

# Two-dimensional Shiba lattices as a possible platform for crystalline topological superconductivity

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Journal Club, 27.02.2026,

Andrea Chieppa

# Scanning Tunnelling Microscopy

$$R_{A \rightarrow B} = \frac{2\pi}{\hbar} \sum_{k, k', \sigma} |T_{kk'}|^2 \{ f(E_A)(1 - f(E_B)) - f(E_B)(1 - f(E_A)) \} \delta(E_B + eV - E_A)$$

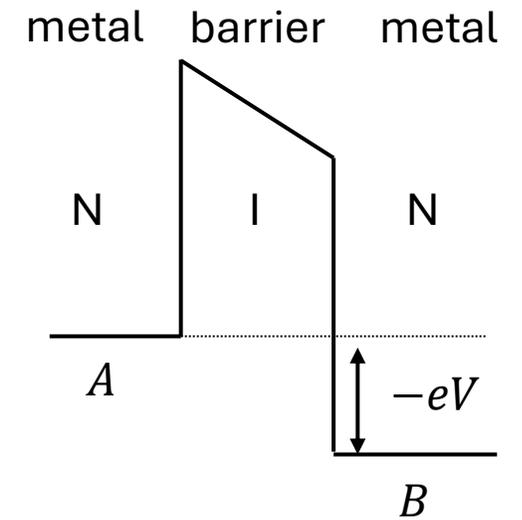
$$R_{A \rightarrow B} = \frac{2\pi}{\hbar} \sum_{\sigma} \int d\epsilon d\epsilon' D_A(\epsilon) D_B(\epsilon') |T(\epsilon, \epsilon')|^2 \{ f(\epsilon)(1 - f(\epsilon')) - f(\epsilon')(1 - f(\epsilon)) \} \delta(\epsilon' - \epsilon + eV)$$

$$R_{A \rightarrow B} \propto I \propto \int d\epsilon' D_B(\epsilon') \{ f(\epsilon' + eV) - f(\epsilon') \}$$

With an STM you can:

1. Measure the density of states locally (LDOS)
2. Fix the current, measure topography  $I \propto e^{-x/l}$

In the following,  $T = 500 \text{ mK} \leftrightarrow 50 \text{ } \mu\text{V}$



# Magnetism and Superconductivity

Phys. Rev. Lett. 1, 92, 1958

## SPIN EXCHANGE IN SUPERCONDUCTORS

B. T. Matthias, H. Suhl, and E. Corenzwit

Bell Telephone Laboratories,  
Murray Hill, New Jersey  
(Received July 15, 1958)

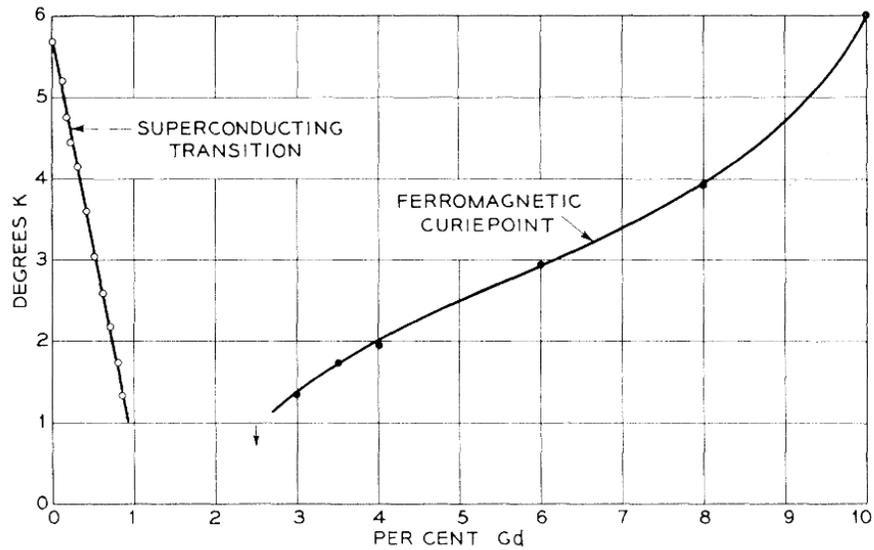


FIG. 3. Ferromagnetic and superconducting transition temperatures of solid solutions of gadolinium in lanthanum.

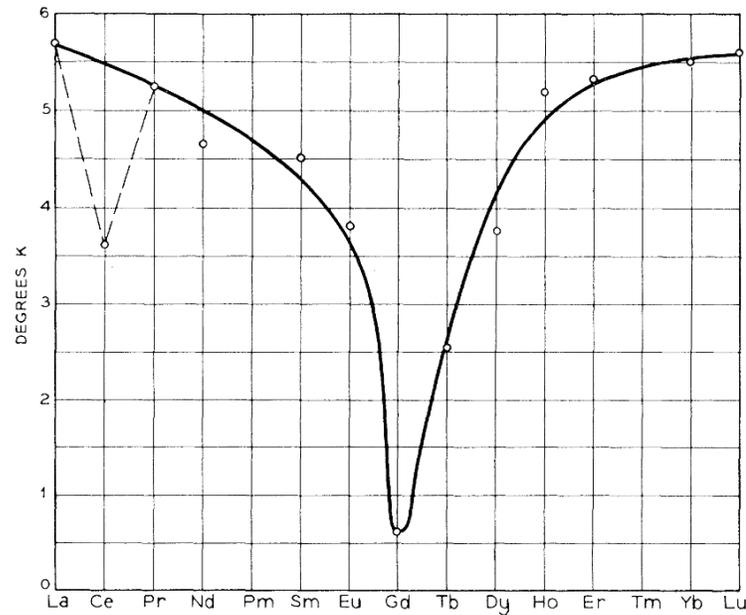


FIG. 2. Superconducting transition temperatures of 1 at % rare earth solid solutions in lanthanum.

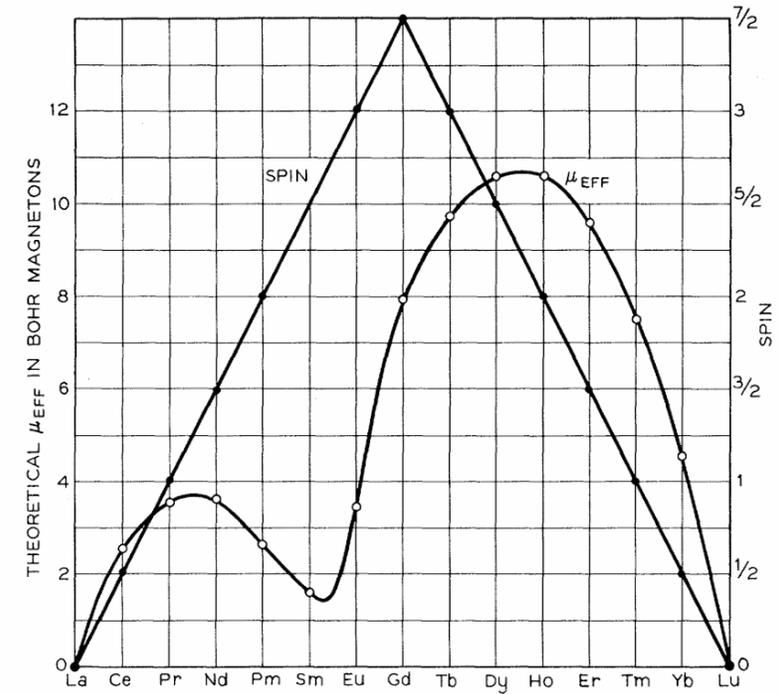


FIG. 1. Effective magnetic moments and spins of the rare earth elements (see reference 2).

