Thermometry Based on a Superconducting Qubit

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Journal Club

Content

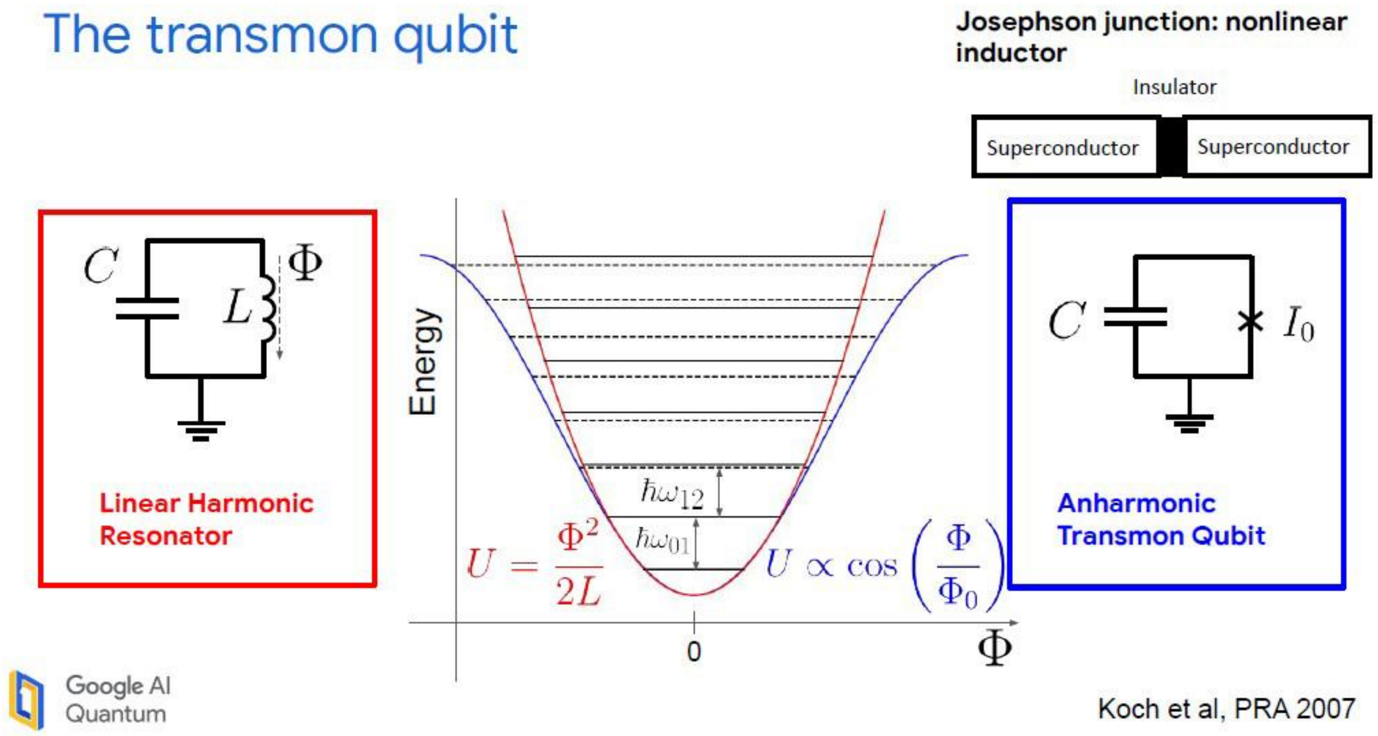
TEMPERATURE OF A SUPERCONDUCTING QUBIT

MEASUREMENT TECHNIQUE

ANALYSIS AND MODELLING

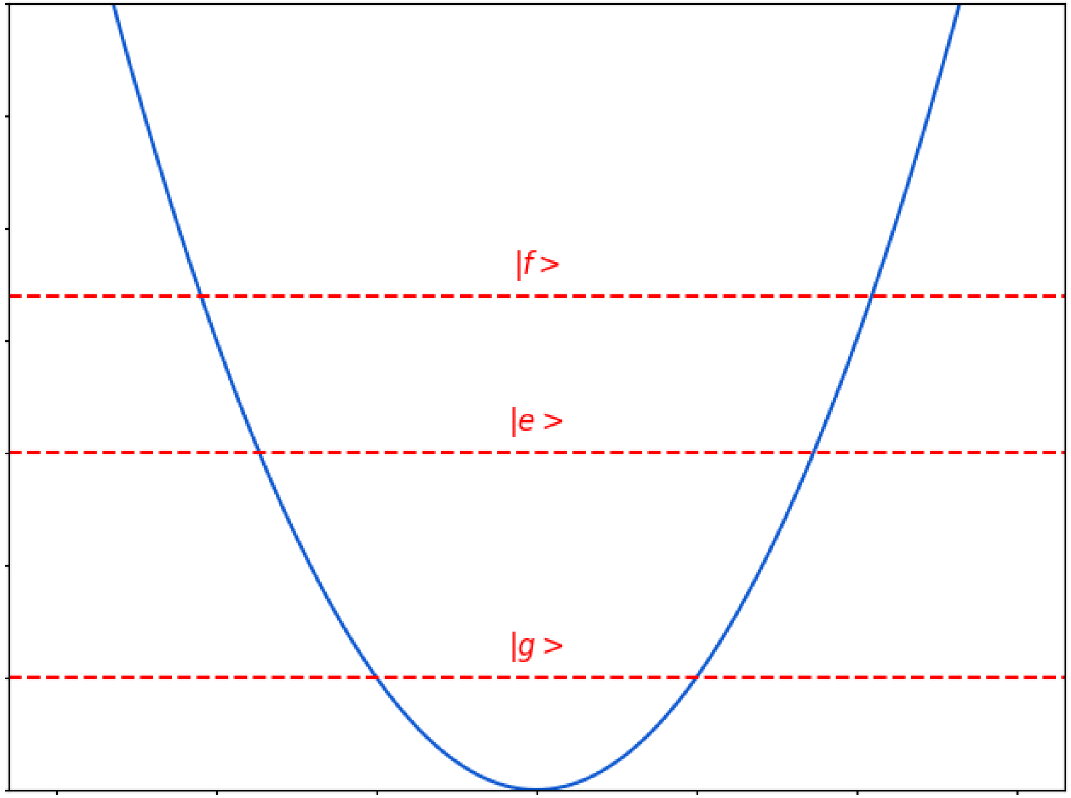
DEVICE DESIGN

Superconducting Qubit



Energy gap between levels is different

Temperature



Occupation by
Boltzmann Distribution

$$\frac{p_e}{p_g} = e^{-\frac{\hbar\omega_{ge}}{k_B T}}$$

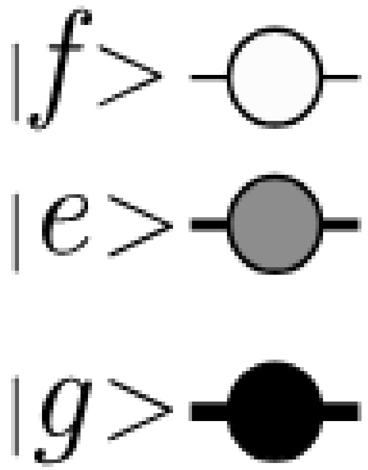
Depends on Energy gap
and Temperature

Measurement Response

φ_{i}- Pure state Response

p_{i}- Pure state Probability

Thermalized State

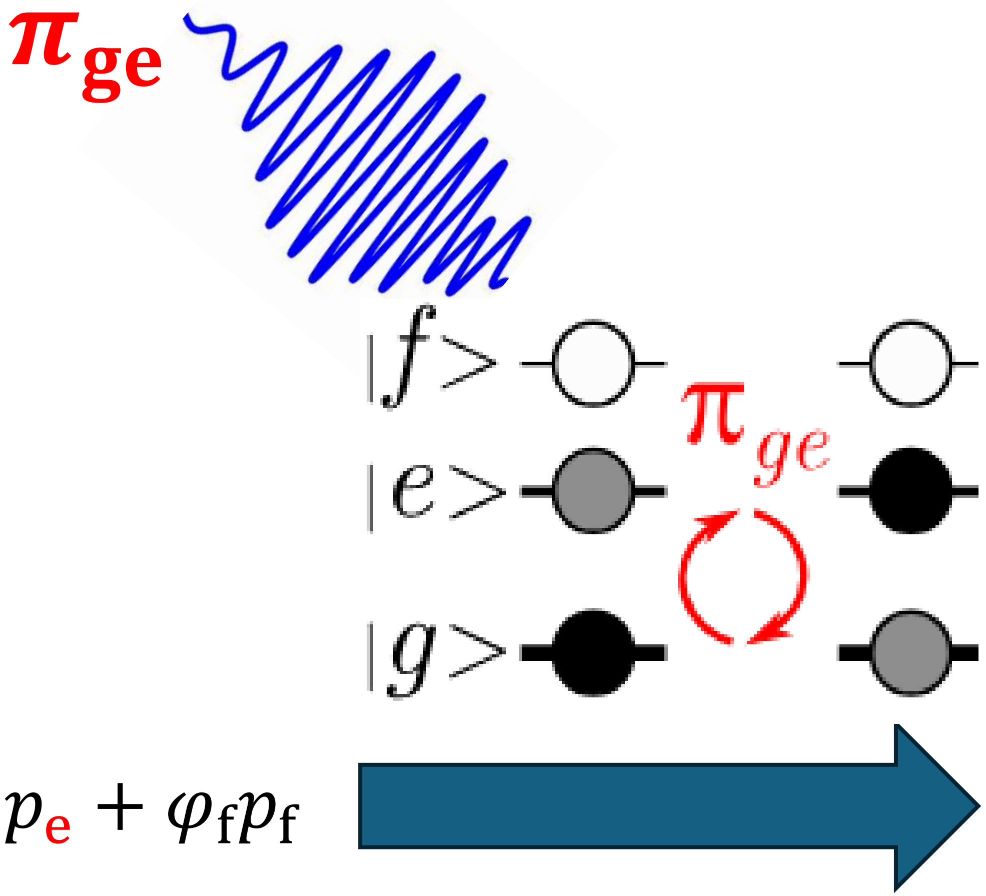


Measurement

$$\varphi = \varphi_{\mathrm{g}} p_{\mathrm{g}} + \varphi_{\mathrm{e}} p_{\mathrm{e}} + \varphi_{\mathrm{f}} p_{\mathrm{f}}$$

*Finite Temperature

Measuring Excited State



Inverting population by applying a pulse

$$x_0 = \varphi_g p_g + \varphi_e p_e + \varphi_f p_f$$

$$x_1 = \varphi_g p_e + \varphi_e p_g + \varphi_f p_f$$

Possible excitations in 3 level systems

$$x_0 = p_g\varphi_g + p_e\varphi_e + p_f\varphi_f, \qquad \text{(no pulse)}$$

$$x_1 = p_e\varphi_g + p_g\varphi_e + p_f\varphi_f, \qquad (\pi_{ge})$$

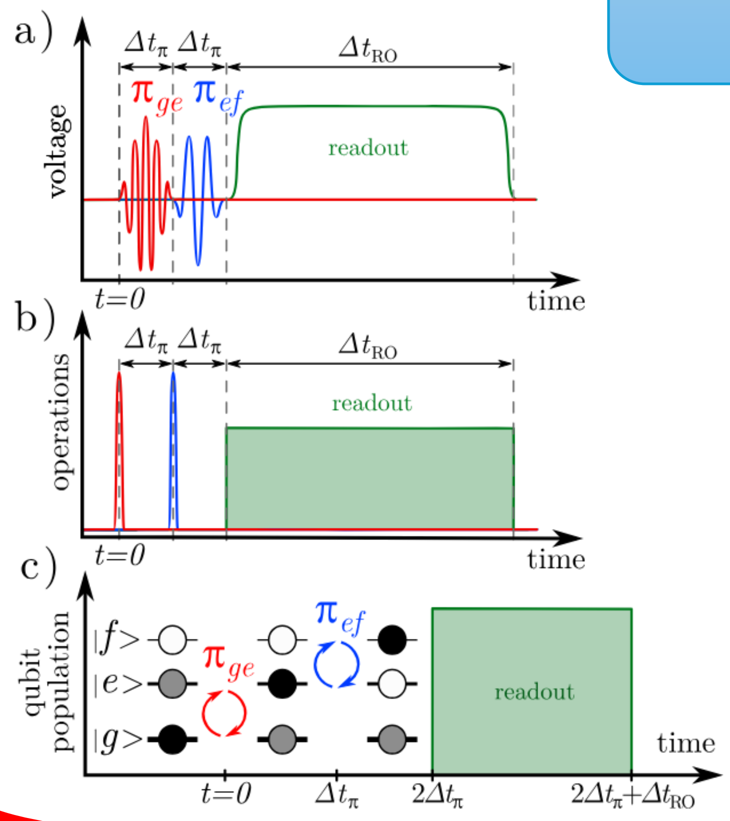
$$x_2 = p_e\varphi_g + p_f\varphi_e + p_g\varphi_f, \qquad (\pi_{ge}\pi_{ef})$$

$$y_0 = p_g\varphi_g + p_f\varphi_e + p_e\varphi_f, \qquad (\pi_{ef})$$

$$y_1 = p_f\varphi_g + p_g\varphi_e + p_e\varphi_f, \qquad (\pi_{ef}\pi_{ge})$$

$$y_2 = p_f\varphi_g + p_e\varphi_e + p_g\varphi_f. \qquad (\pi_{ef}\pi_{ge}\pi_{ef})$$

