 4.32(a), this setup corresponds to measuring $V_{4 c}-V_{4 d}$. True four-probe setups are not always possible, though. It is then difficult or even impossible to discriminate between the contact resistances, which may be quite high, and the resistance of the sample. Note that in a two-probe measurement, we measure $R_{x x}+R_{x y}$. The individual components of the resistivity tensor can be measured only with the corresponding four-probe configurations.

A conductance measurement, on the other hand, is usually a 2-terminal experiment. Here, the current meter is in series with the sample. This setup is preferable for samples with a high resistance above $\approx 1 \mathrm{M} \Omega$.

Sometimes, it is convenient to be able to measure a differential quantity, such as the differential conductance $d I / d V$, or the transconductance $t=d I / d V_{G}$, of a transistor. Here, $I$ is the source-drain current, $V$ the source drain voltage, and $V_{G}$ denotes the gate voltage. This can be done by superposing a small AC voltage to the DC voltage that is tuned, and detect only those signals with the superimposed frequency with a lock-in amplifier. Schematic setups are shown in Fig. 4.33. Such differential measurements often give a higher resolution and a lower noise level, since the lock-in technique can be used where absolute measurements must be performed DC. Of course, we obtain the current as a function of the gate voltage, or of the source-drain voltage, by simple numerical integration of the measured differential trace.


Figure 4.33: Setup for measuring the differential conductance (a), and the transconductance of a field effect transistor (b).