# Matthias rules

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[Read](#) [Ed](#)

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From Wikipedia, the free encyclopedia

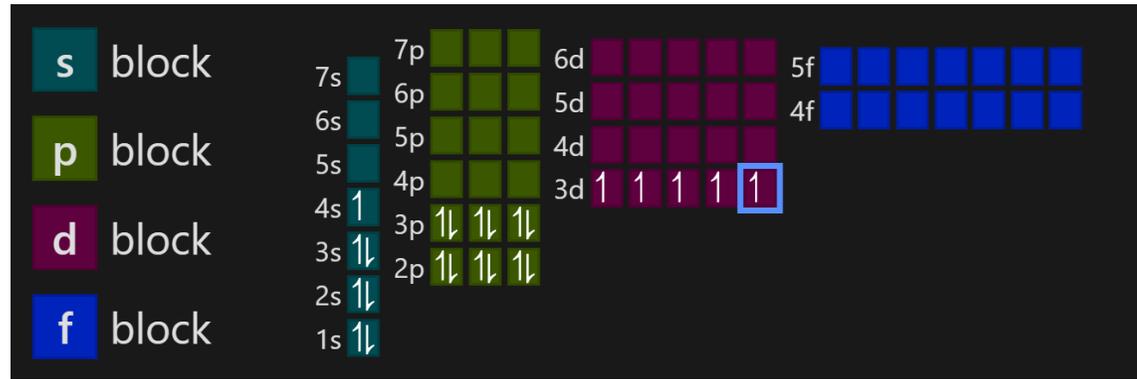
In [physics](#), the **Matthias rules** refers to a historical set of empirical guidelines on how to find <sup>conventional</sup> [superconductors](#).

1. High [symmetry](#) is good, [cubic symmetry](#) is the best.
2. High density of electronic states is good.
3. Stay away from [oxygen](#).
4. Stay away from [magnetism](#)
5. Stay away from [insulators](#).
6. Stay away from [theorists](#)!

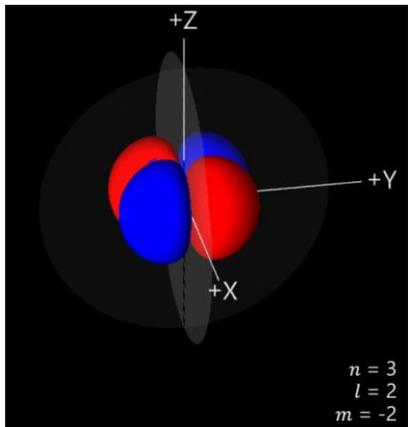
# Structure of the Journal Club

- Single atom of Cr on Nb
  - Lattices of Cr atoms on Nb
  - Theory work
- Yu-Shiba-Rusinov states
  - Yu-Shiba-Rusinov bands
  - Crystalline Topological Superconductivity