6 possible outcomes possible



Systematic Measurements

1. Photon Temperature T_{MXC} using $\mathrm{RuO_x}$
2. Population (all 6 iterations)
3. Qubit Relaxation time τ_1
4. Ramsey oscillations τ_2^{R}
5. Hahn echo τ_2^{E}

Extracting Temperature

Solving System of Linear Equations

$$\begin{aligned}
&\sout{x_0 = p_g\varphi_g + p_e\varphi_e + p_f\varphi_f, \quad (\text{no pulse})}\\
&x_1 = p_e\varphi_g + p_g\varphi_e + p_f\varphi_f, \quad (\pi_{ge})\\
&x_2 = p_e\varphi_g + p_f\varphi_e + p_g\varphi_f, \quad (\pi_{ge}\pi_{ef})\\
&y_0 = p_g\varphi_g + p_f\varphi_e + p_e\varphi_f, \quad (\pi_{ef})\\
&y_1 = p_f\varphi_g + p_g\varphi_e + p_e\varphi_f, \quad (\pi_{ef}\pi_{ge})\\
&\sout{y_2 = p_f\varphi_g + p_e\varphi_e + p_g\varphi_f. \quad (\pi_{ef}\pi_{ge}\pi_{ef})}
\end{aligned}$$

$$B_2 = \frac{x_1 - y_1}{y_0 - x_2} = \cdots = \frac{p_\mathrm{e} - p_\mathrm{f}}{p_\mathrm{g} - p_\mathrm{e}} = \cdots = \frac{1 - e^{-\frac{\hbar\omega_\mathrm{fe}}{k_\mathrm{B}T}}}{e^{-\frac{\hbar\omega_\mathrm{ge}}{k_\mathrm{B}T}} - 1}$$

Extracting Temperature

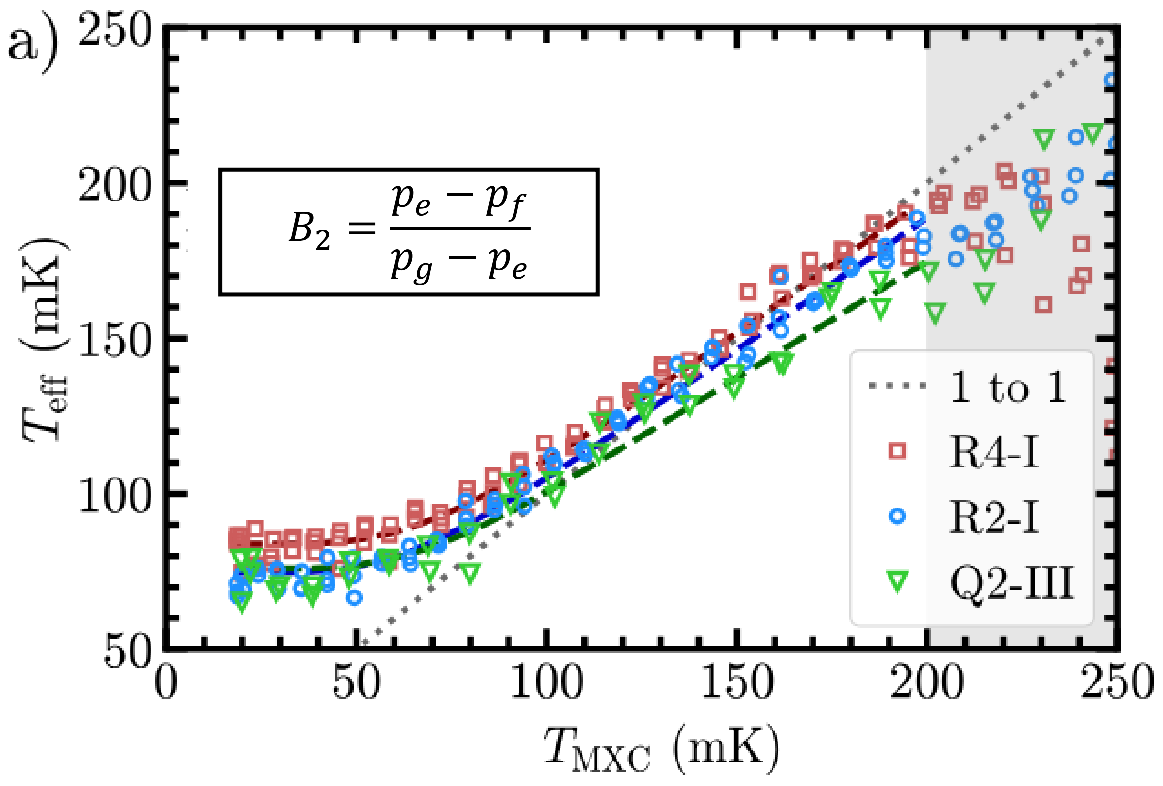
9 Ways to Calculate Temperature

TABLE I. Different ways of the qubit effective temperature calculation from the qubit state readout results.

Ratio	Calculation method 1	2	3	Result	Equation
A	$\frac{x_0-x_1}{y_0-y_1}$	$\frac{y_0-x_2}{x_0-y_2}$	$\frac{y_1-y_2}{x_1-x_2}$	$\frac{p_g-p_e}{p_g-p_f}$	$A=\frac{1-\exp\left(-\hbar\omega_{ge}/k_BT\right)}{1-\exp\left(-\hbar\omega_{gf}/k_BT\right)}$
B	$\frac{x_2-y_2}{x_0-x_1}$	$\frac{x_1-y_1}{y_0-x_2}$	$\frac{x_0-y_0}{y_1-y_2}$	$\frac{p_e-p_f}{p_g-p_e}$	$B=\frac{\exp\left(-\hbar\omega_{ge}/k_BT\right)-\exp\left(-\hbar\omega_{gf}/k_BT\right)}{1-\exp\left(-\hbar\omega_{ge}/k_BT\right)}$
C	$\frac{x_2-y_2}{y_0-y_1}$	$\frac{x_1-y_1}{x_0-y_2}$	$\frac{x_0-y_0}{x_1-x_2}$	$\frac{p_e-p_f}{p_g-p_f}$	$C=\frac{\exp\left(-\hbar\omega_{ge}/k_BT\right)-\exp\left(-\hbar\omega_{gf}/k_BT\right)}{1-\exp\left(-\hbar\omega_{gf}/k_BT\right)}$

Extracted Temperature

- Gray dashed line $T_{\mathrm{eff}} = T_{\mathrm{MXC}}$
- Red/Blue/Green: Different devices
- Axes: linear, y origin offset



