Beating the thermal limit of qubit initialization with a Bayesian ”Maxwell’s Demon”

Mark A. I. Johnson,¹,² Mateusz T. Mądzik,¹,² Fay E. Hudson,¹ Kohei M. Itoh,³ Alexander M. Jakob,⁴,² David N. Jamieson,⁴,² Andrew Dzurak,¹ and Andrea Morello¹,²,*

¹School of Electrical Engineering and Telecommunications, UNSW Sydney, Sydney, NSW 2052, Australia
²Centre of Excellence for Quantum Computation & Communication Technology
³School of Fundamental Science and Technology, Keio University, Kohoku-ku, Yokohama, Japan
⁴School of Physics, University of Melbourne, Melbourne, VIC 3010, Australia

(Dated: October 6, 2021)
Motivation

Fidelity requirements of Surface Code

• single- and two qubit gate fidelities: 1 >99%
• state preparation and measurement (SPAM) fidelity: 1 >99%

• systems based on energy-selective tunnelling: ultimately limited by temperature

• readout fidelity: can be improved by QND measurement\(^2\)

• here: overcome temperature limit of initialization
  • for high-temperature operation
  • or higher fidelity at lower temperature

2J. Yoneda et al., Nat. Commun. 11, 1144 (2020)
Basic idea

Maxwell’s Demon

• thought experiment: use a “demon” to separate hot and cold gas particles

„qubit initialization“ demon

• load cold electrons into the QD and leave warm electrons in the reservoir
• beat theoretical limit for qubit initialization fidelity

Device: donor QD

- $^{31}\text{P}$ donor in enriched $^{28}\text{Si}$
- MW antenna for ESR and NMR
- SET as reservoir and charge sensor (50kHz)
- control setup via FPGA at 100MHz

MW antenna

SET

~20mK

DG3

DG4

SPL

B $\sim$1.4T

E

$E_{C1}$ $E_{C2}$ $E_F$
Qubit initialization

Spin-dependent tunneling

- Probability for loading $|\downarrow\rangle$ or $|\uparrow\rangle$ is given by Fermi distribution:
  \[
  f(E) = \left(1 + \exp \left(\frac{E - E_F}{k_B T_e}\right)\right)^{-1}
  \]
- If $E_z \gg k_B T \rightarrow$ mostly $|\downarrow\rangle$ is loaded
- Probability to load $|\downarrow\rangle$ is limited by finite $T$
  \[P(\downarrow) = f(E_{\downarrow})\]
Bayesian Maxwell’s Demon

Negative-outcome measurement

- if $|\uparrow\rangle$ is loaded, electron tunnels out fast since $\Gamma_{\uparrow,\text{out}} \gg \Gamma_{\downarrow,\text{out}}$

- absence of a tunneling event provides information on spin state without destroying the state

---

Bayesian Maxwell’s Demon

Discrete Bayesian model

- integration time $T_s = 10\mu s$
- $N$ number of measurements
- measurement outcome
  - $B =$ tunneling detected
  - $\neg B =$ no tunneling detected
- prior probability $P(\downarrow) = f(E_\downarrow)$
- probability to not observe a tunneling event:
  \[
  \mathcal{L}(\neg B_1 | \uparrow) = e^{-T_s \Gamma_{\uparrow,\text{out}}}
  \]
  \[
  \mathcal{L}(\neg B_1 | \downarrow) = e^{-T_s \Gamma_{\downarrow,\text{out}}}
  \]
- after $N$ measurements with outcome $\neg B$
  posterior probability is
  \[
  P(\downarrow | \neg B^N) = \frac{\mathcal{L}(\neg B^N | \downarrow) P(\downarrow)}{\mathcal{L}(\neg B^N | \downarrow) P(\downarrow) + \mathcal{L}(\neg B^N | \uparrow) P(\uparrow)}
  \]
  \[
  = \frac{1}{1 + \frac{\mathcal{L}(\neg B^N | \uparrow) P(\uparrow)}{\mathcal{L}(\neg B^N | \downarrow) P(\downarrow)}}
  \]
  \[
  = \left(1 + \frac{1 - P(\downarrow)}{P(\downarrow)} e^{-NT_s (\Gamma_{\uparrow,\text{out}} - \Gamma_{\downarrow,\text{out}})}\right)^{-1},
  \]
  goes towards 1 for large $N$.
Bayesian Maxwell’s Demon

(a) Empty and Reload

(b) i) $|\uparrow\rangle, |\uparrow\rangle$

ii) $|\uparrow\rangle, |\uparrow\rangle$

iii) $\text{Reload electron}$

iv) $|\uparrow\rangle, |\uparrow\rangle$

$\sim 20\text{ms}$

$P(1|\neg B^n)$
$P(\downarrow|\neg B^n)$

SET current
Virtual gate
Total fidelity $F = F_I F_C F_R$

- initialization fidelity: $F_I$
- NMR control\(^1\) fidelity: $F_C \sim 99.5\%$
- QND readout\(^2\) fidelity: $F_R \sim 99.99\%$
- additionally: missed blips on detector $P_M$

Fidelity increase:
$F_I(0) \sim 78\%-80\%$
$F_I = 98.9\%$

\(^1\)S. Freer \textit{et al.}, Quantum Science and Technology \textbf{2}, 015009 (2017)
Fidelity robustness

- detune initialization level $\mu_D$
- initialization with feedback shows plateau of width $\sim E_Z/2$

Fidelity robustness against detuning increases
Temperature

„Effective“ temperature

\[
\Gamma_{\uparrow,\text{in}}(T_{\text{eff}}) = \frac{2\pi}{\hbar} |\langle \uparrow | H' | 0 \rangle|^2 n(E_{\uparrow}) f(E_{\uparrow})
\]

\[
\Gamma_{\downarrow,\text{in}}(T_{\text{eff}}) = \frac{2\pi}{\hbar} |\langle \downarrow | H' | 0 \rangle|^2 n(E_{\downarrow}) f(E_{\downarrow})
\]

- fails to reproduce fidelity
- phenomenological parameter

\[
\chi = \frac{n(E_{\uparrow}) |\langle \uparrow | H' | 0 \rangle|^2}{n(E_{\downarrow}) |\langle \downarrow | H' | 0 \rangle|^2} \approx 0.388
\]

\[
\mathcal{F}_{\text{NF}}^{1} = \frac{1}{1 + R_{\text{in}}} = \frac{1}{1 + \chi \frac{f(E_{\uparrow})}{f(E_{\downarrow})}}
\]
Limitations

Limiting factors of the detector\textsuperscript{1}

- sample rate: 100MS/s
- signal-to-noise: $>>1$
- LP filter: 50kHz

- 99.9% initialization fidelity possible with
  $\sim300$kHz bandwidth
  or $\sim880$Hz electron tunneling rate