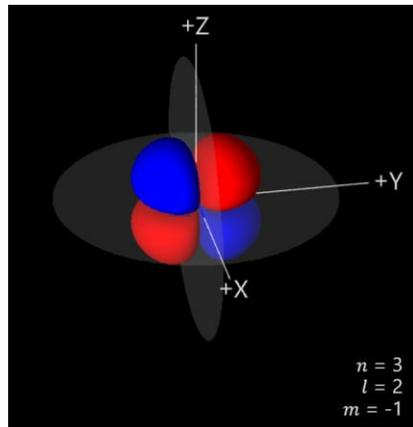
# The isolated chromium atom



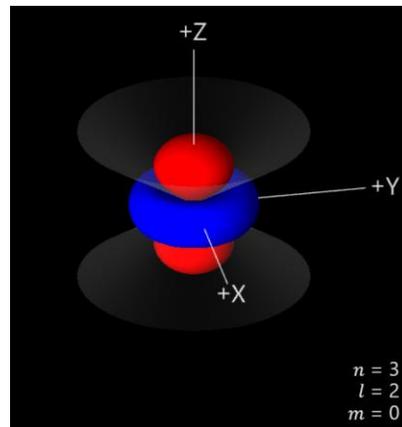
$d_{xy}$



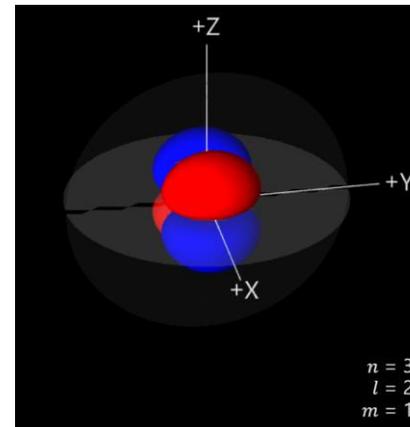
$d_{yz}$



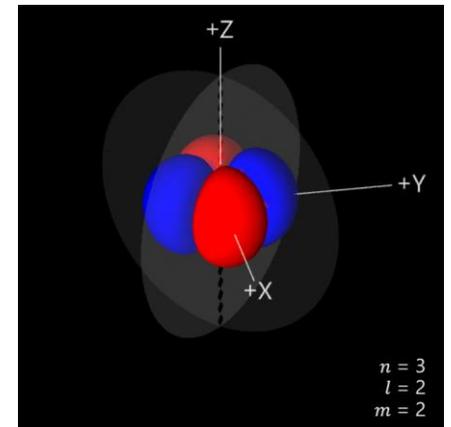
$d_{z^2}$



$d_{xz}$

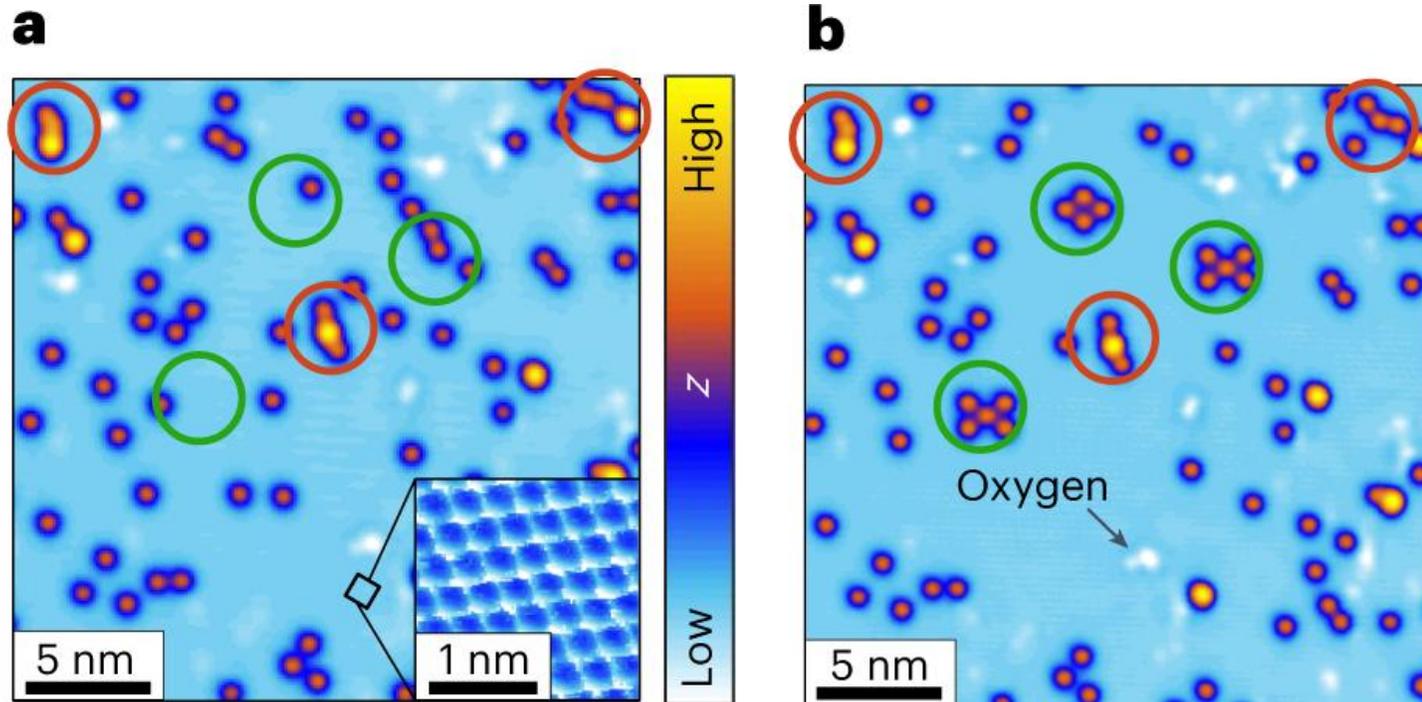


$d_{x^2-y^2}$



# Surface preparation and Cr deposition

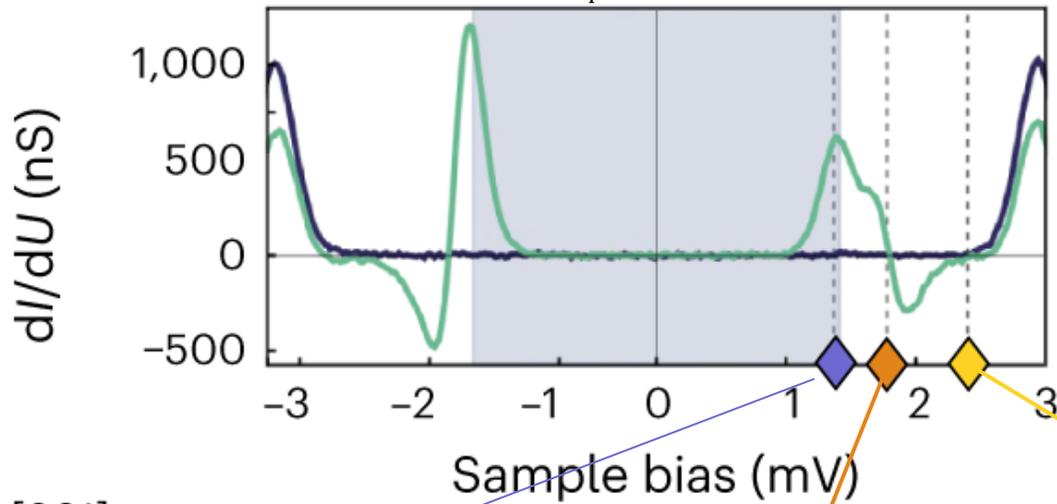
Topographic contrast (constant current)



- Cr atoms (bright spots) on Nb(110) (blue surface)
- Preparation of the surface:  $> 2000\text{ }^{\circ}\text{C}$  [1]
- Atomic manipulation: constant current 70 nA,  $V_S = -5\text{ mV}$ ,

# Single Cr atom on Nb (110)

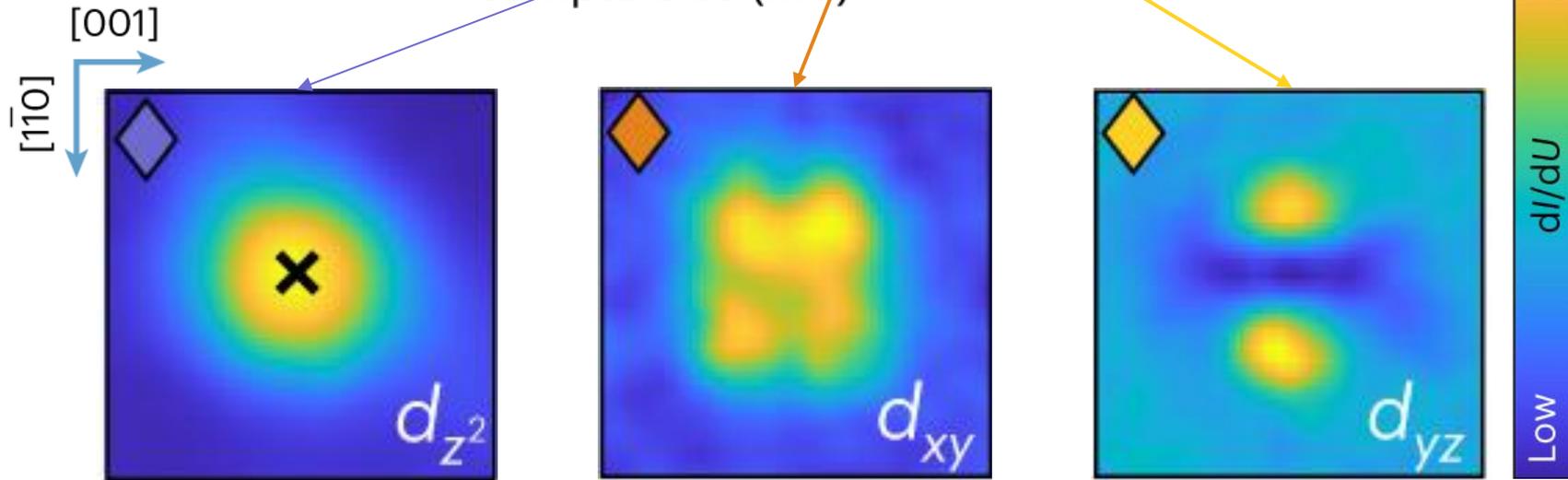
$I = 500 \text{ pA}; V_{\text{sample}} = -5 \text{ mV}; \delta V = 50 \text{ } \mu\text{V}$



- Spectra on Cr atom
- Spectra on Nb surface

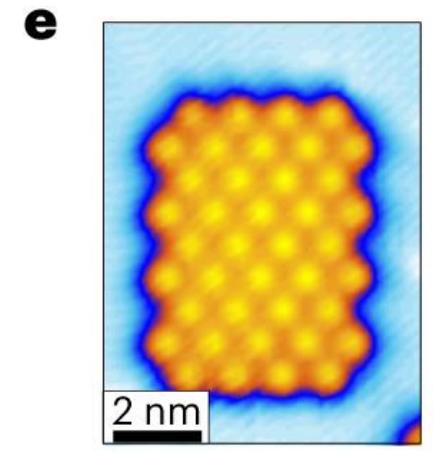
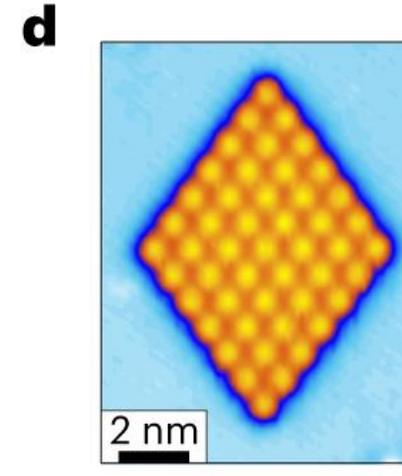
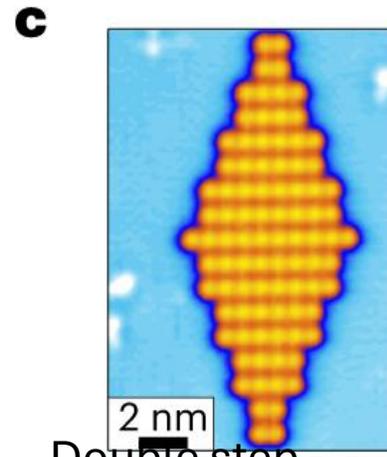
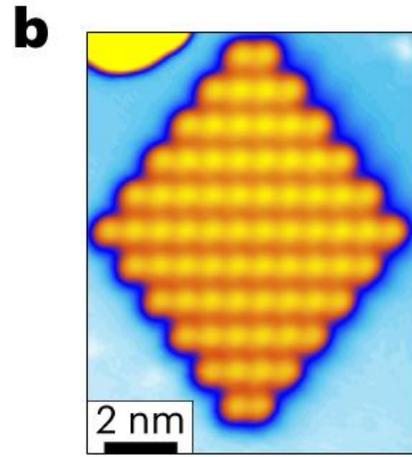
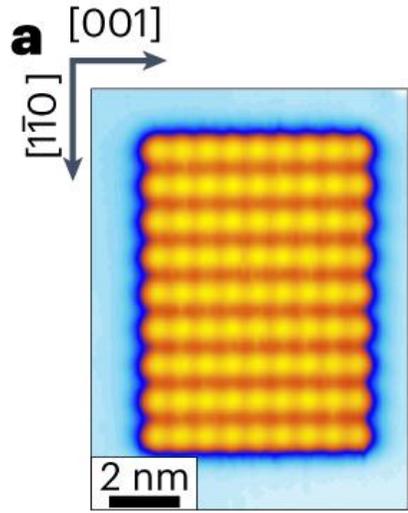
Spatial character can be obtained by fixing the sample bias and scanning the region nearby the magnetic atom