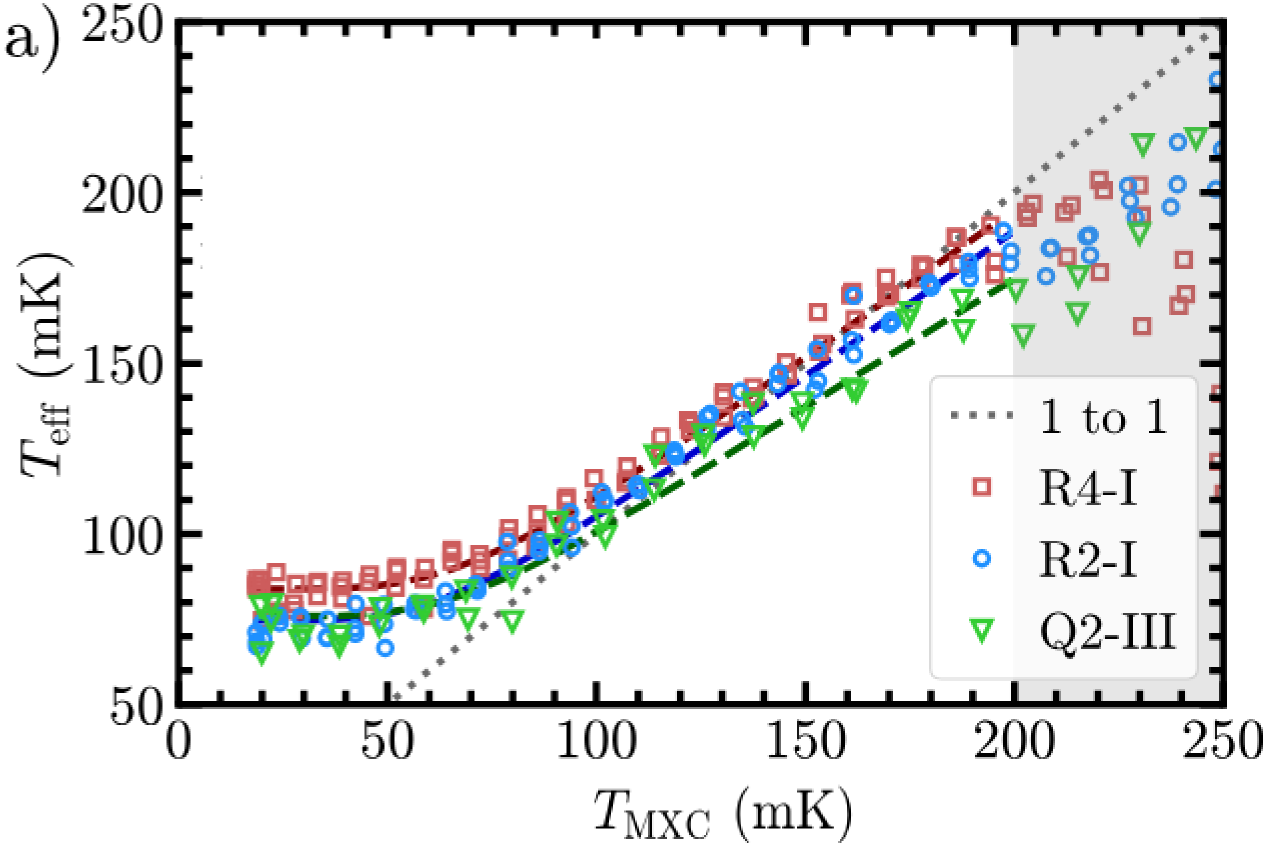
Deviation Mechanisms

LOW T

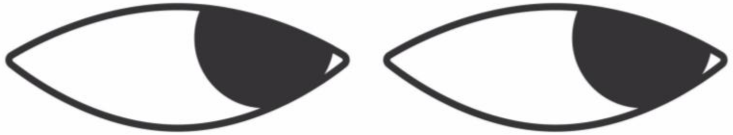
Multiple Heat Sources

HIGH T

Quasiparticles

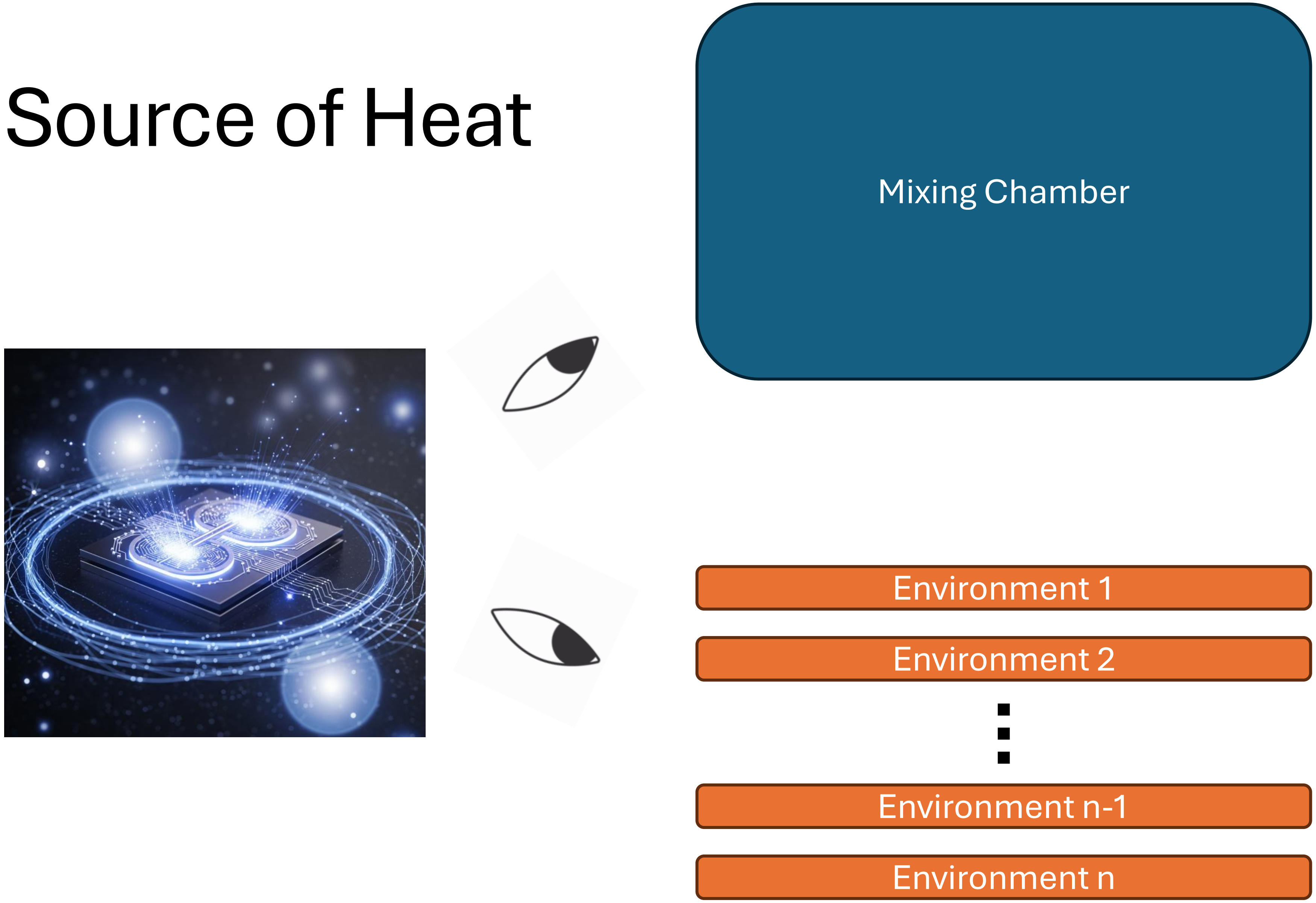


Source of Heat



Mixing Chamber

Source of Heat



Multiple Bath Model

$$\gamma_1 = \sum_i \left(\Gamma_\downarrow^{(i)} + \Gamma_\uparrow^{(i)}\right) = \sum_i \gamma^{(i)} \left(2n_\mathrm{i} + 1\right)$$

$$n_\mathrm{i} = \frac{1}{e^{\frac{h\omega}{k_\mathrm{B}T^{(i)}}} - 1}$$

$$n_\mathrm{eff} = \frac{1}{\gamma_1^0} \sum_i \gamma^{(i)} n_i$$

γ_1
Qubit energy relaxation rate

n_eff
Average photon number over different baths, weighted by their coupling

Two Bath Model

- γ_1
  Qubit energy relaxation rate
- n_{eff}
  Average photon number

Mixing Chamber

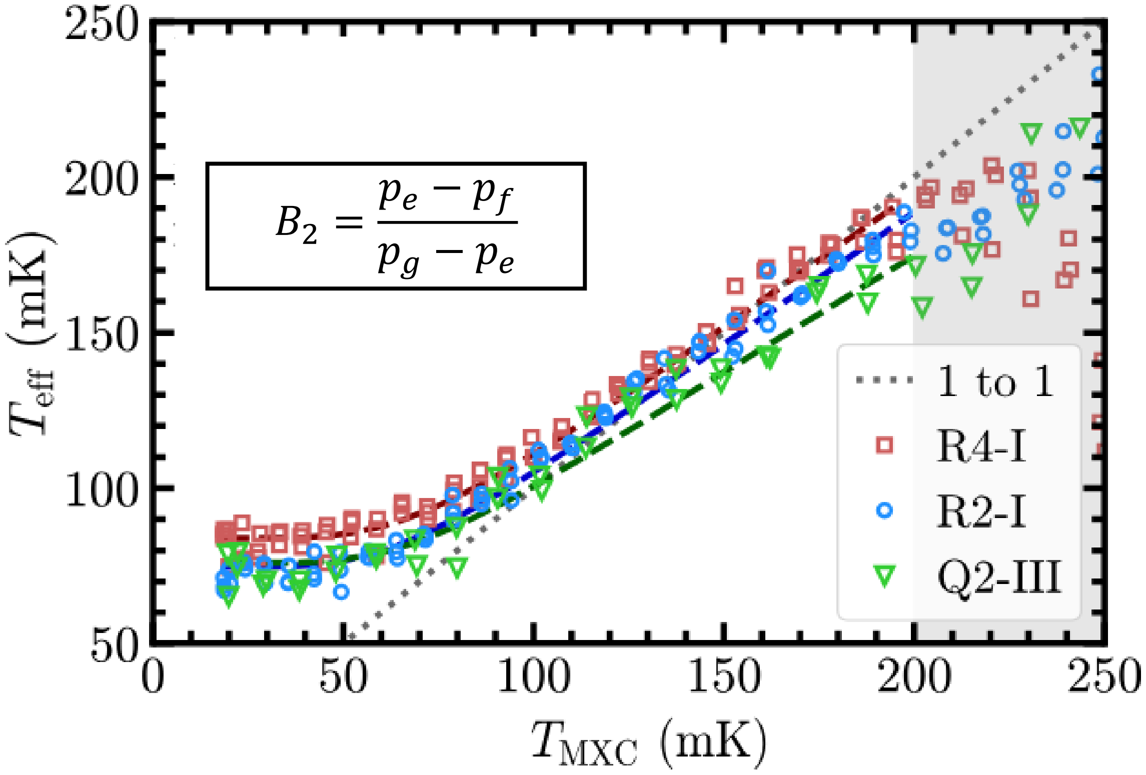
Environment

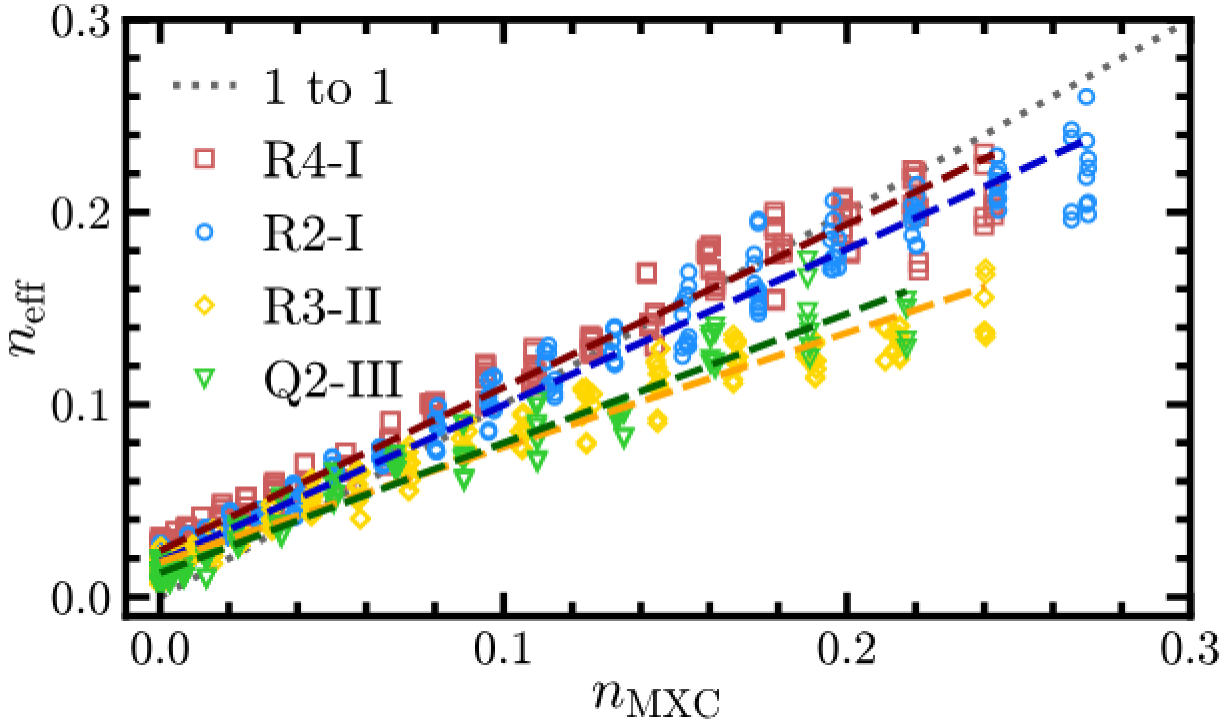
$$n_{\text{eff}} = \frac{1}{\gamma_1^0}\sum_i \gamma^{(i)} n_i \;\Rightarrow\; n_{\text{eff}} = \frac{\gamma_{\text{MXC}}}{\gamma_1^0} n_{\text{MXC}} + \frac{\gamma_{\text{env}}}{\gamma_1^0} n_{\text{env}}$$

$$\gamma_1 = \sum_i \gamma^{(i)}\,(2n_{\text{i}} + 1) \;\Rightarrow\; \gamma_1 = \gamma_{\text{MXC}}(2n_{\text{MXC}} + 1) + \gamma_{\text{env}}(2n_{\text{env}} + 1) = $$
$$= \gamma_1^0(2n_{\text{eff}} + 1)$$

