Fault-tolerant thresholds for the surface code in excess of 5% under biased noise

David K. Tuckett, Stephen D. Bartlett, Steven T. Flammia, and Benjamin J. Brown
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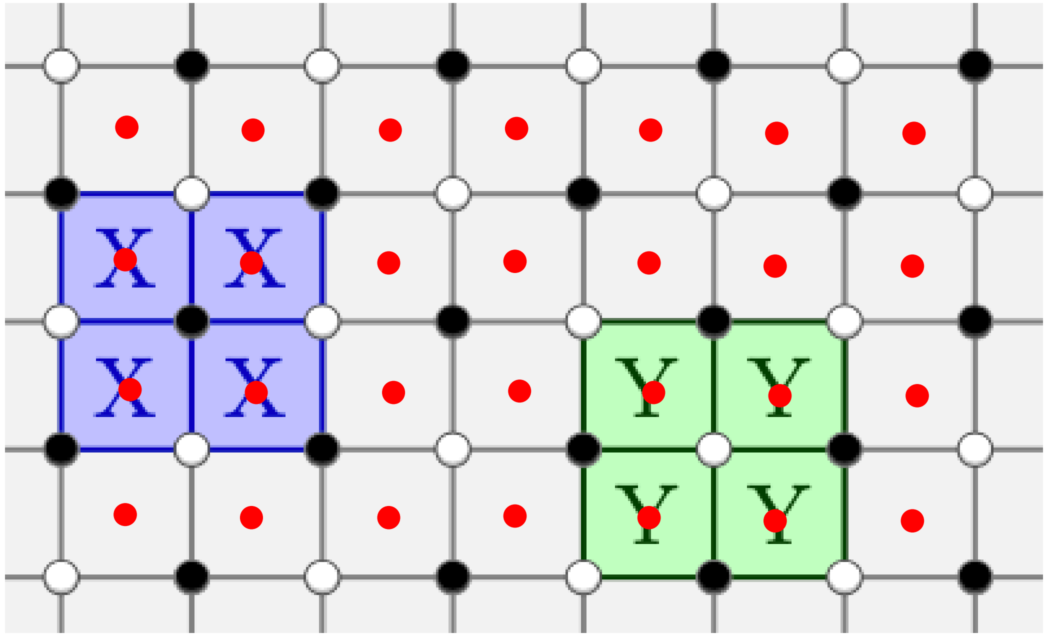
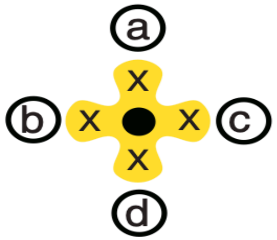
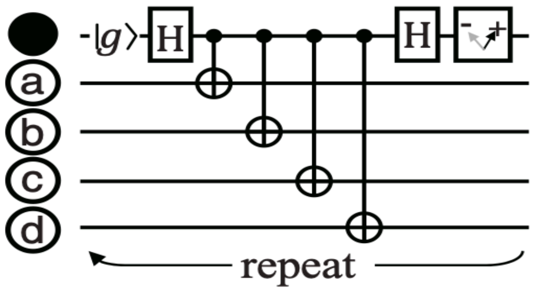
Surface code 101

- d^2 data qubits
- d^2-1 ancillas (always initialized)
  - 2 group: **X** and $\mathbb{Y}$ stabilizers
  - e.g., $S_{14} = X_1X_2X_3X_4$
- Stabilizer measurements
  - measure every stabilizer (mutually commuting)
- Computational subspace: all stabilizers are +1



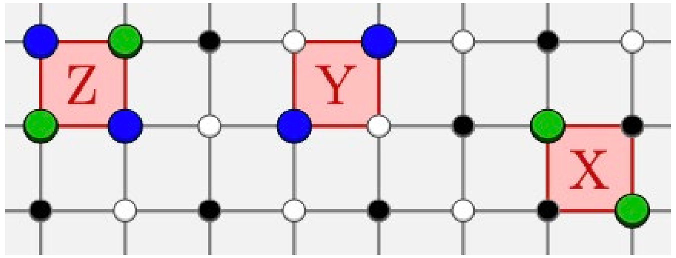


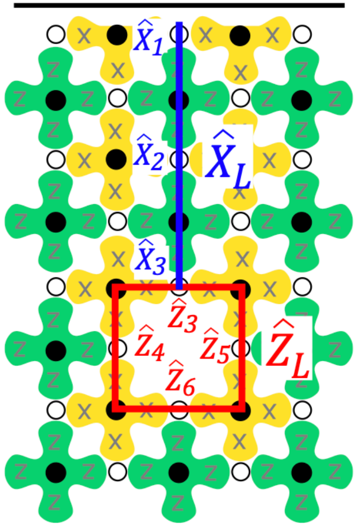


A.G. Fowler *et al.*, PRA **86**, 032324 (2012)