**c**



\*no scale reported

# Crystal lattices explored



Termination along  $[001]$   
 $[1\bar{1}0]$

$[1\bar{1}\bar{1}]$   
 $[1\bar{1}1]$

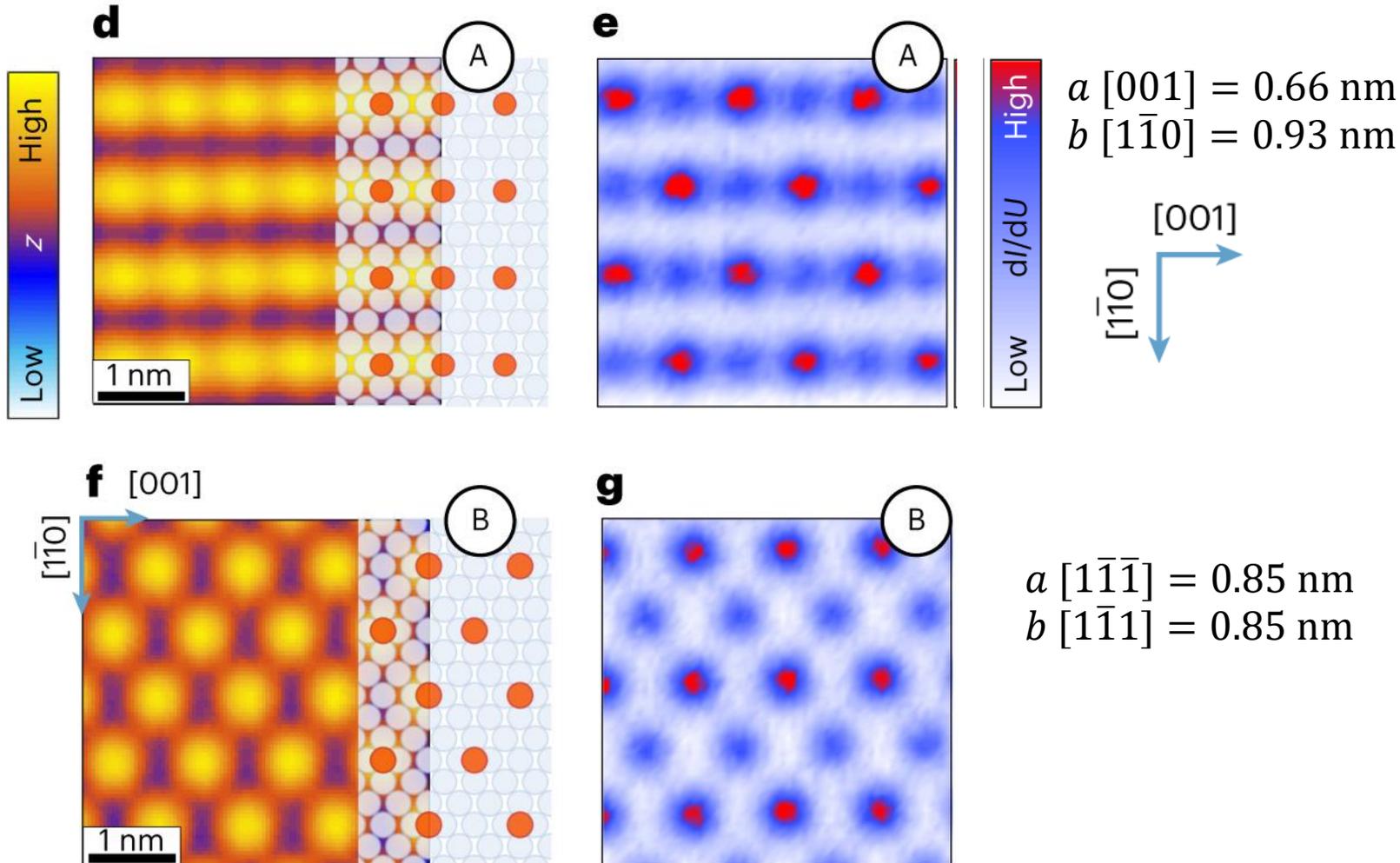
$[2\bar{2}1]$   
 $[2\bar{2}\bar{1}]$

$[1\bar{1}\bar{1}]$   
 $[1\bar{1}1]$

$[001]$   
 $[1\bar{1}0]$

# Two lattices created with Cr atoms

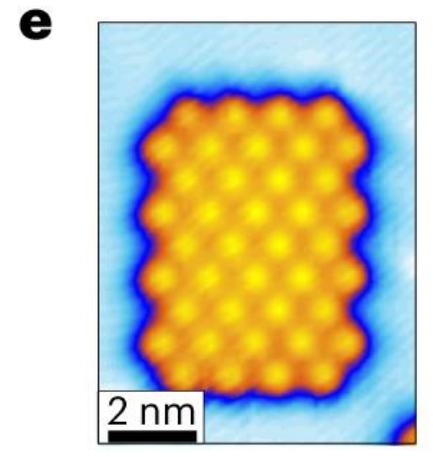
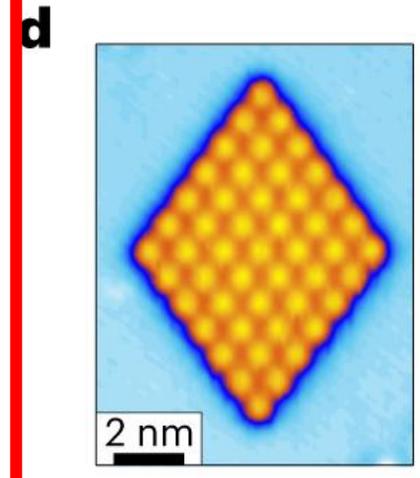
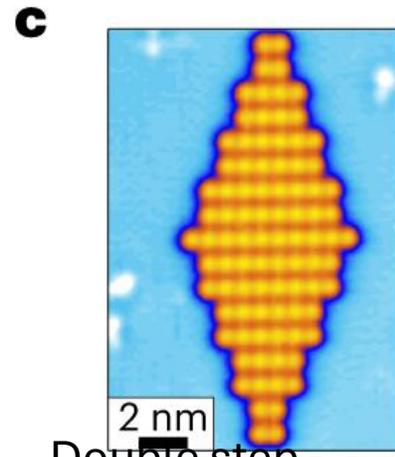
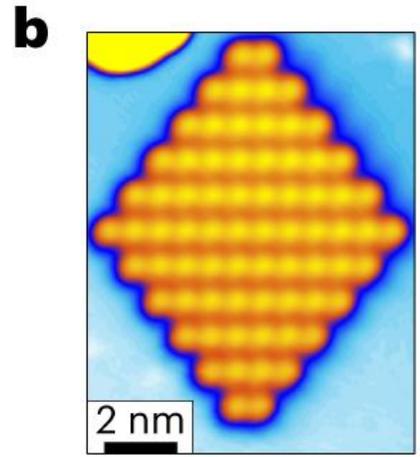
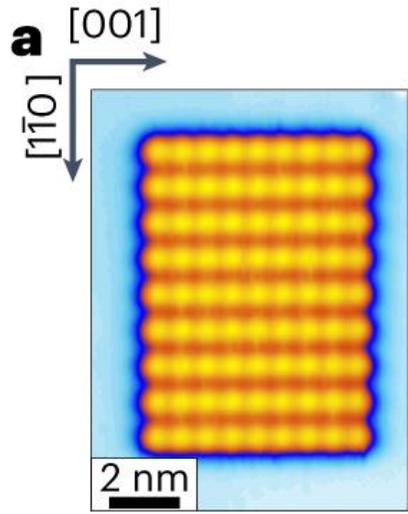
Spin resolved STM in constant height mode,  $B = 0.7$  T



$I = 500$  pA;  $V_{\text{sample}} = -3$  mV;  $V_{\text{tip}} = 0.33$  mV;  $\delta V = 80$   $\mu$ V

- Spin polarized STM
- Alternating spin contrast
- Antiferromagnetic ground state