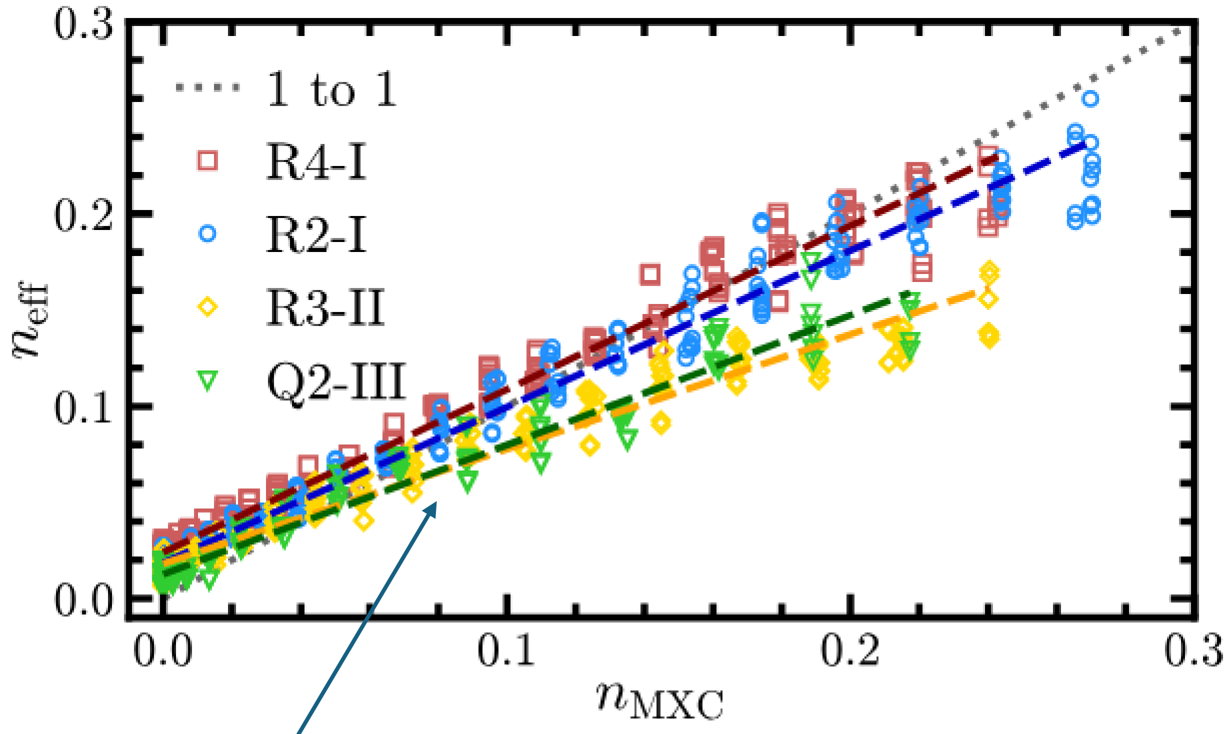
$$n_{\text{eff}} = \frac{1}{e^{\frac{h\omega}{k_B T_{\text{eff}}}} - 1}$$

Temperature – Photon Number





Coupling Strength

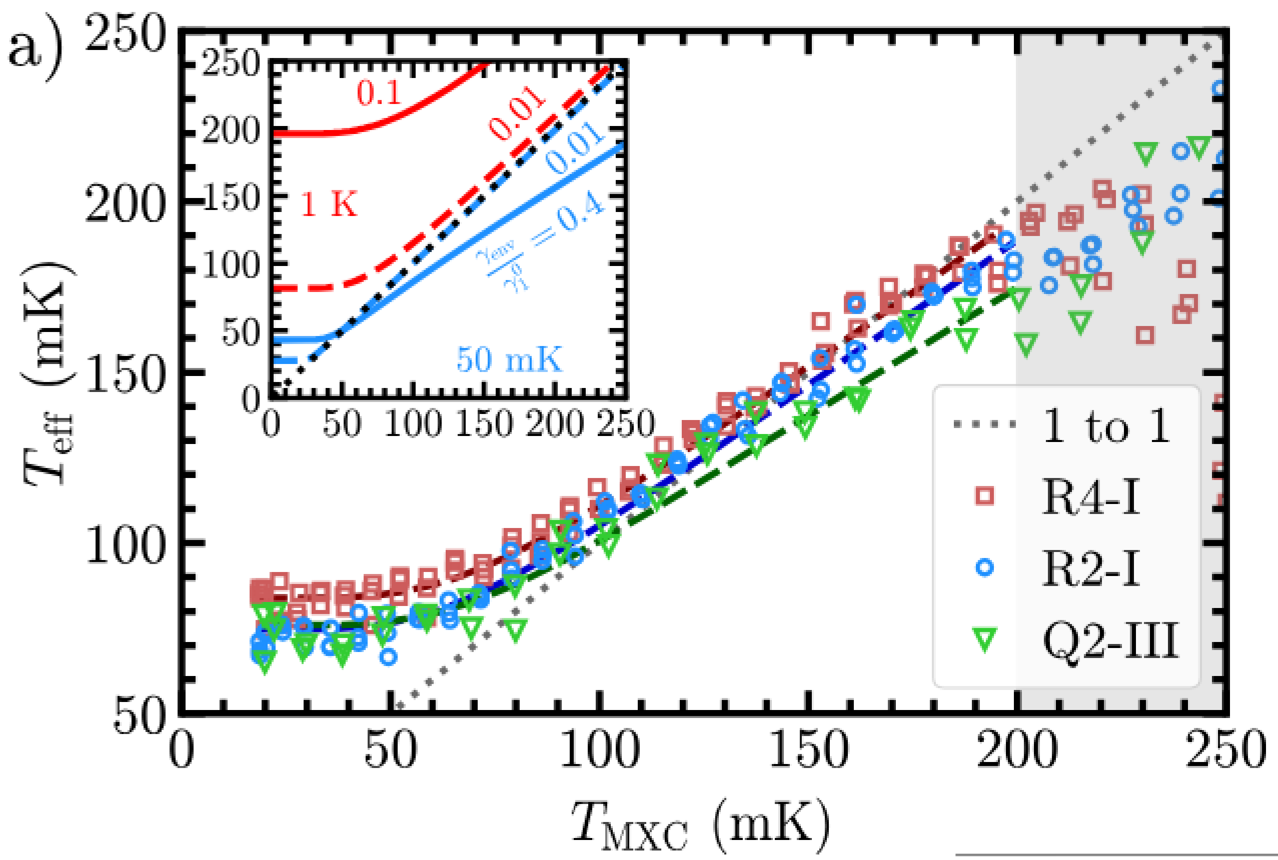


$$n_{\mathrm{eff}} = \frac{\gamma_{\mathrm{MXC}}}{\gamma_1^0} n_{\mathrm{MXC}} + \frac{\gamma_{\mathrm{env}}}{\gamma_1^0} n_{\mathrm{env}}$$

Ratio of coupling strength is deduced from the slope

Guessing Environment

$$n_{\text{eff}} = \frac{\gamma_{\text{MXC}}}{\gamma_1^0} n_{\text{MXC}} + \frac{\gamma_{\text{env}}}{\gamma_1^0} n_{\text{env}}$$

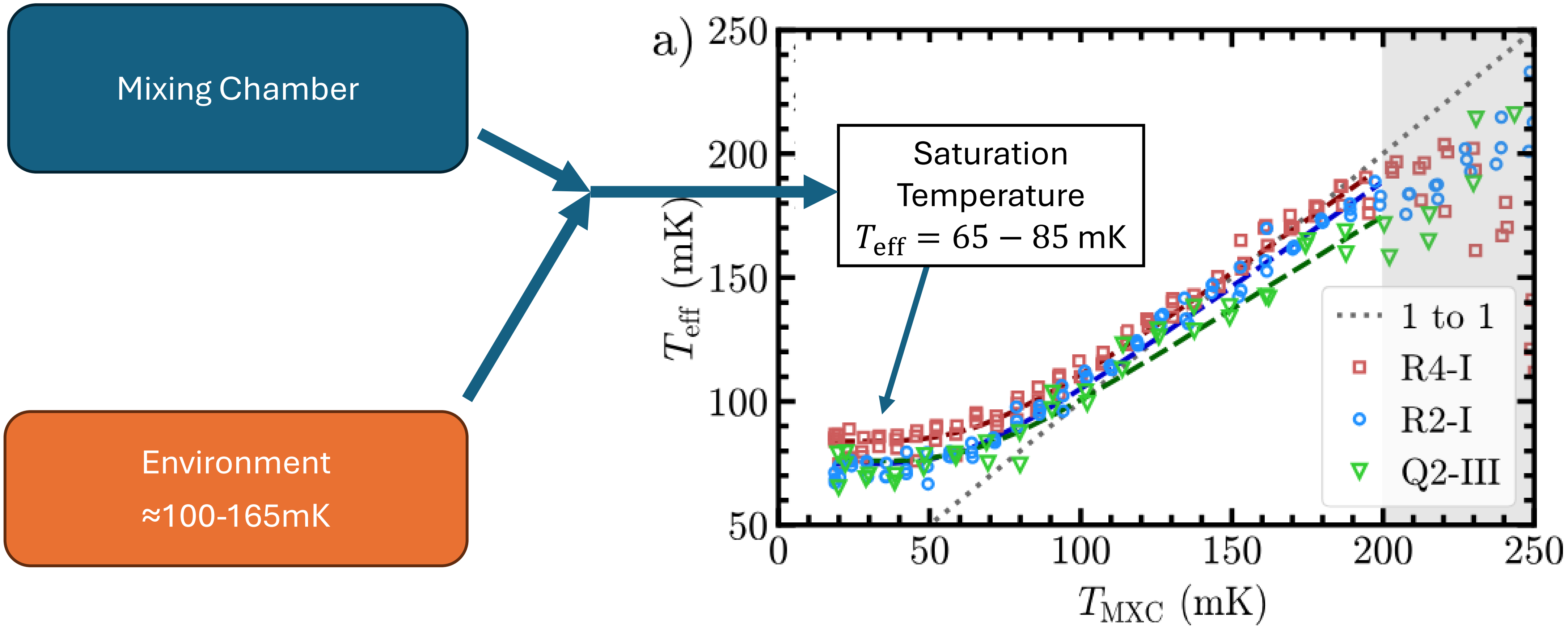


Environment coupling and temperature are obtained from "Best Fit"

TABLE III. Parameters of the linear fits shown in Fig. 3c.

Fitting Parameters	Sample R4-I	R2-I	R3-II	Q2-III
$dn_{\text{eff}}/dn_{\text{MXC}}$	0.849 ± 0.014	0.812 ± 0.012	0.596 ± 0.012	0.673 ± 0.024
n^0_{eff}, $\times 10^{-3}$	24.0 ± 0.7	18.5 ± 0.7	18.1 ± 0.6	12.6 ± 1.1
T_{env}, mK	102.0 ± 1.6	162.9 ± 3.4	127.0 ± 1.7	100.3 ± 0.7