Surface code 101

- Error syndromes: stabilizer measurement “-1”
  - single errors → out of the computational subspace
- Decoders: error map from the syndromes
  - fault tolerance threshold
  - 3.3% for no error bias
- Logical qubit is encoded in a multiqubit state
  - d^2 data qubits and $d^2 - 1$ constraints…
  - switch off stabilizers → extra degrees of freedom
  - logical operations





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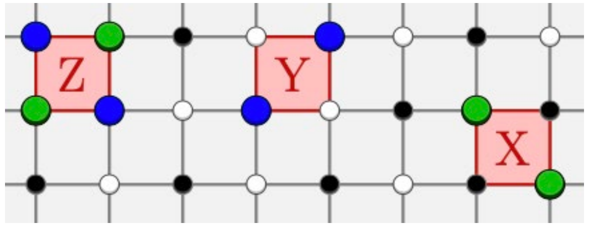
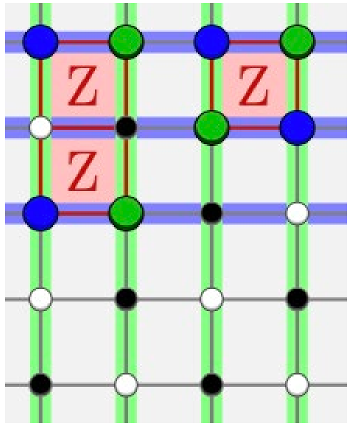
Error syndromes and symmetries



- Z error is detected by both X and Y stabilizers

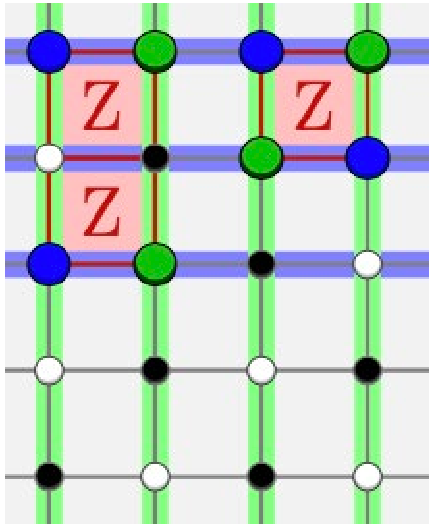
- 1D symmetries (infinite bias)
  - E-insensitive stabilizers: $SE|\psi\rangle = (+1)E|\psi\rangle \quad E \in \mathcal{E}_Z$
  - conserved quantities: defect parity per column (row)



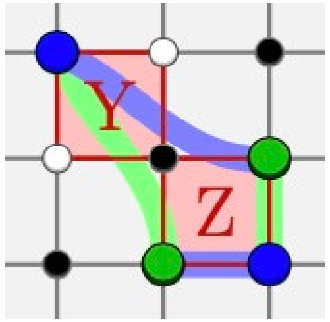
- Symmetry breaking:
  - finite error bias $\eta = p_z/(p_x + p_y)$
  - measurement error p
  - finite lattice (different syndromes on the edge)

Decoding algorithm

- Infinite bias + periodic boundary conditions
  - “1D” parity conservation
  - Horizontal (H) and Vertical (V) vertices
  - minimum-weight perfect matching (MWPM)

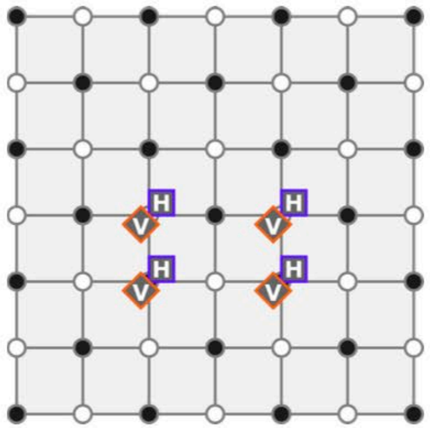


- Accounting for symmetry breaking
  - measurement errors
    - syndrome disappears in the next cycle
  - X, Y errors
    - high-weight diagonal paths



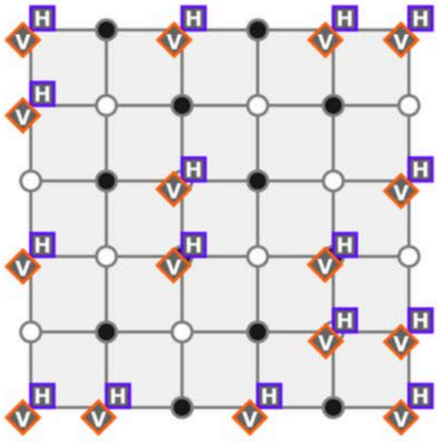
Simple example of syndrome decoding

- Infinite bias (dephasing only)
- No measurement errors
- Periodic lattice



Not quite so simple example of decoding (overlaping syndromes)

- Infinite bias
- No measurement errors
- Finite lattice



Noise model

- Noise in terms of error probability and bias

$$p_Z = p\eta/(\eta + 1)$$
$$p_{XY} = p/2(\eta + 1)$$

- Measurement noise: phenomenological noise model
  - Z errors on the ancillas do not cause X-Y errors on the data qubits

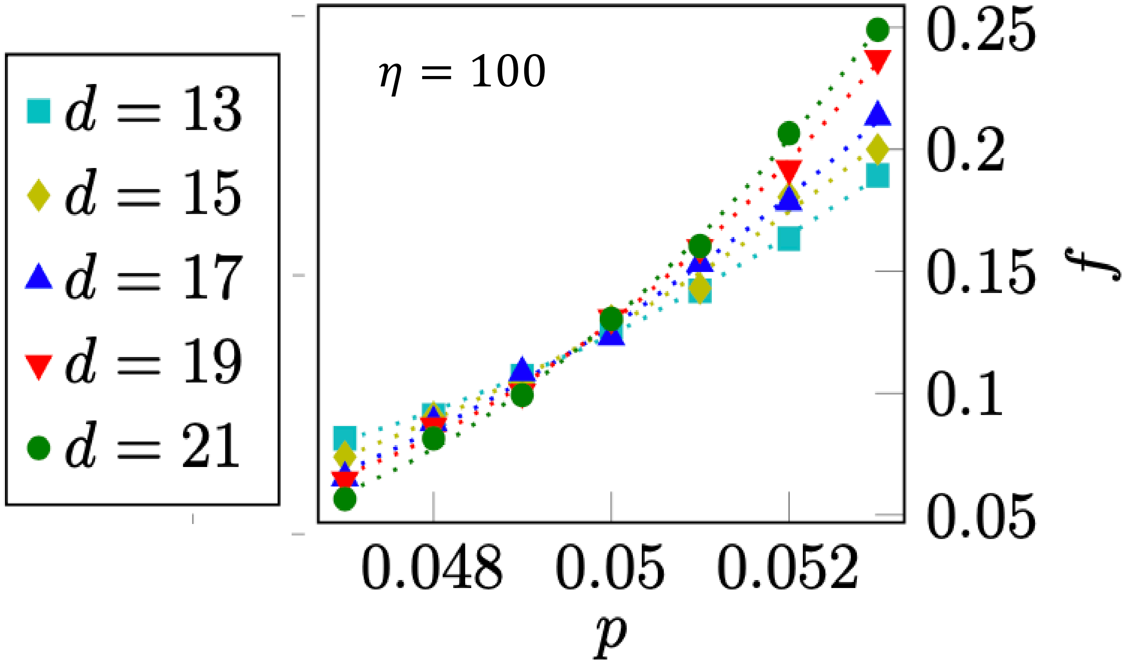
$$p_M = p$$

Results – fault tolerance threshold

- How to obtain the error threshold?
- Logical failure rate

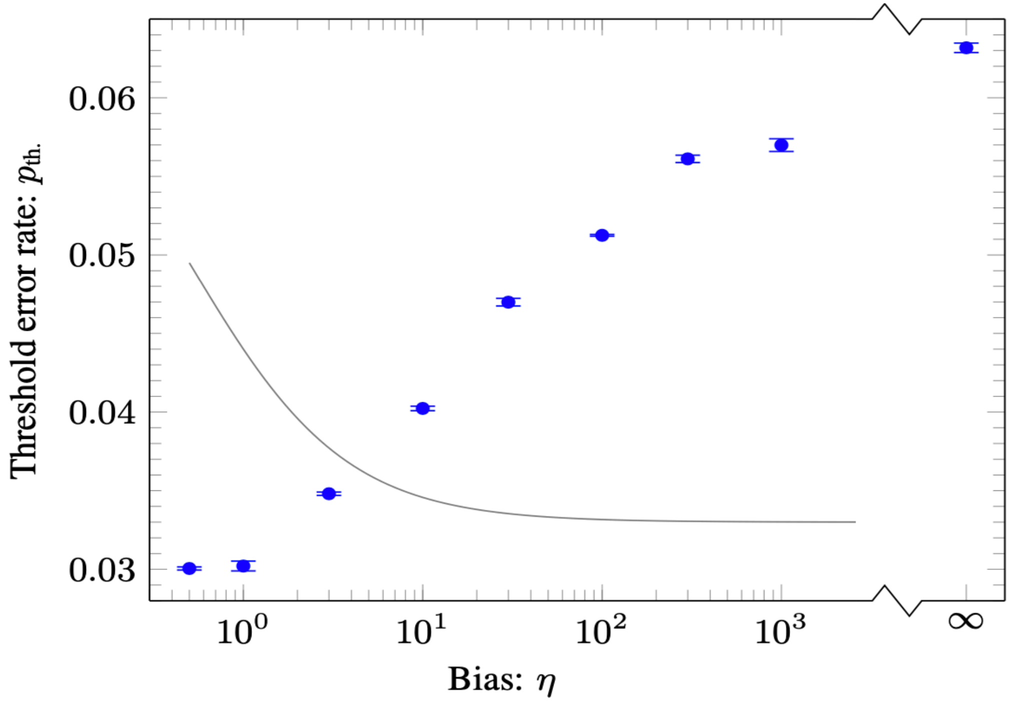
$$f = f_{th.}\, e^{-a(p-p_{th.})^{\nu} d}$$

- f close to threshold
    - expand to quadratic order
    - fit the dependence on p



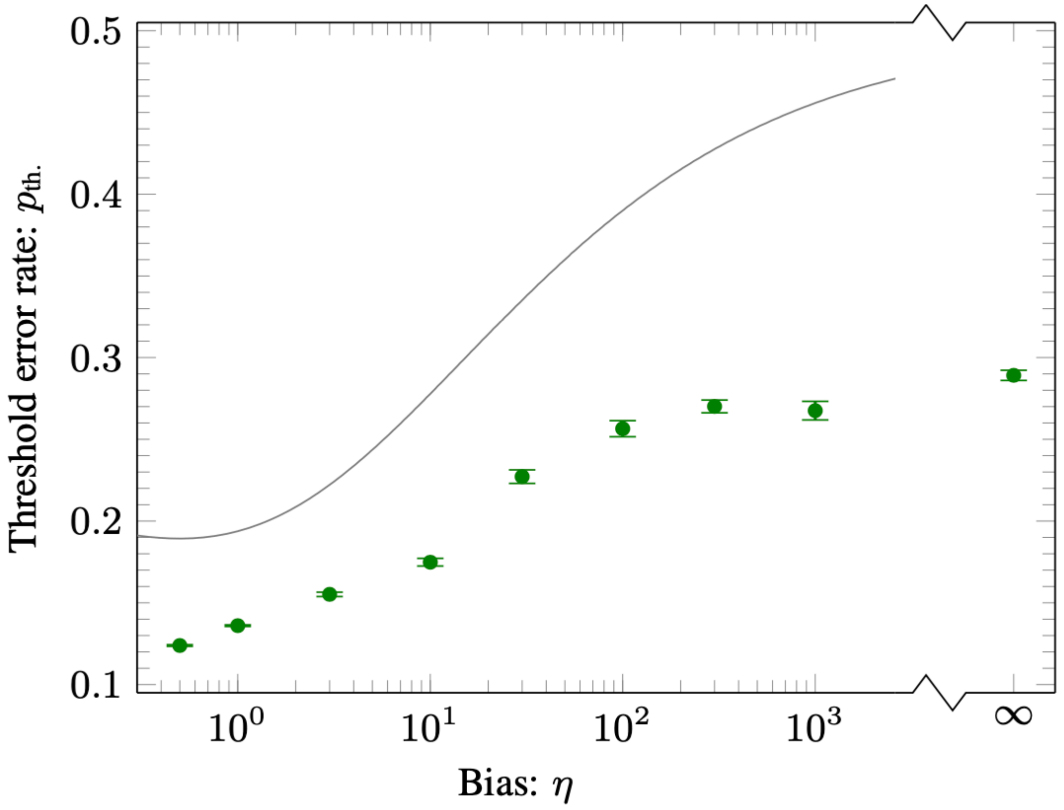
Results – bias dependence of the error threshold

- Fault tolerance threshold: p_{th}.
  - with measurement errors
  - periodic boundary conditions
- New decoder outperforms the unbiased algorithm (solid)
- Boundaries slightly change p_{th}.



Results – optimal fault tolerance threshold

- No measurement errors + periodic BC
- Maximum likelihood decoder (solid)
  - 50% for infinite error bias
  - 18% for no bias
- Room for improvement for this MWPM decoder



Logical failure for low error rate ($p \ll 1/d$)

- Conventional decoders $f \sim \mathcal{O}(p^{\delta d})$
- High threshold at infinite bias:
  - might tolerate up to $n/2$ errors
  - can improve the scaling to $f \sim \mathcal{O}(p_Z^{\alpha d^2})$
  - for large d, the scaling will be dominated by $f \sim \mathcal{O}(p_{XY}^{\delta d})$

My thoughts/comments

- External boundaries do not change the threshold
  - What about internal boundaries (logical qubit)?

- Improvement in low-error logical failure scaling is (admittedly) speculative
  - In my opinion: just as important as the threshold