# Crystal lattices explored



Termination along  $[001]$   
 $[1\bar{1}0]$

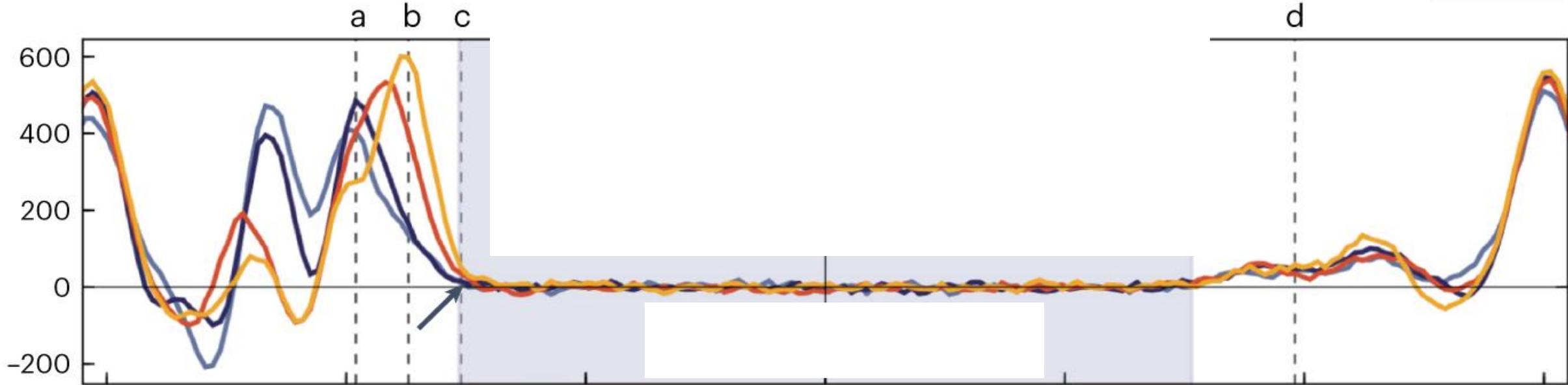
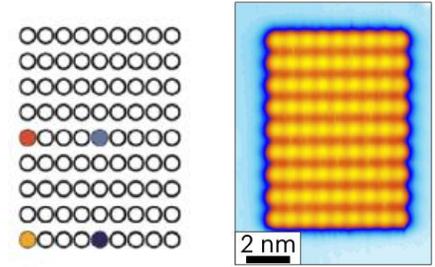
$[1\bar{1}\bar{1}]$   
 $[1\bar{1}1]$

$[2\bar{2}1]$   
 $[2\bar{2}\bar{1}]$

$[1\bar{1}\bar{1}]$   
 $[1\bar{1}1]$

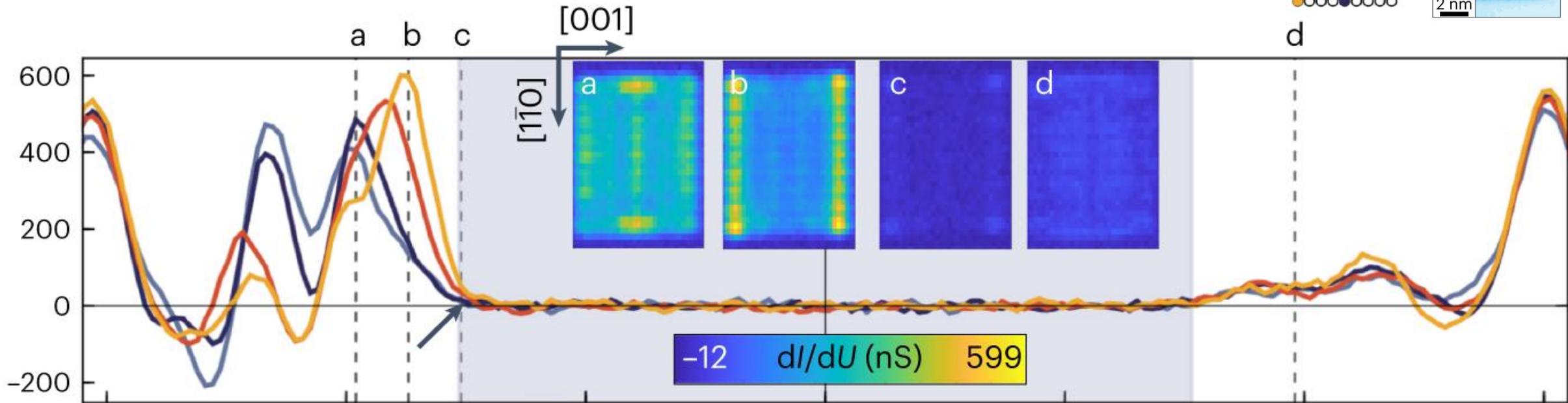
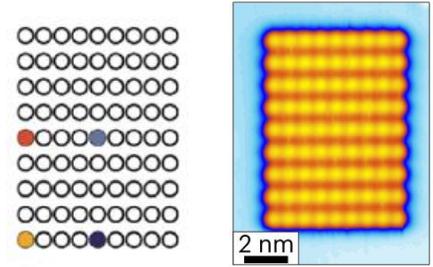
$[001]$   
 $[1\bar{1}0]$

# Corners, Edges, Bulk: rectangular lattice



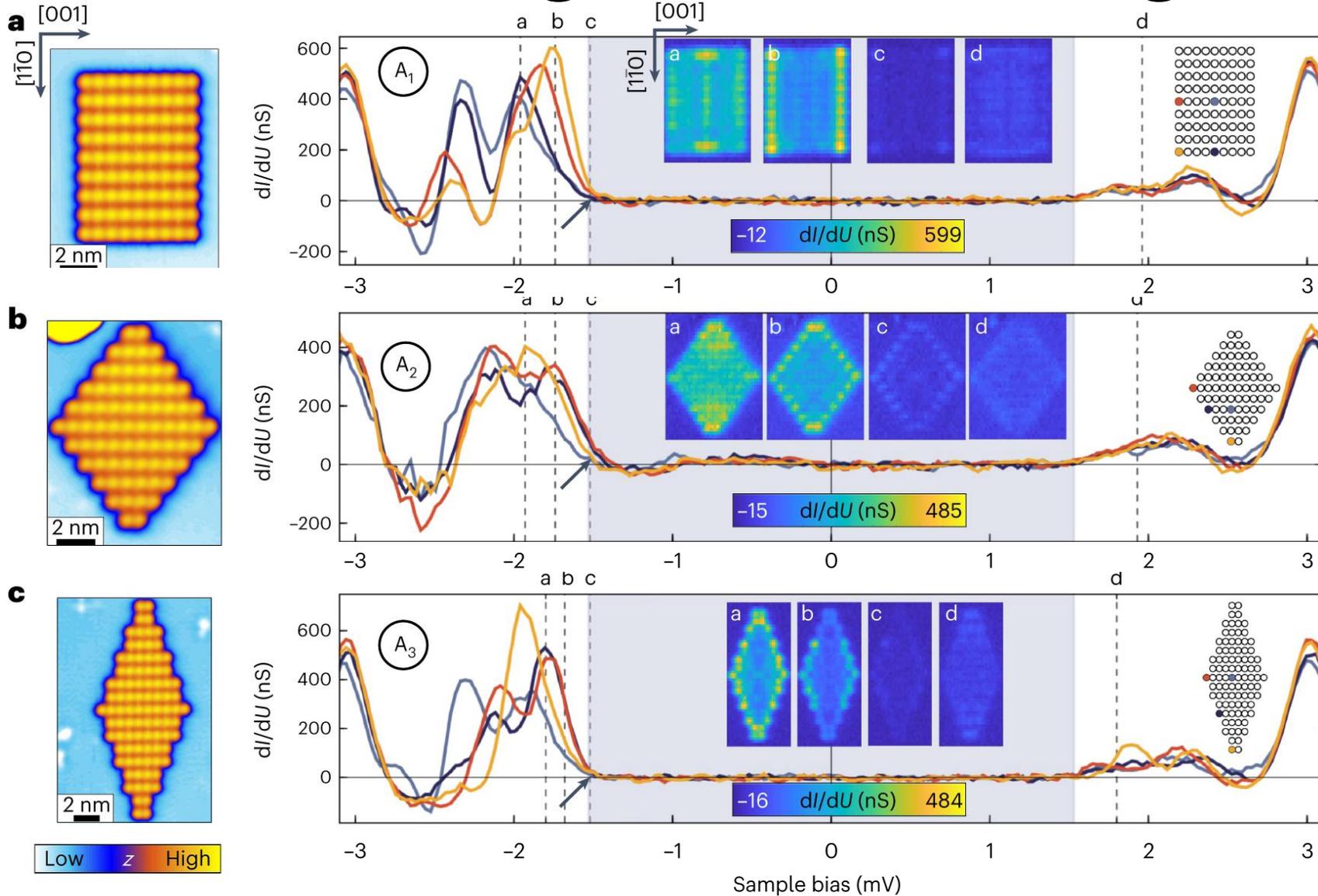
- Position dependent spectrum
- Toward edge stronger spectral intensity at low energies with respect to the bulk
- Asymmetric around  $E_F$ : particle-hole asymmetric