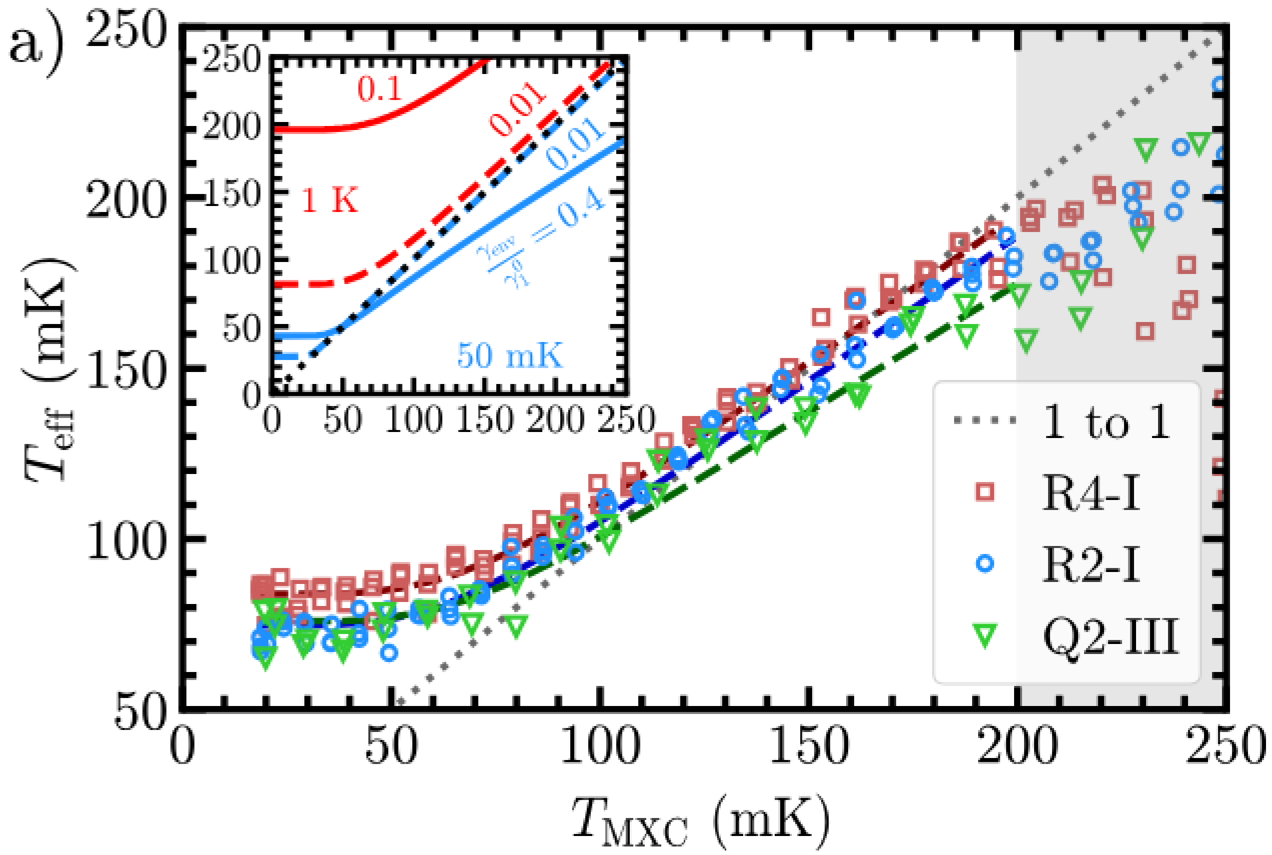
Effective Environment



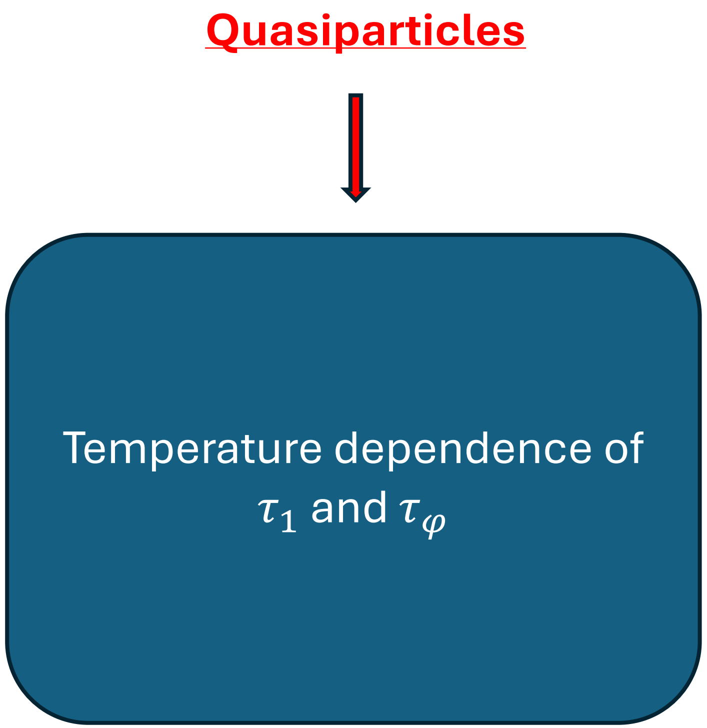
Theoretical Confirmation

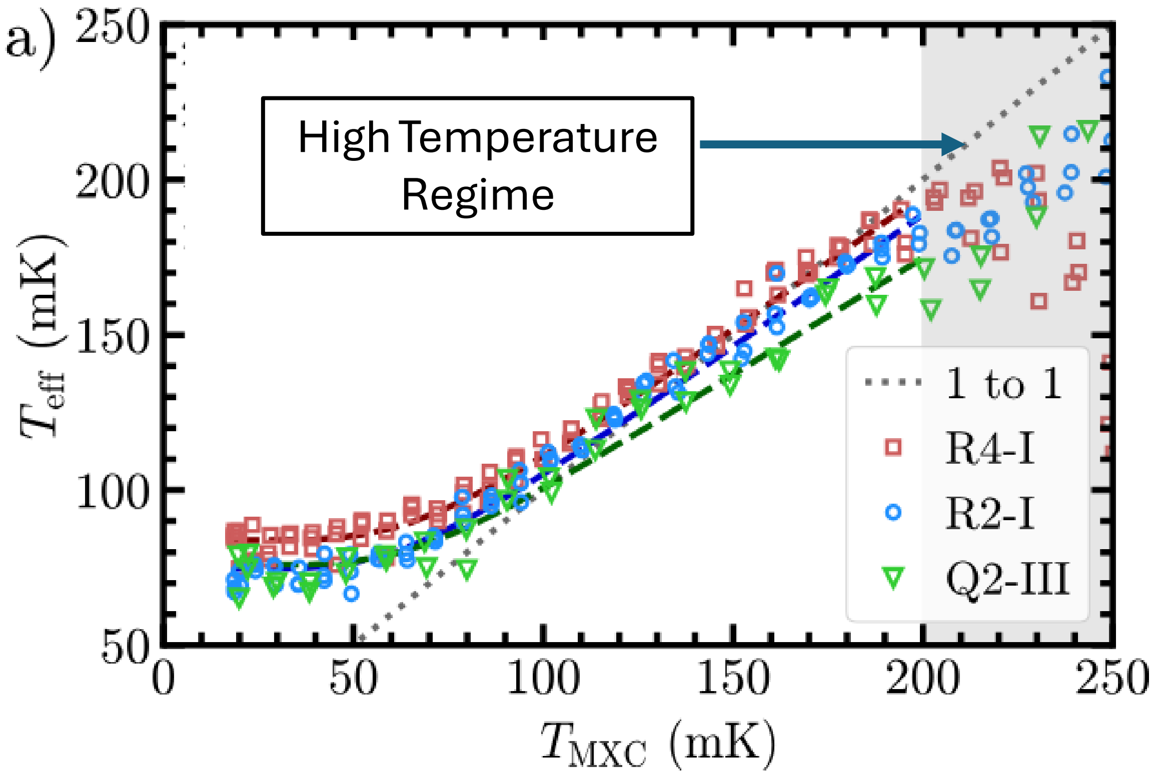
- Hot environment
  $$\frac{h\omega}{k_B T_{env}} \ll 1$$
- Strong coupling to MXC
  $$\frac{\gamma_{MXC}}{\gamma_1^0} \sim 1$$

$$T_{eff} \approx \frac{\gamma_{env}}{\gamma_1^0} T_{env}$$



High Temperatures



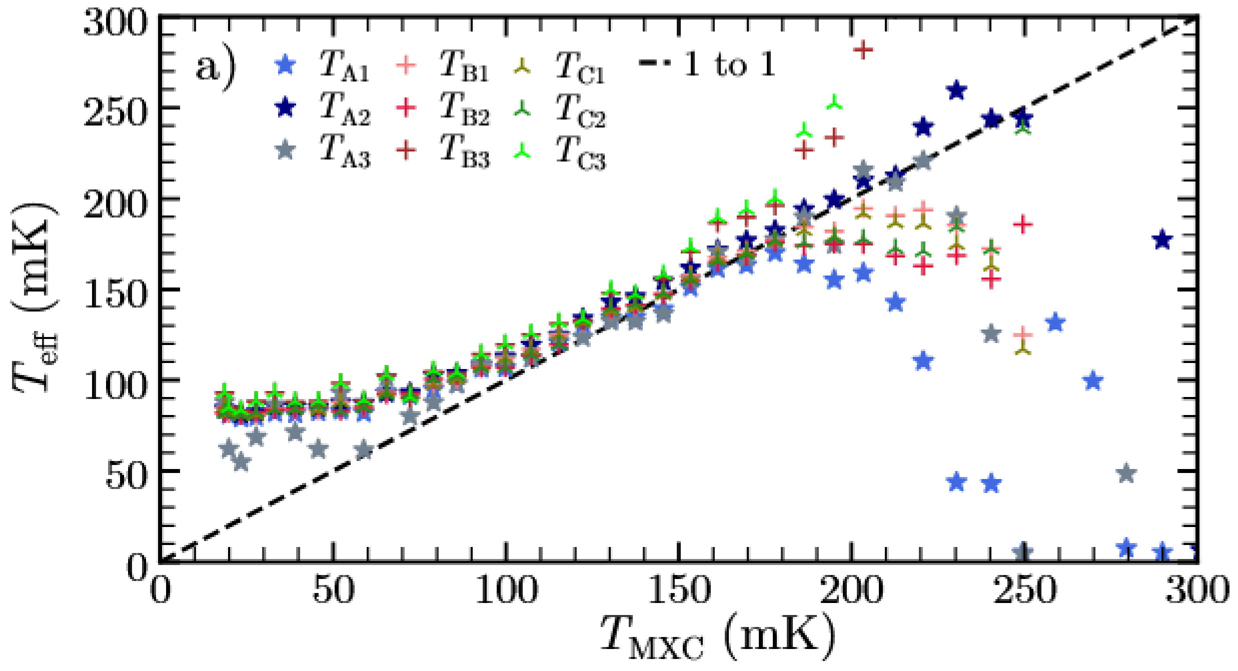


Quasiparticles

$$x_0 = p_g\varphi_g + p_e\varphi_e + p_f\varphi_f, \qquad \text{(no pulse)}$$
$$x_1 = p_e\varphi_g + p_g\varphi_e + p_f\varphi_f, \qquad (\pi_{ge})$$
$$x_2 = p_e\varphi_g + p_f\varphi_e + p_g\varphi_f, \qquad (\pi_{ge}\pi_{ef})$$
$$y_0 = p_g\varphi_g + p_f\varphi_e + p_e\varphi_f, \qquad (\pi_{ef})$$
$$y_1 = p_f\varphi_g + p_g\varphi_e + p_e\varphi_f, \qquad (\pi_{ef}\pi_{ge})$$
$$y_2 = p_f\varphi_g + p_e\varphi_e + p_g\varphi_f. \qquad (\pi_{ef}\pi_{ge}\pi_{ef})$$

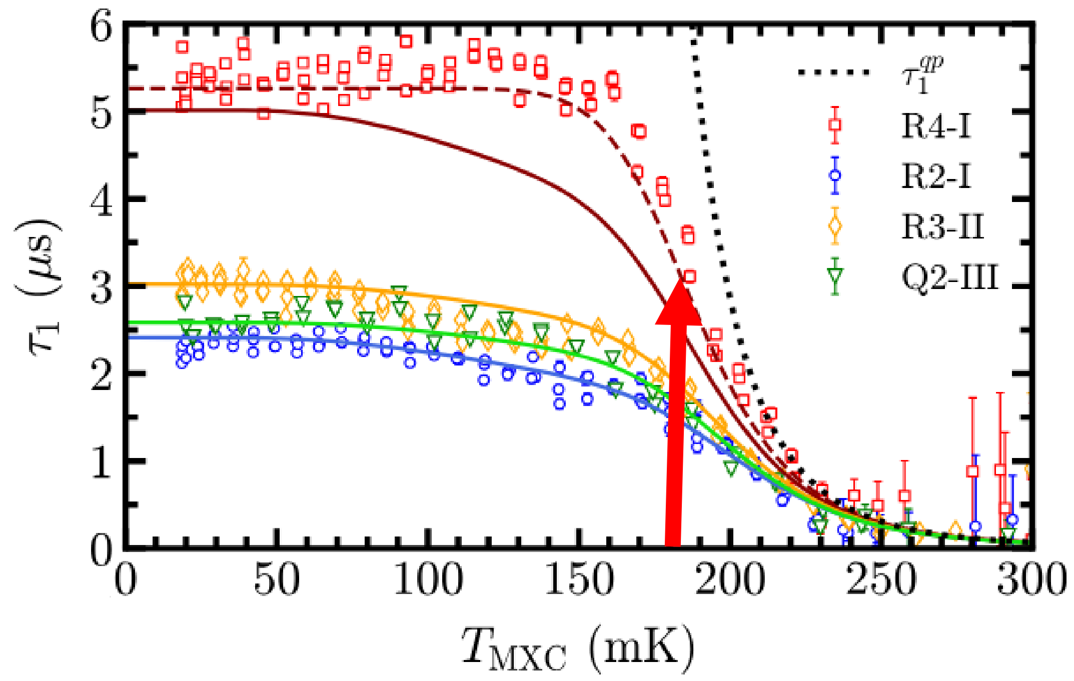
- One sample
- Different ways to measure temperature

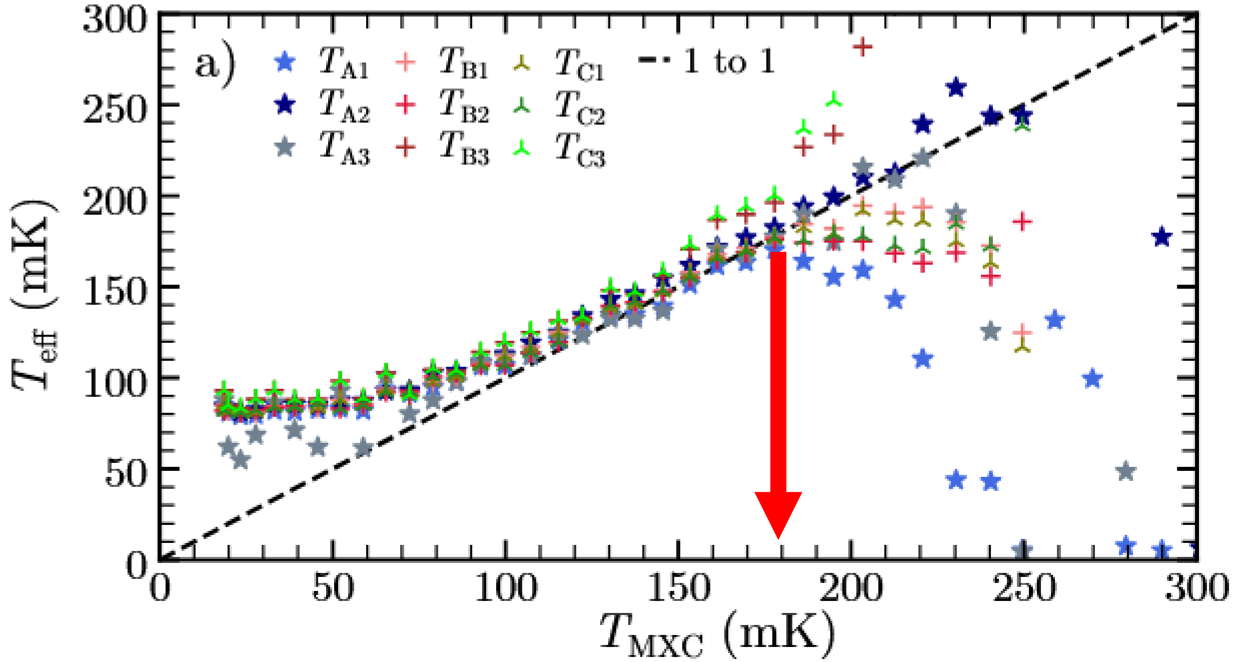
Ratio	Calculation method		
	1	2	3
A	$\frac{x_0-x_1}{y_0-y_1}$	$\frac{y_0-x_2}{x_0-y_2}$	$\frac{y_1-y_2}{x_1-x_2}$
B	$\frac{x_2-y_2}{x_0-x_1}$	$\frac{x_1-y_1}{y_0-x_2}$	$\frac{x_0-y_0}{y_1-y_2}$
C	$\frac{x_2-y_2}{y_0-y_1}$	$\frac{x_1-y_1}{x_0-y_2}$	$\frac{x_0-y_0}{x_1-x_2}$



Relaxation Time

- Significant changes at T > 180mK



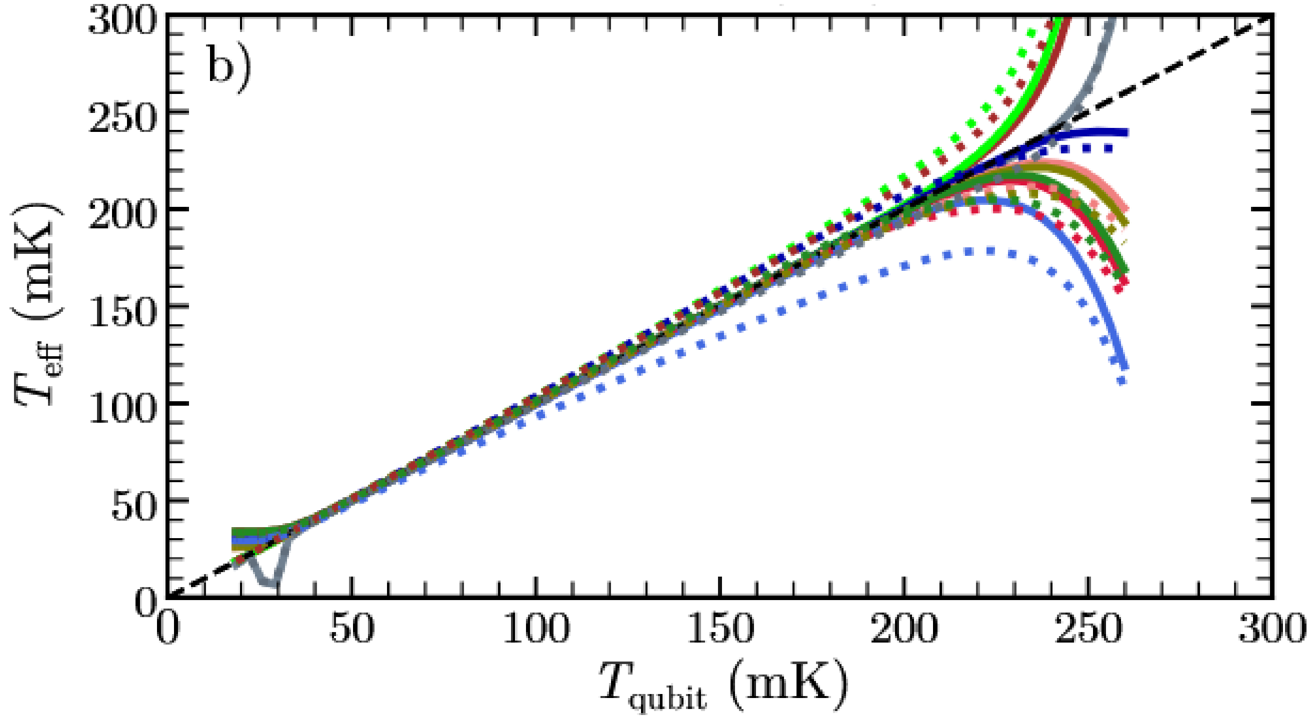


$$\tau_1 = 1/\left\{\gamma_1^{qp}\left(T_{\rm MXC}\right) + \gamma_1^0\left[2n\left(\omega_{ge}, T_{\rm eff}\right) + 1\right]\right\},$$

$$\gamma_1^{qp} = \frac{1}{\pi}\frac{\omega_p^2}{\omega_{ge}}\left\{x_{qp}\sqrt{\frac{2\Delta}{\hbar\omega_{ge}}} + 4e^{-\frac{\Delta}{k_B T}}\cosh\left(\frac{\hbar\omega_{ge}}{2k_B T}\right)K_0\left(\frac{\hbar\omega_{ge}}{2k_B T}\right)\right\}$$

Including Time Evolution of states

Theoretical Model

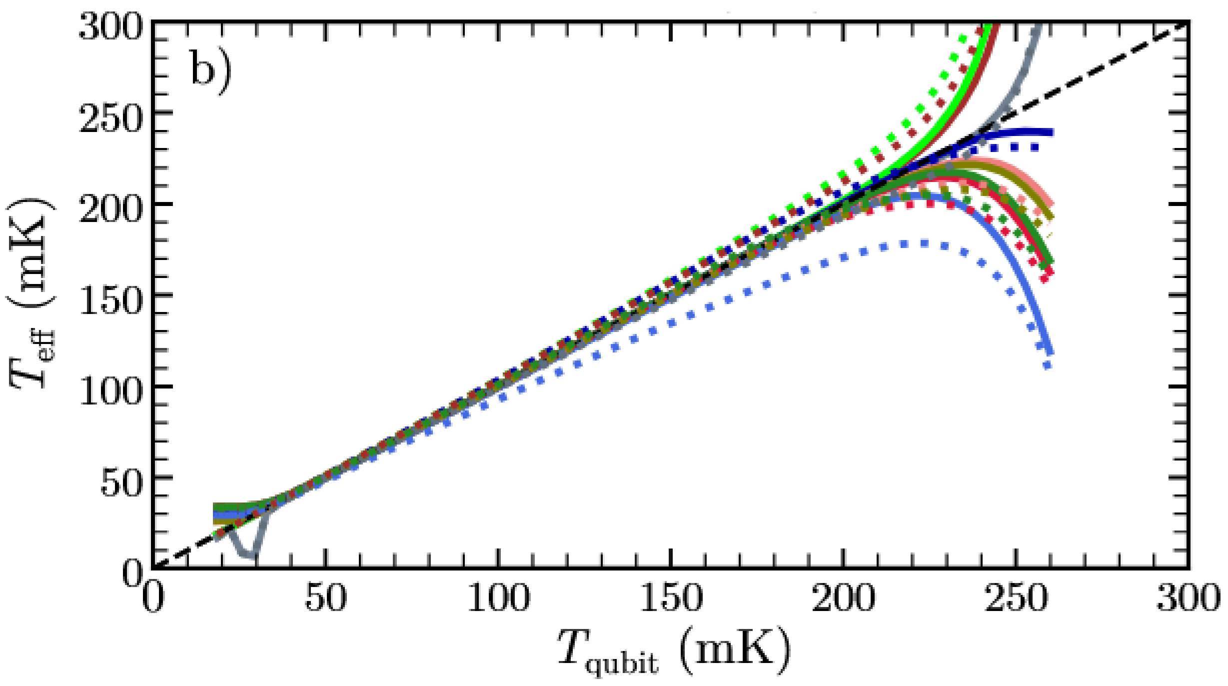


- Solid line: Time evolution during readout
- Dotted line: Time evolution during readout + imperfect pulsing