# Corners, Edges, Bulk: rectangular lattice



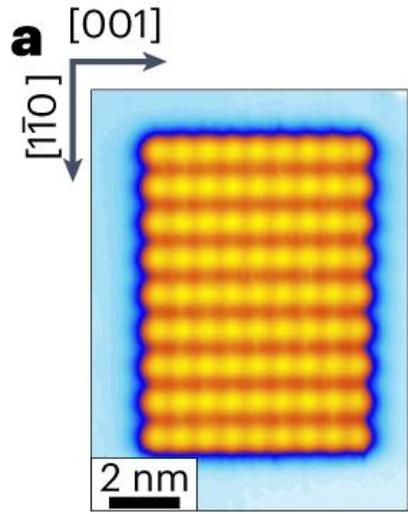
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# Corners, Edges, Bulk: rectangular lattice

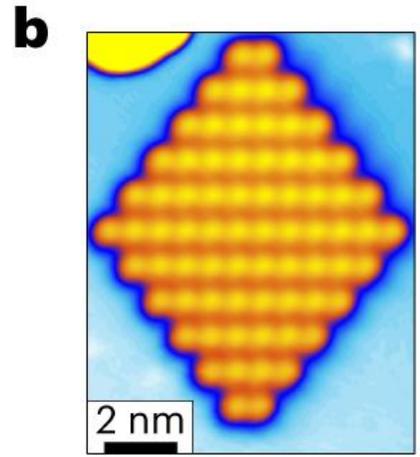


- Bulk properties termination independent
- Edge modes are termination dependent
- Gapped spectrum

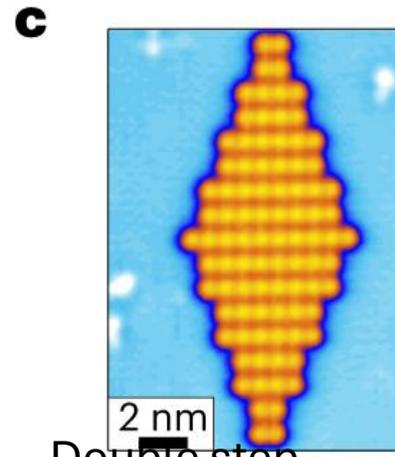
# Crystal lattices explored



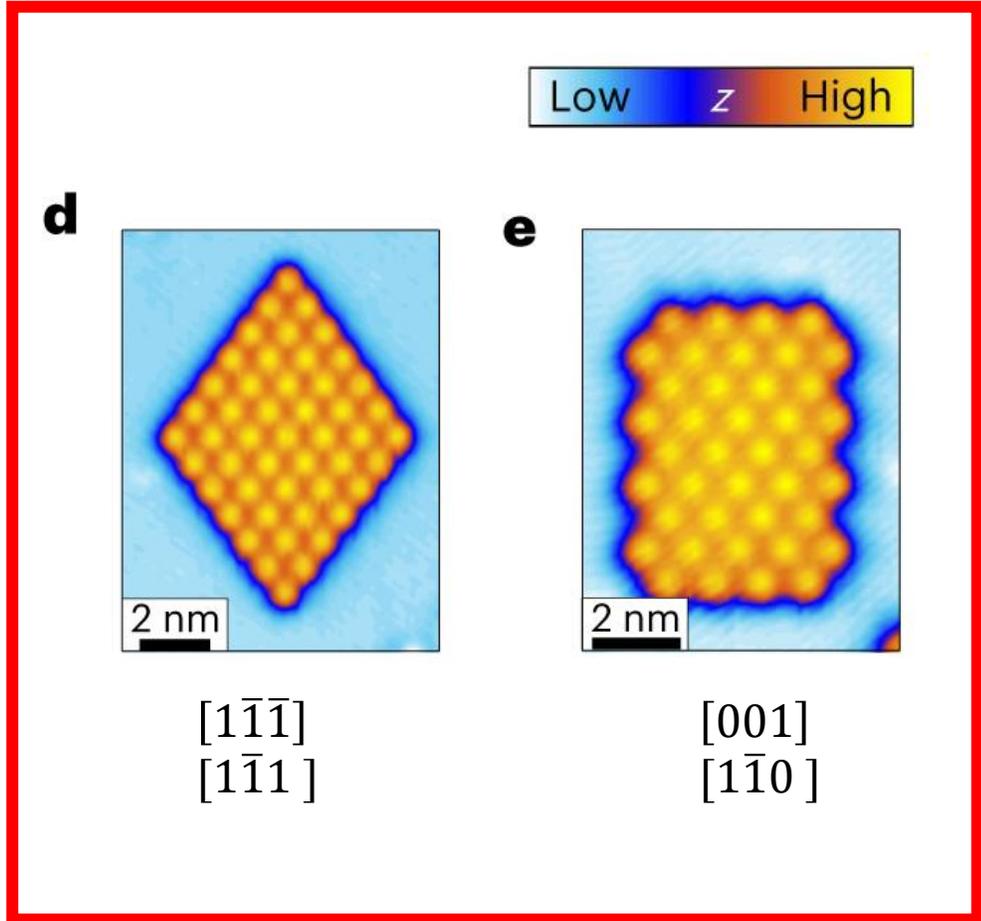
Termination along [001] [110]



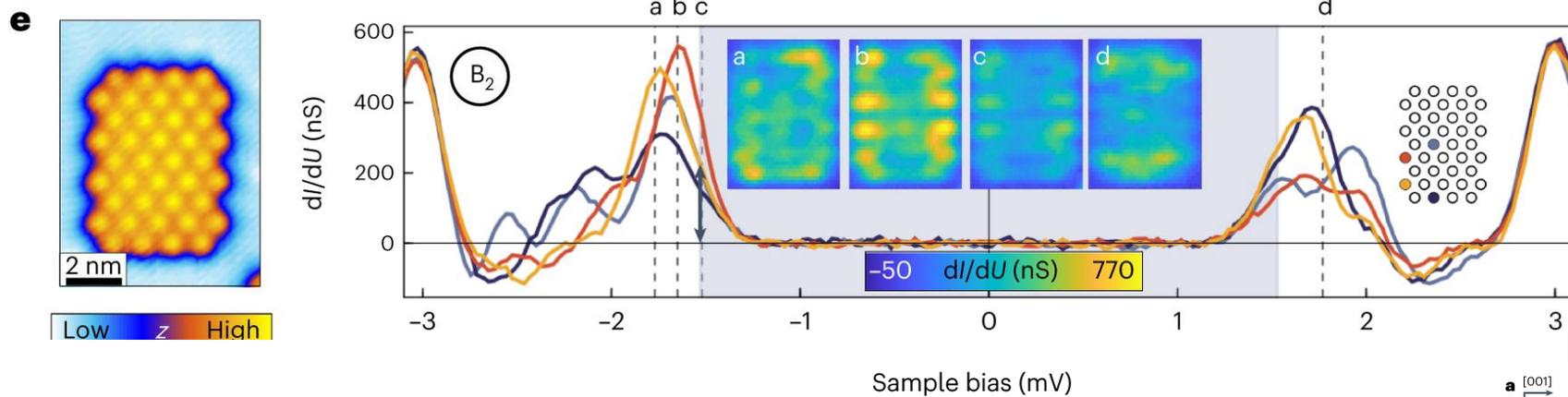
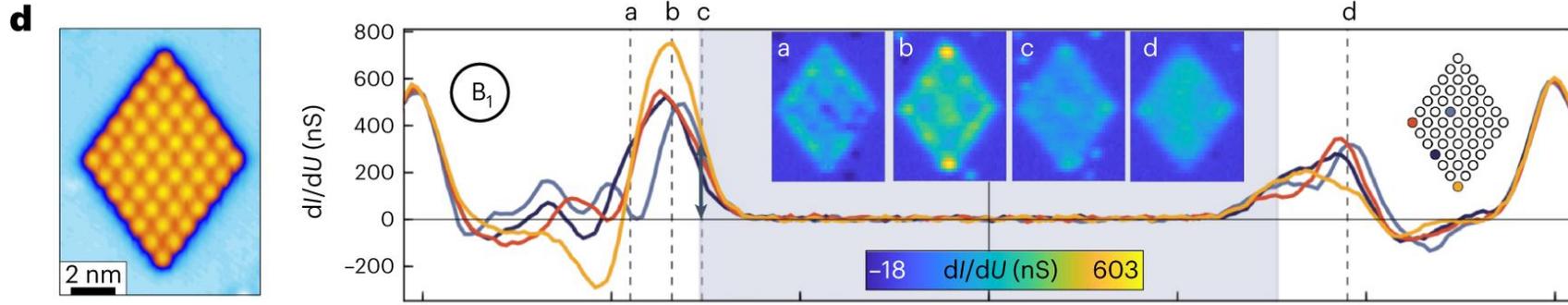
[111]  
[111]