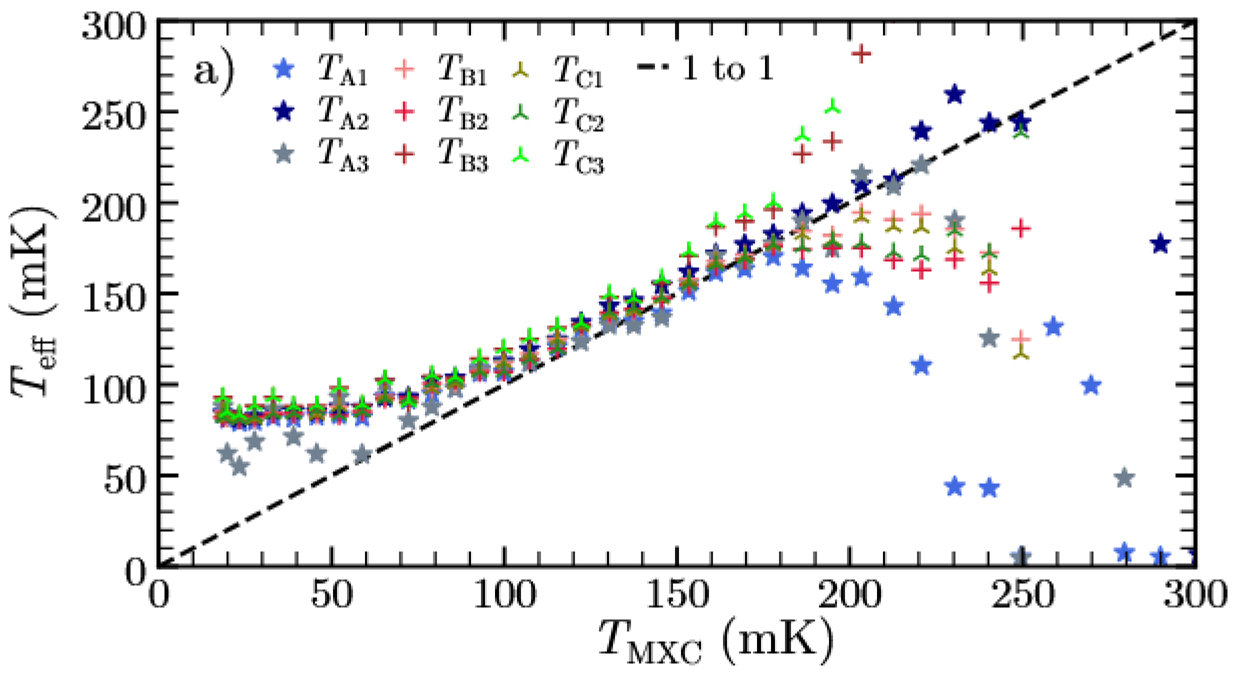
Model-Experiment

- Model predicts the error seen in experiment

Theoretical Model

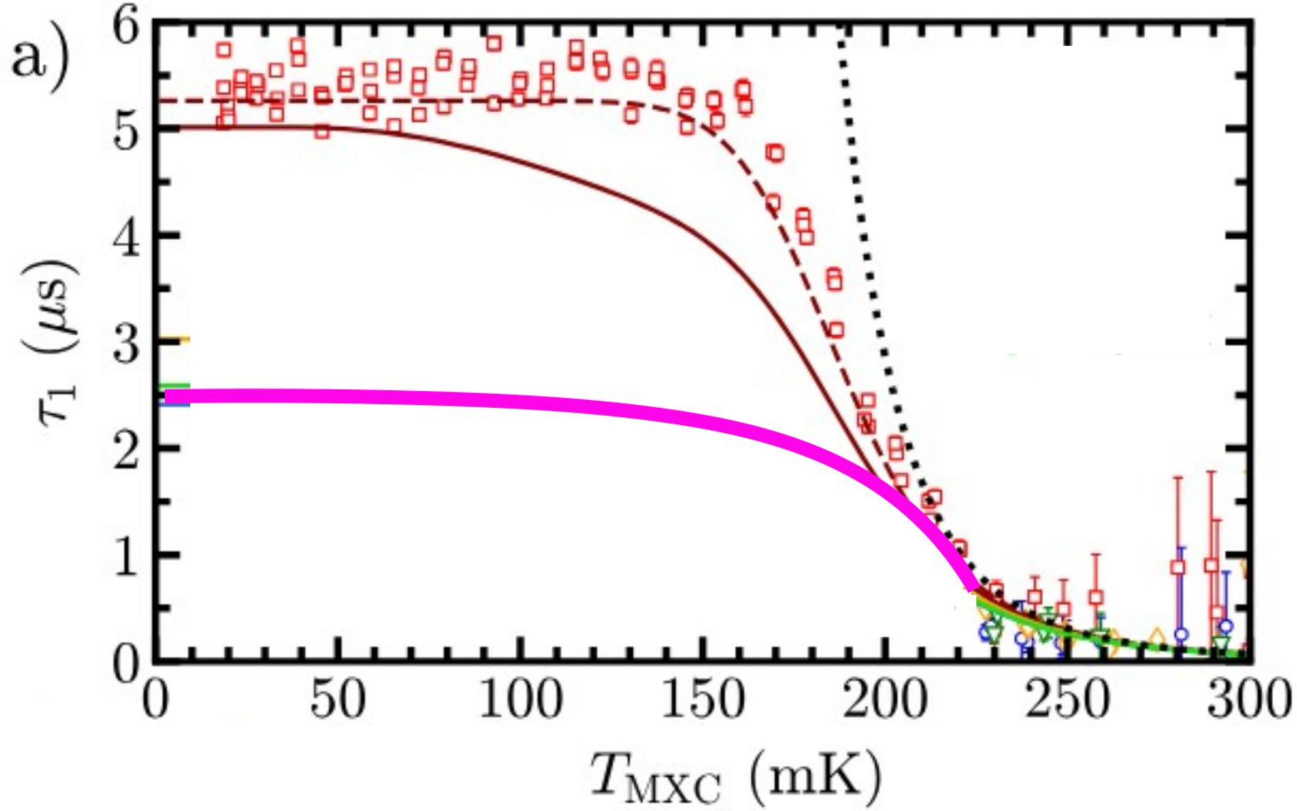


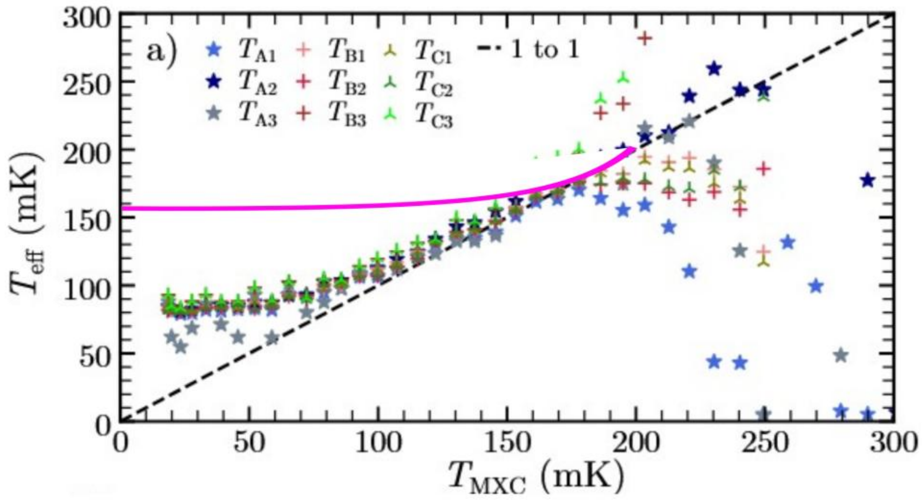
Experiment



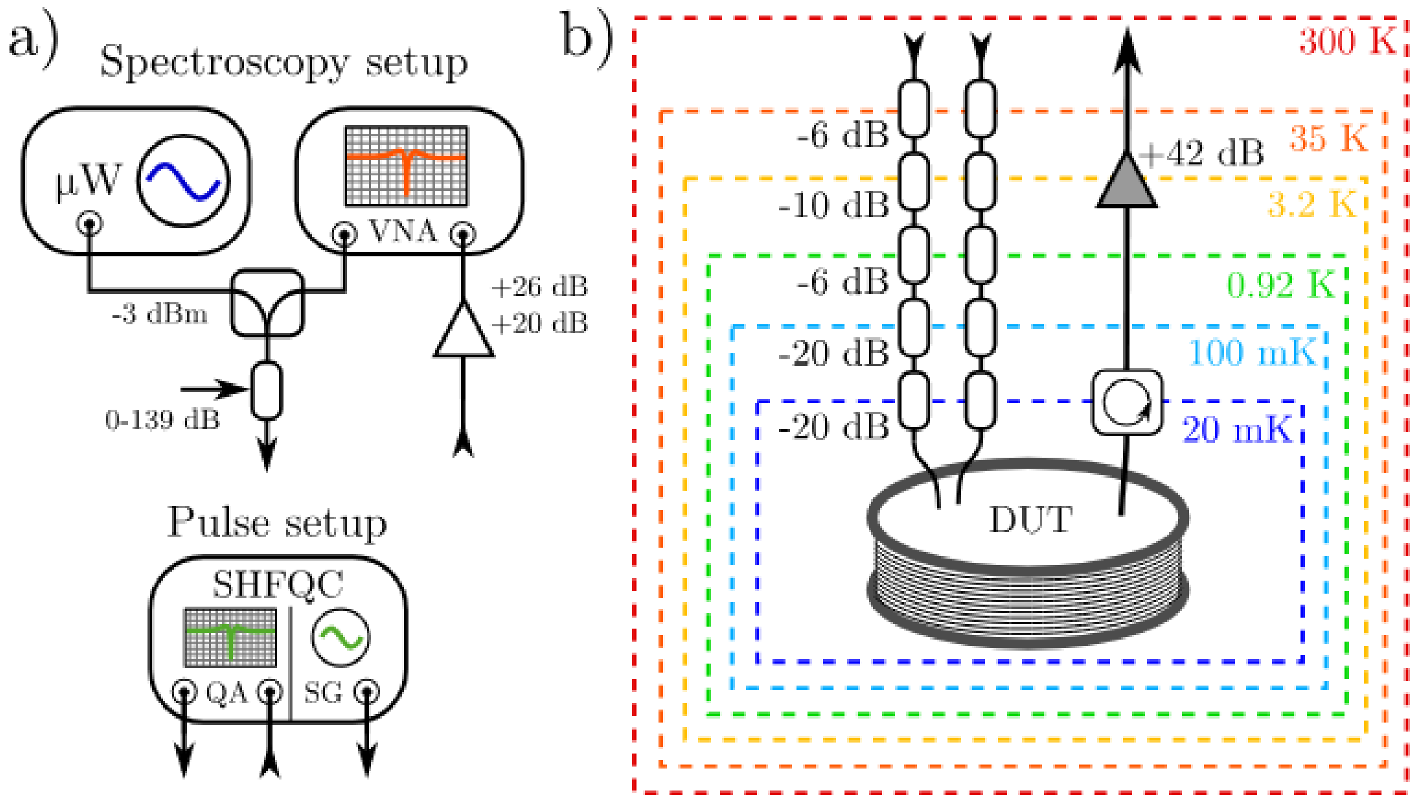
Probing Environment

- Could be bad qubit
- Could be hot environment





Setup

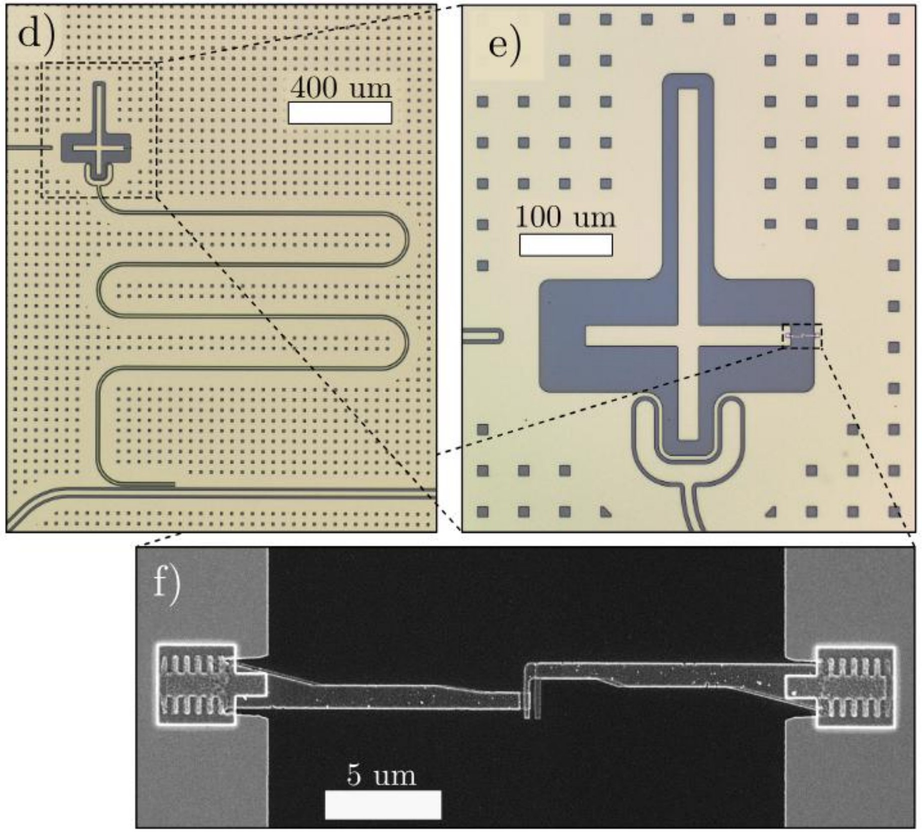


a)
Spectroscopy setup
μW
VNA
-3 dBm
+26 dB
+20 dB
0-139 dB
Pulse setup
SHFQC
QA
SG
b)
300 K
35 K
3.2 K
0.92 K
100 mK
20 mK
-6 dB
-10 dB
-6 dB
-20 dB
-20 dB
+42 dB
DUT

Devices

TABLE II. Parameters of the devices.

Device	$\omega_{ge}/2\pi$, GHz	$\omega_{ef}/2\pi$, GHz	E_c/h, MHz	$\Delta_r/2\pi$, GHz	$g/2\pi$, MHz
R2-I	6.422	6.221	201	1.876	34
R4-I	6.649	6.417	232	1.753	38
R3-II	6.732	6.513	219	0.765	44
Q2-III	7.042	6.835	207	2.151	37



Applications

- Microscopic thermometer
- Probing environment temperature and coupling

Conclusion

Thermometry done based on 3 levels of Transmon Qubit

→ Range 60 – 200 mK

→ Deviations well explained by two-bath model and Quasiparticles

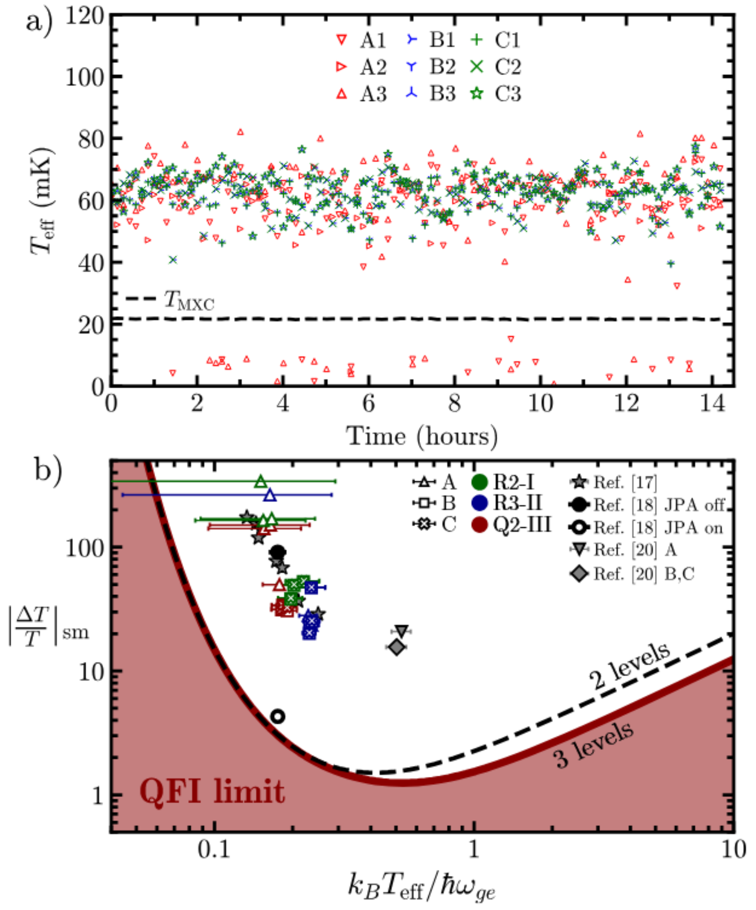
→ Analysis determined effective environment temperature as well as its coupling strength

→ Niche Applications

Thank you!

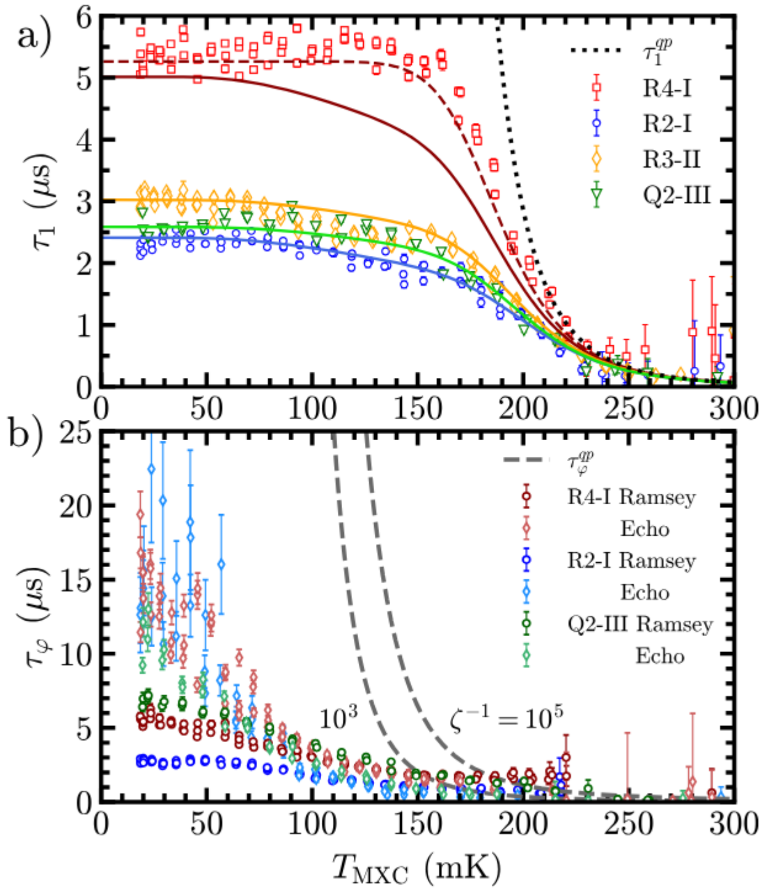
Appendix

Limits



a)
A1
A2
A3
B1
B2
B3
C1
C2
C3
T_{eff} (mK)
T_{MXC}
Time (hours)
b)
A
B
C
R2-I
R3-II
Q2-III
Ref. [17]
Ref. [18] JPA off
Ref. [18] JPA on
Ref. [20] A
Ref. [20] B,C
$\left|\frac{\Delta T}{T}\right|_{\mathrm{sm}}$
2 levels
3 levels
QFI limit
$k_B T_{\mathrm{eff}}/\hbar\omega_{ge}$

Ramsey Echo



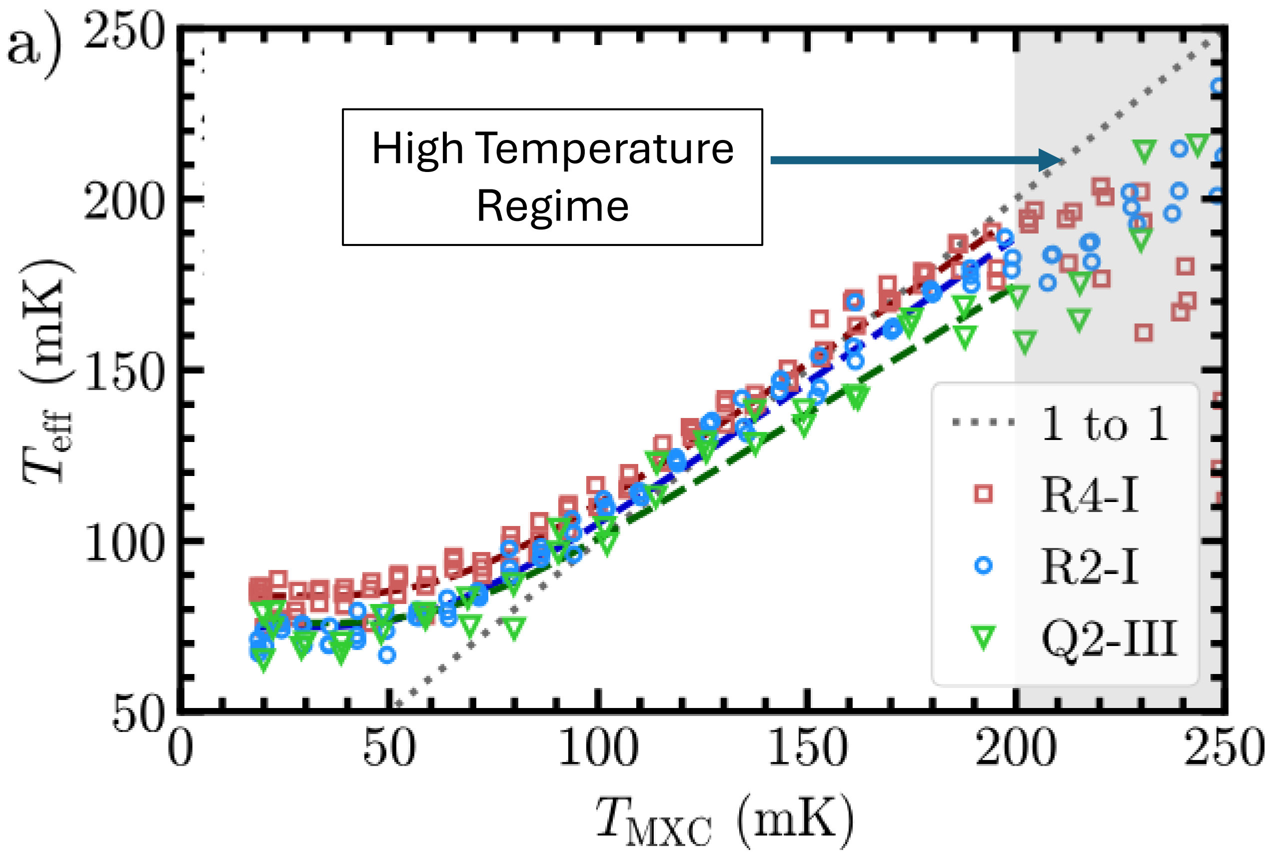
a)
b)
τ_1^{qp}
R4-I
R2-I
R3-II
Q2-III
τ_1 (μs)
τ_φ^{qp}
R4-I Ramsey
Echo
R2-I Ramsey
Echo
Q2-III Ramsey
Echo
τ_φ (μs)
10^3
$\zeta^{-1} = 10^5$
T_{MXC} (mK)

HIGH TXMC

2 Components to consider

Multiple Heat Baths

Quasiparticles



HIGH TXMC

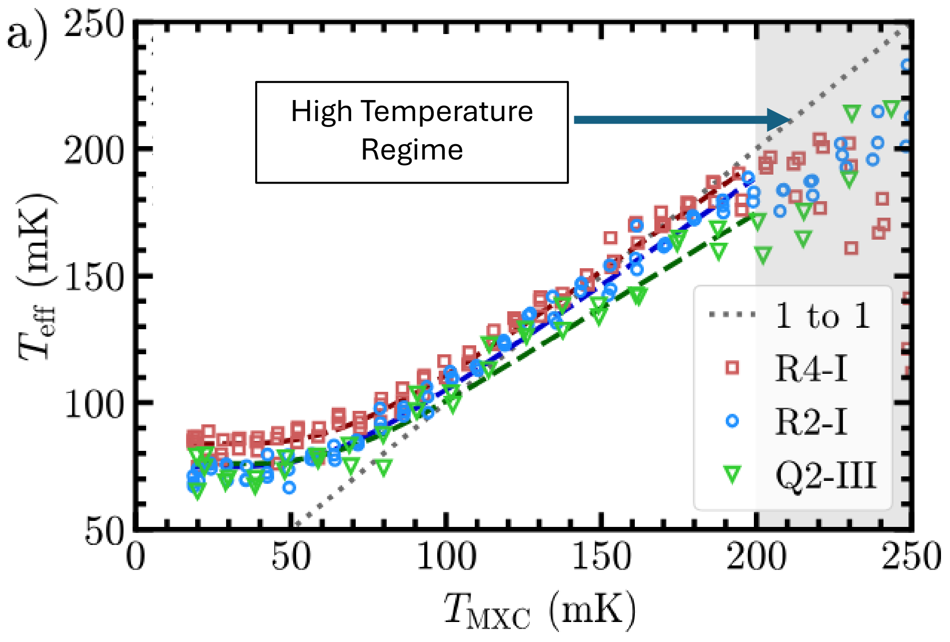
2 Components to consider

Multiple Heat Baths

Quasiparticles

RESULT

$$T_{\mathrm{eff}} \approx \frac{\gamma_{\mathrm{MXC}}}{\gamma_1^0} T_{\mathrm{MXC}}$$



- 2 Taylor expansions assuming $\frac{\hbar\omega}{k_B T} \ll 1$
- Weak coupling to environment

Occupation Probabilities