[221]  
[221]

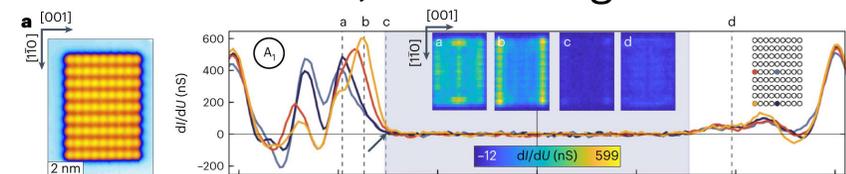


# Corners, Edges, Bulk: rhombic lattice



- Gapless spectrum

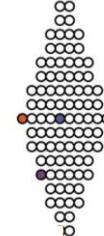
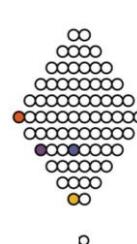
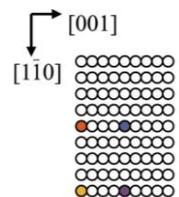
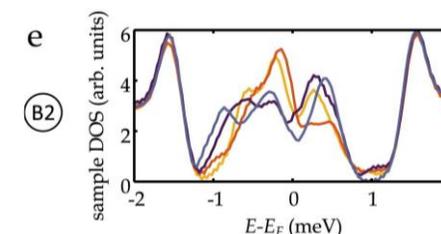
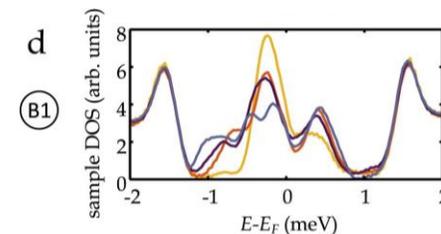
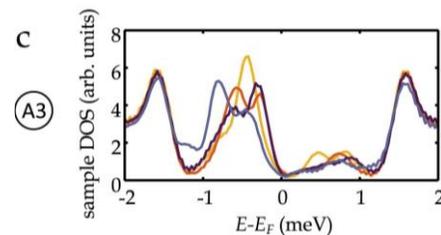
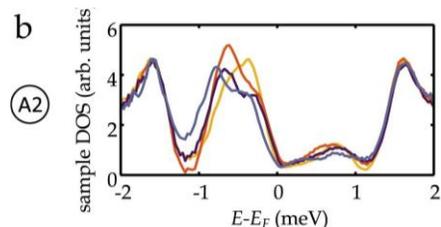
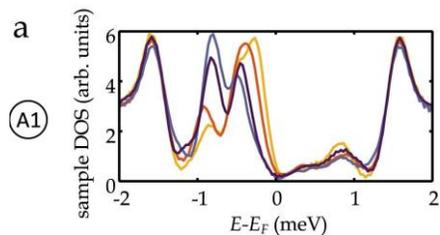
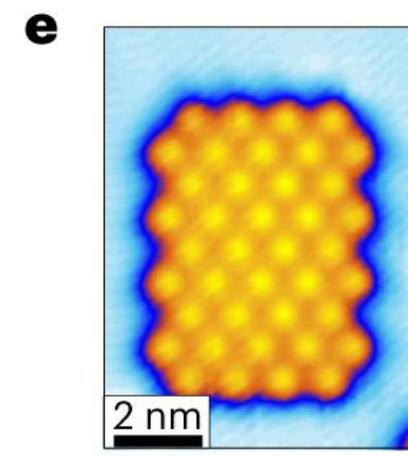
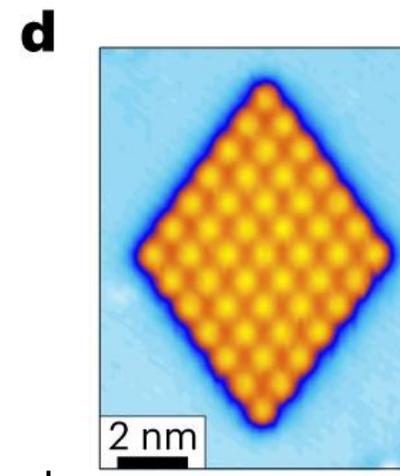
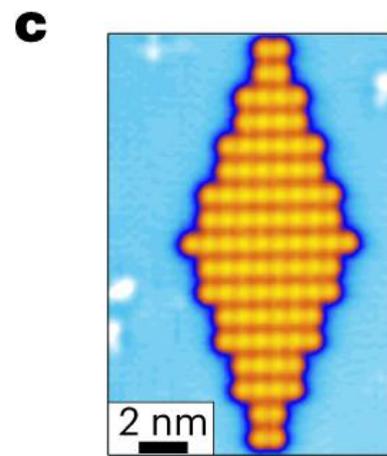
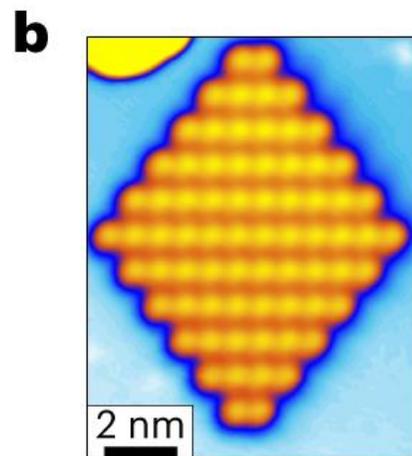
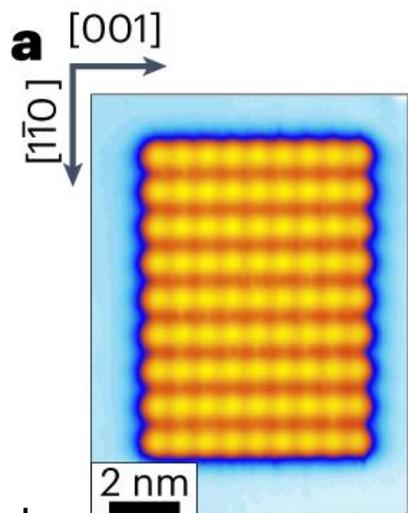
For reference, the rectangular lattice



# Lattice effect on the bulk properties

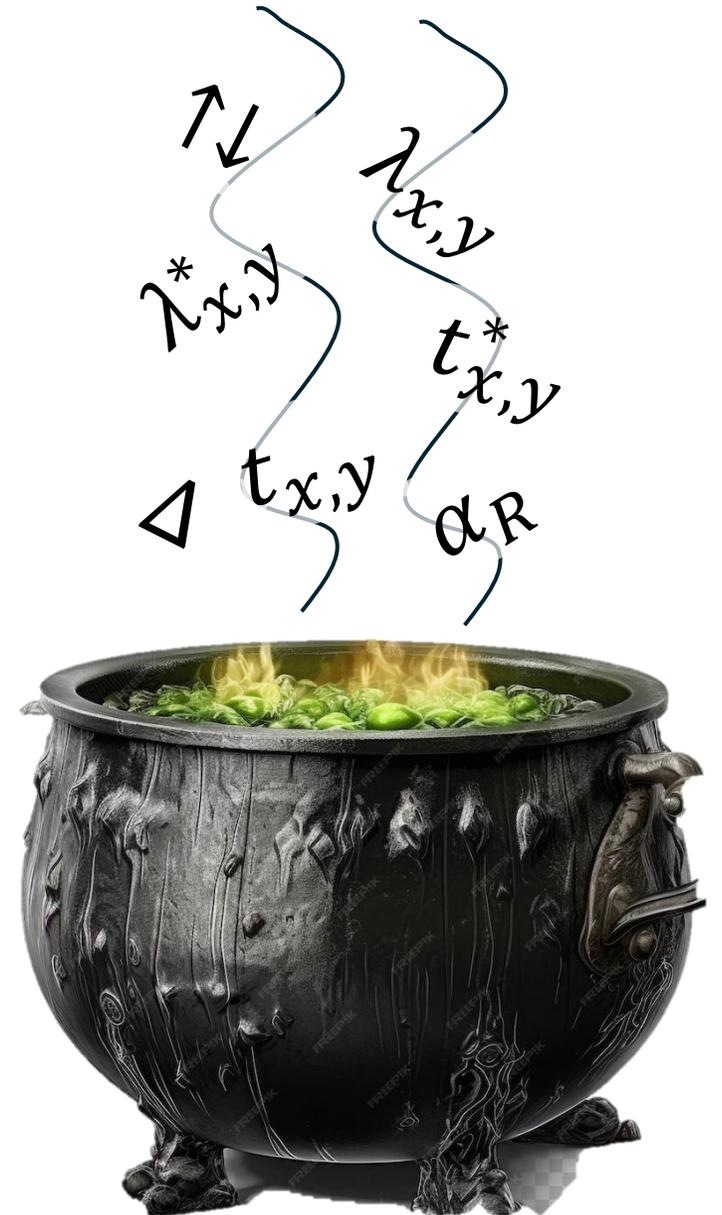
gapped

gapless



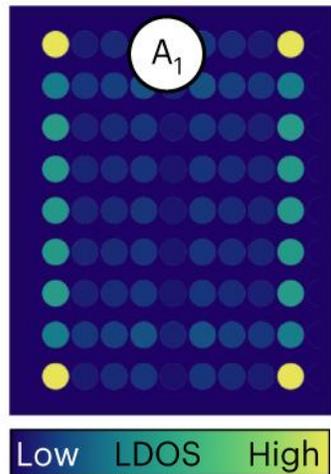
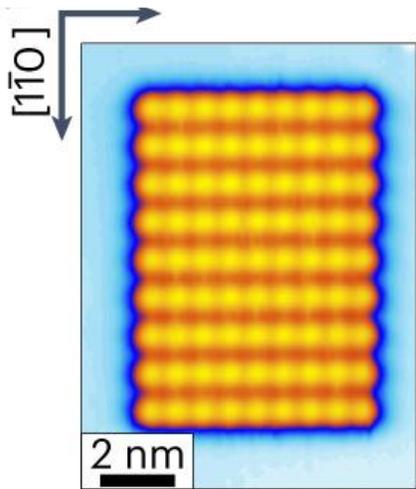
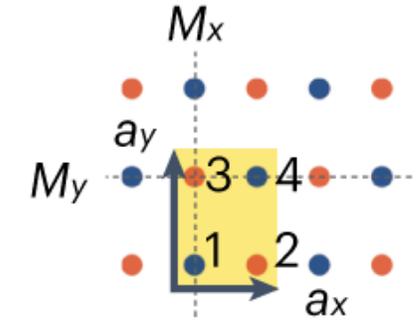
# Theory

- Single orbital tight binding models ( $d_{z^2}$ )
- Nearest-neighbour
- Next nearest-neighbour
- Rashba spin orbit coupling
- s-wave superconductivity of the substrate
- antiferromagnetic ordering of the magnetic moments of the Cr adatom

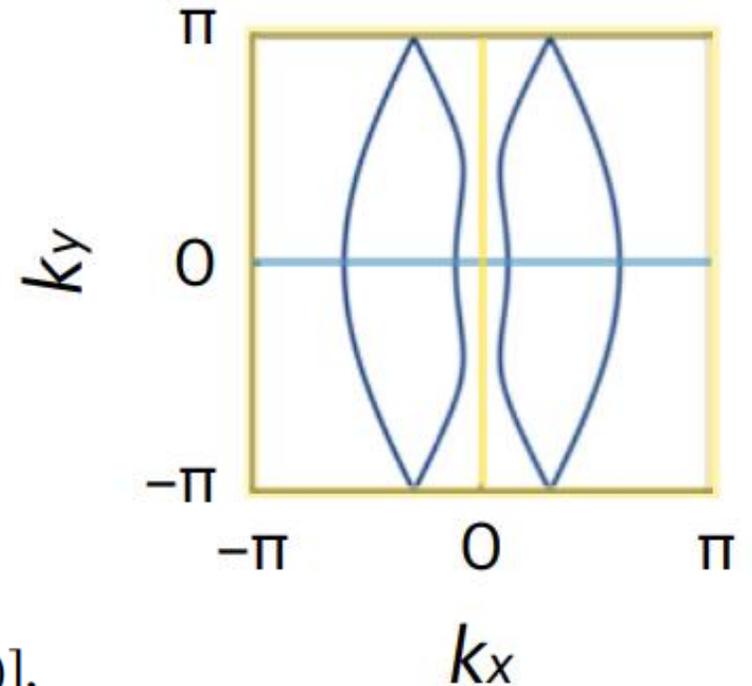
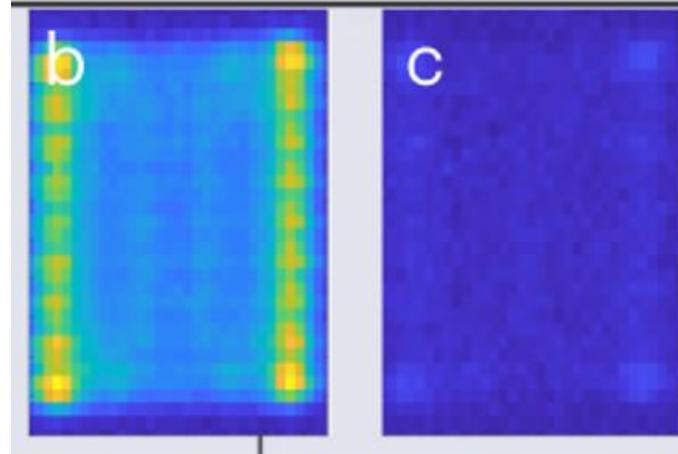


# Symmetries

- Symmetries:  $(M_x, M_y)\mathcal{T}, \mathcal{C}$  ( $C_2 = M_x M_y$ )
- Four paths are left invariant under action of the symmetries (BDI -  $\mathbb{Z}$ )
- Geometry dependent bulk-boundary correspondence



Zero energy DOS

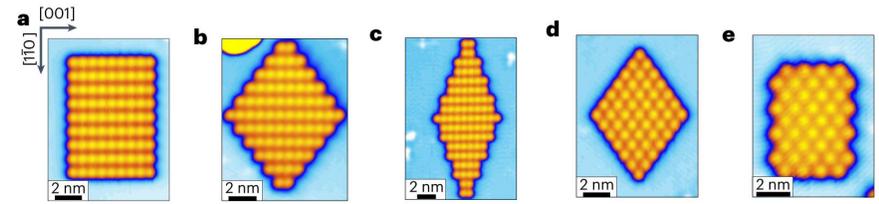


$$H(\mathbf{k}) = \begin{pmatrix} 0 & q(\mathbf{k}) \\ q^\dagger(\mathbf{k}) & 0 \end{pmatrix},$$

$$\nu = -\frac{i}{2\pi} \int_{-\pi}^{\pi} dk \text{Tr}[q^\dagger(k) \partial_k q(k)],$$

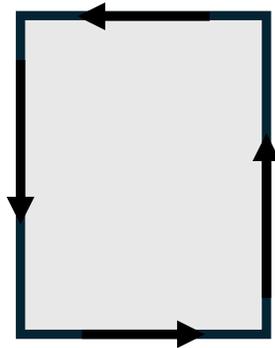
# Take home messages

but not last slide



- Lattice terminations play a role in the emergence of distinct boundary modes
- If crystal symmetry is involved, a geometry dependent bulk–boundary correspondence arises

This does not hold always



Example:

