Universal anyon tunnelling in a chiral Luttinger liquid

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The edge modes of fractional quantum Hall liquids are described by chiral Luttinger liquid theory. Despite many years of experimental investigation, fractional quantum Hall edge modes are not fully understood, and clear discrepancies between experimental observations and detailed predictions of chiral Luttinger liquid theory remain. Here we report the measurements of tunnelling conductance between counterpropagating edge modes at a filling factor of 1/3 across a quantum point contact. We present evidence for the tunnelling of anyons through an incompressible liquid that exhibits universal scaling behaviour with respect to temperature, source–drain bias and barrier transmission, as originally proposed by prior theoretical work. For large transmission through the quantum point contact, we measured the tunnelling exponent $\bar{g} = 0.333 \pm 0.005$ averaged over 29 independent datasets, consistent with the scaling dimension of 1/6 for a Laughlin quasiparticle at the edge. When combined with the measurements of the fractional charge and the recently observed anyonic statistical angle, the measured tunnelling exponent fully characterizes the topological order of the primary Laughlin state at the filling factor of 1/3.

Universal anyon tunnelling in a chiral Luttinger liquid

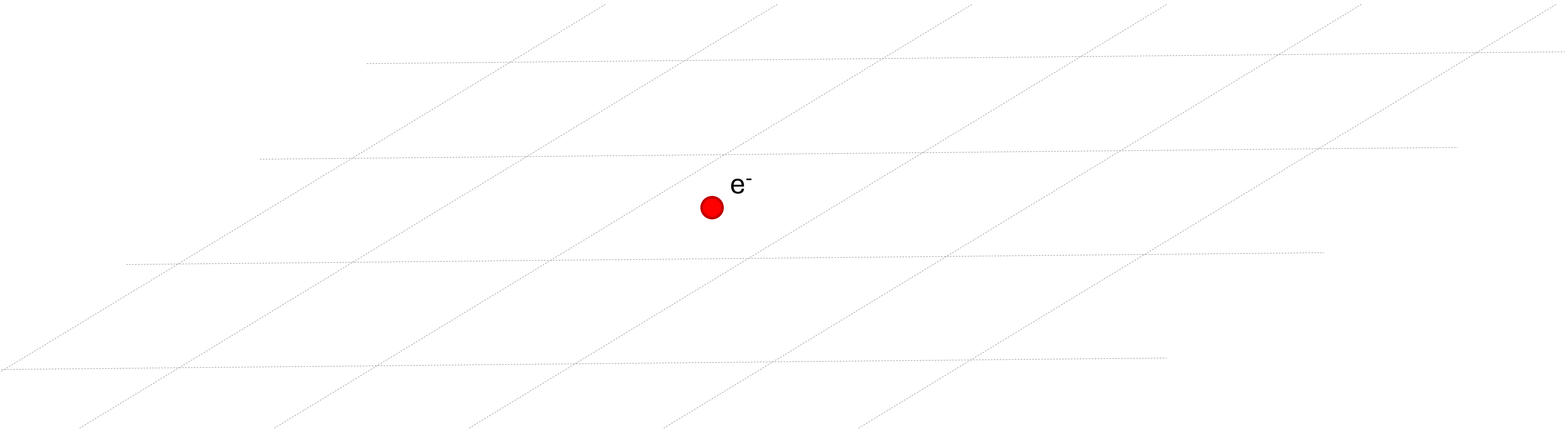
Journal Club

2025-10-03

Outline

- Introduction: from electrons in a GaAs 2DEG to anyons in a chiral Luttinger liquid
- Device and description of the experiment
- Results and analysis with respect to expected tunnelling behaviour (universal scaling behaviour)
- Conclusions

Quantum Hall effect



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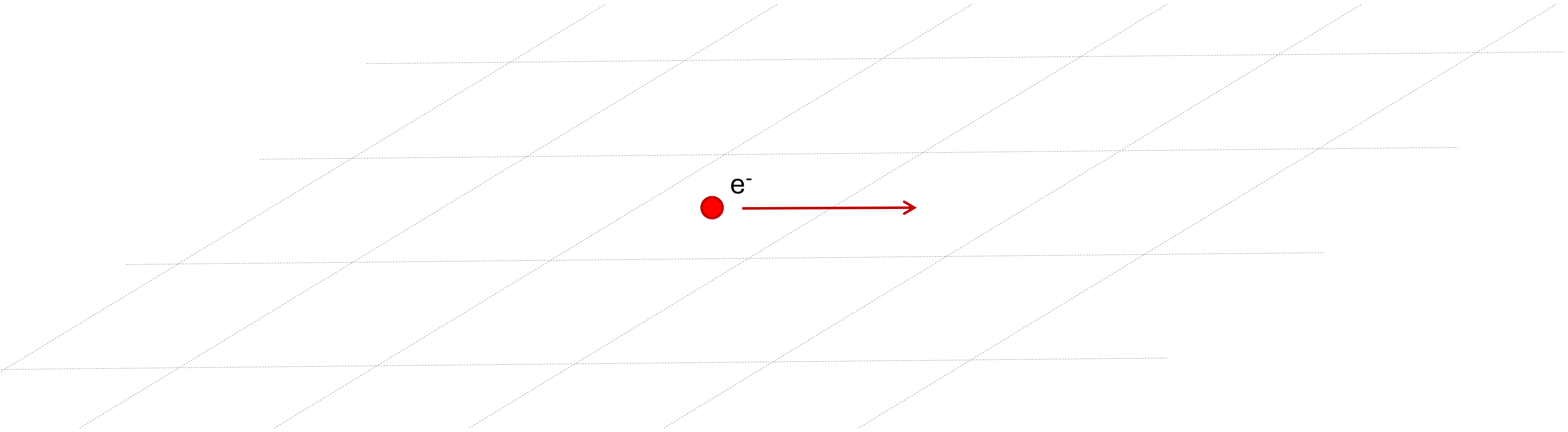

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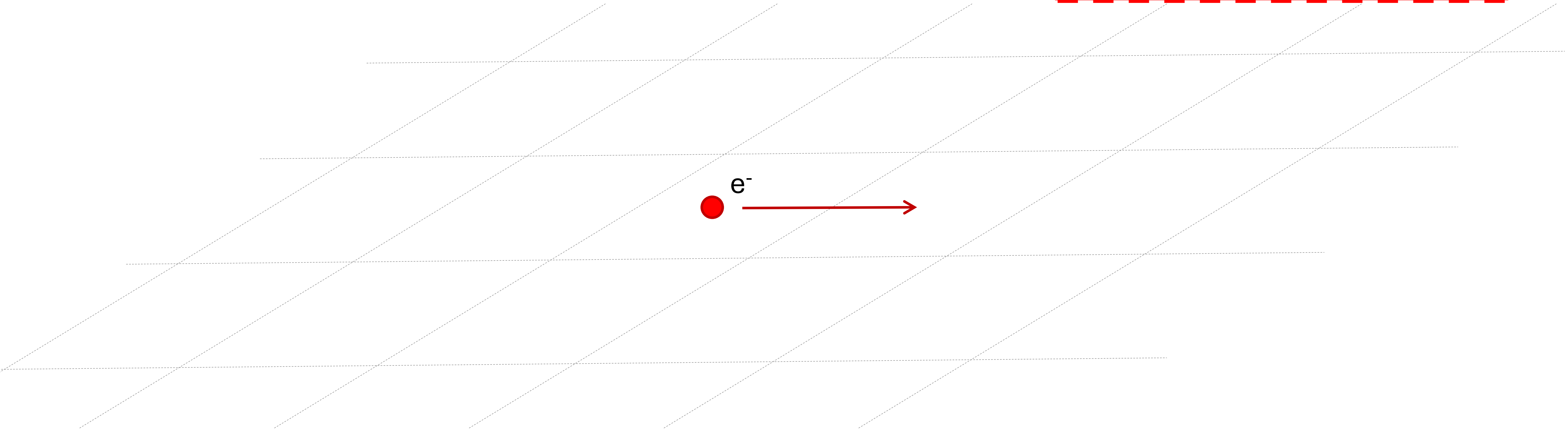

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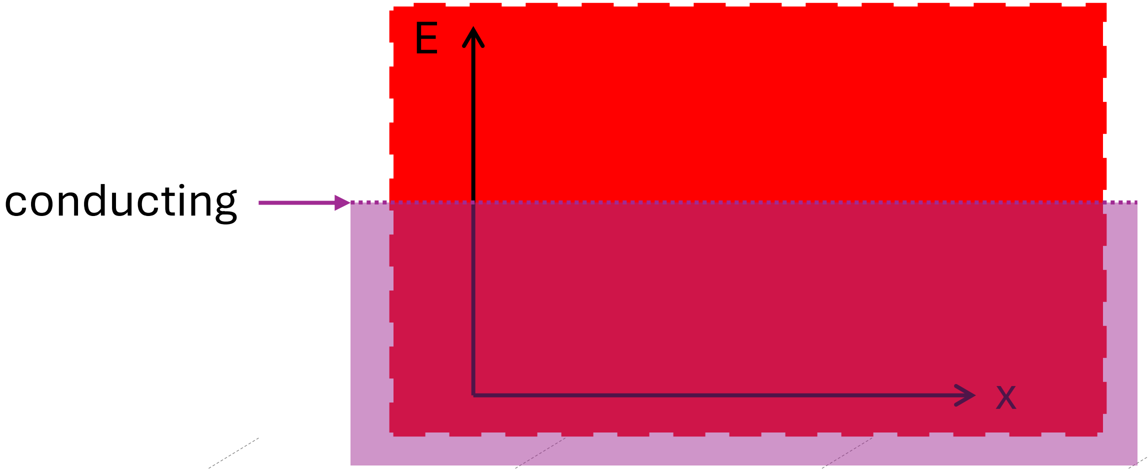

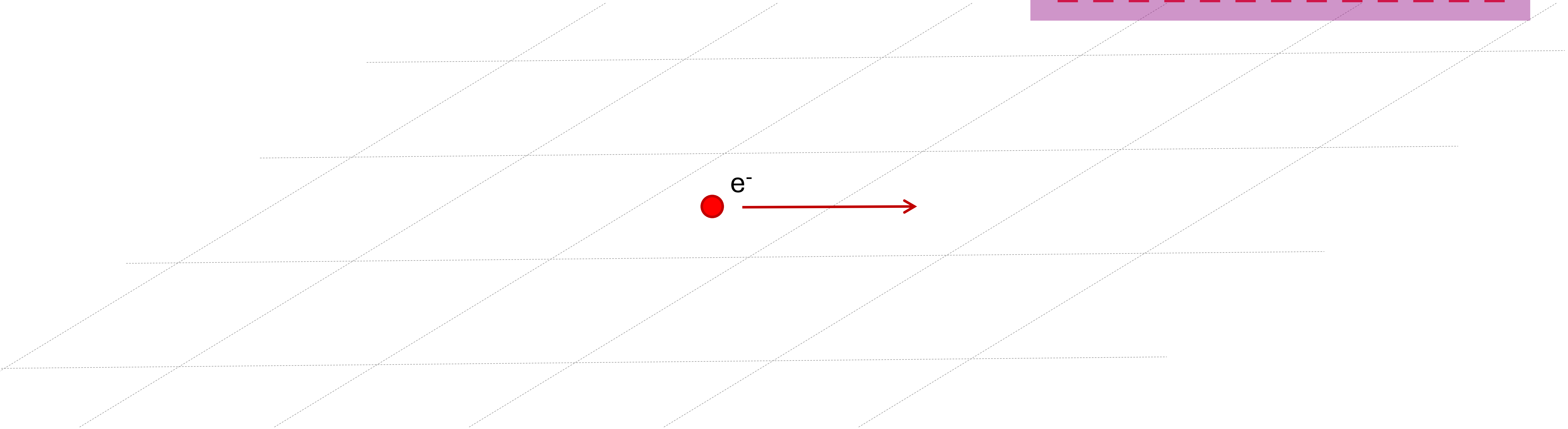

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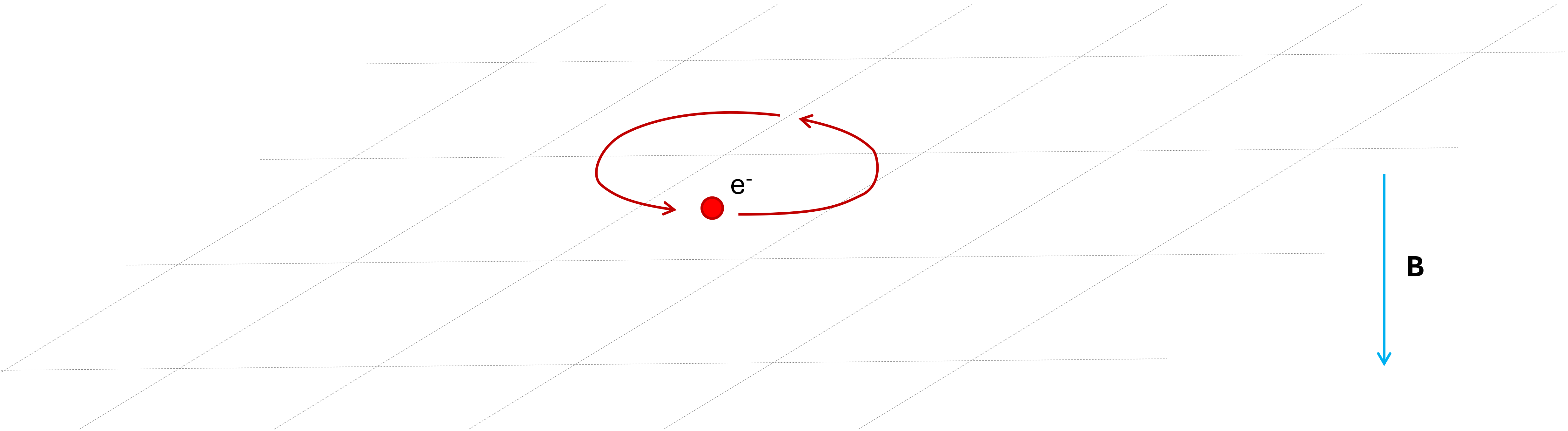

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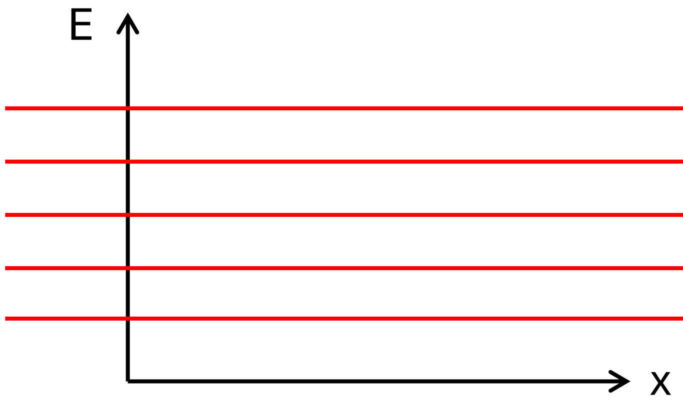

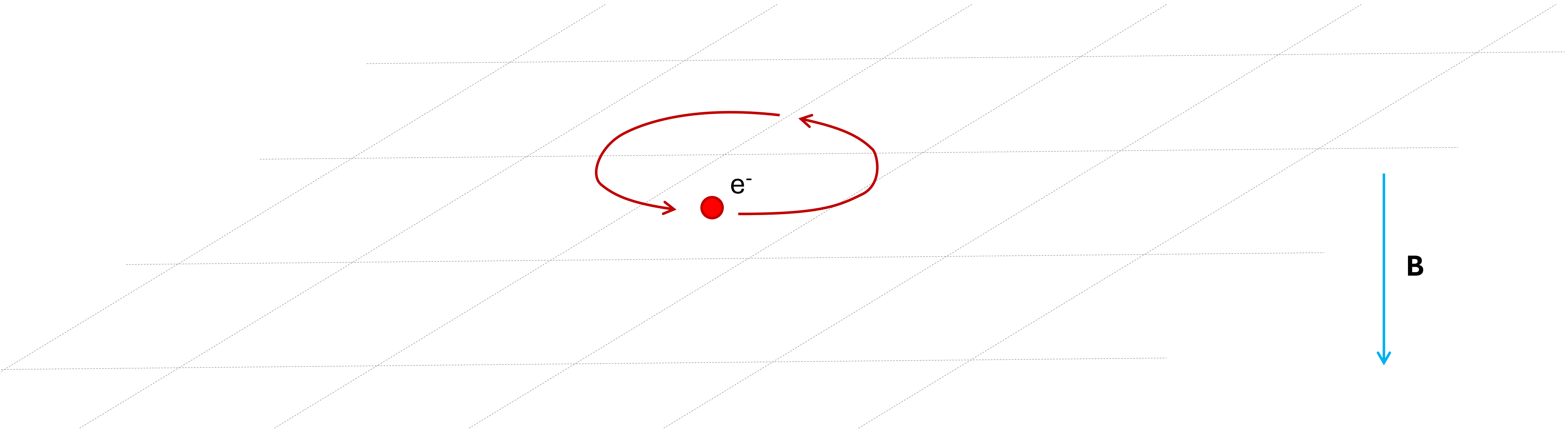

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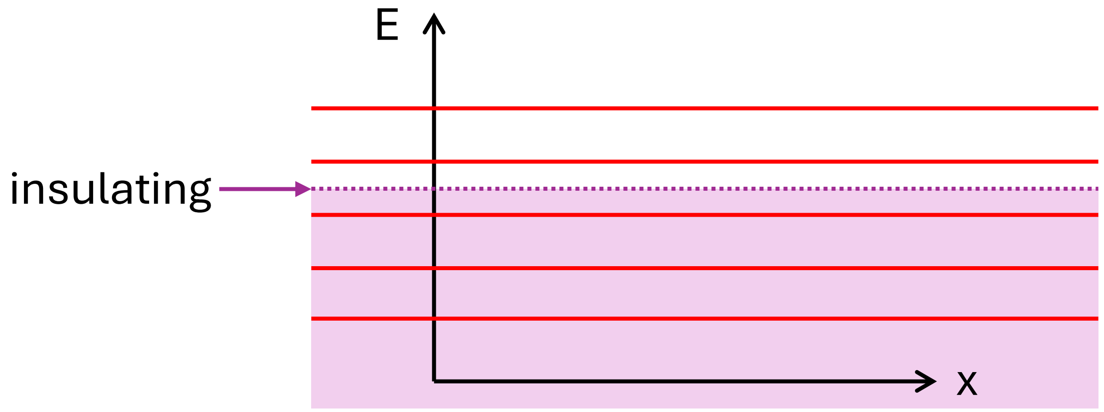

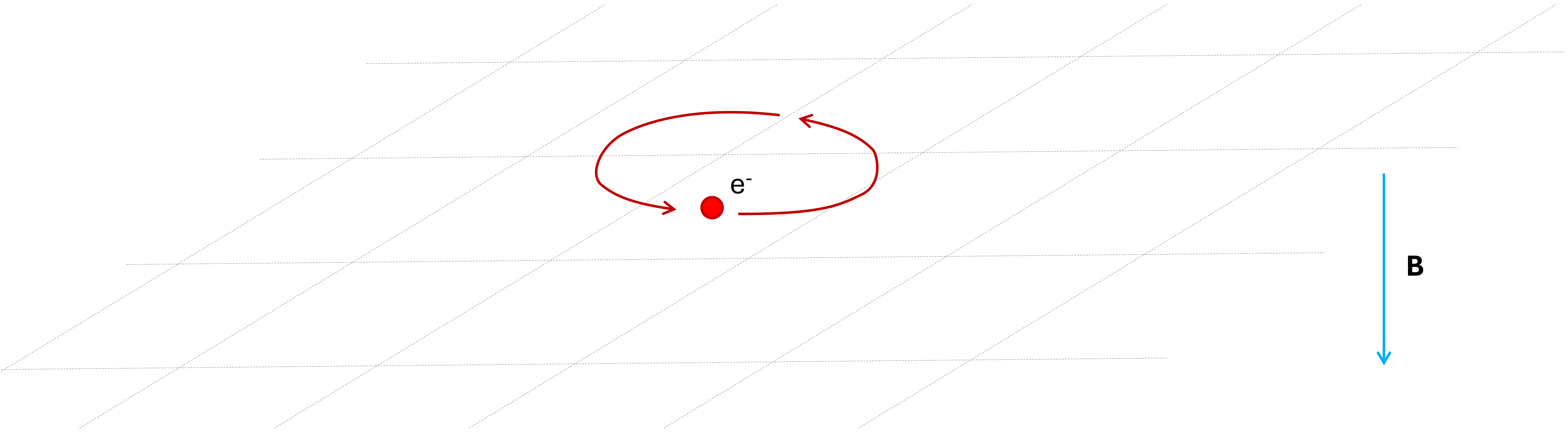

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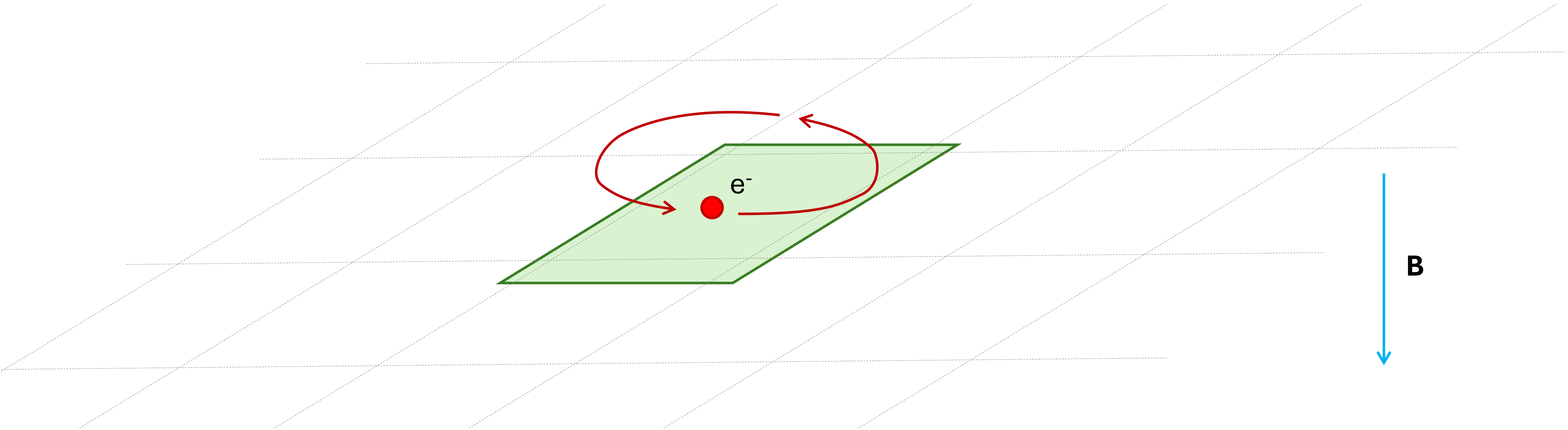

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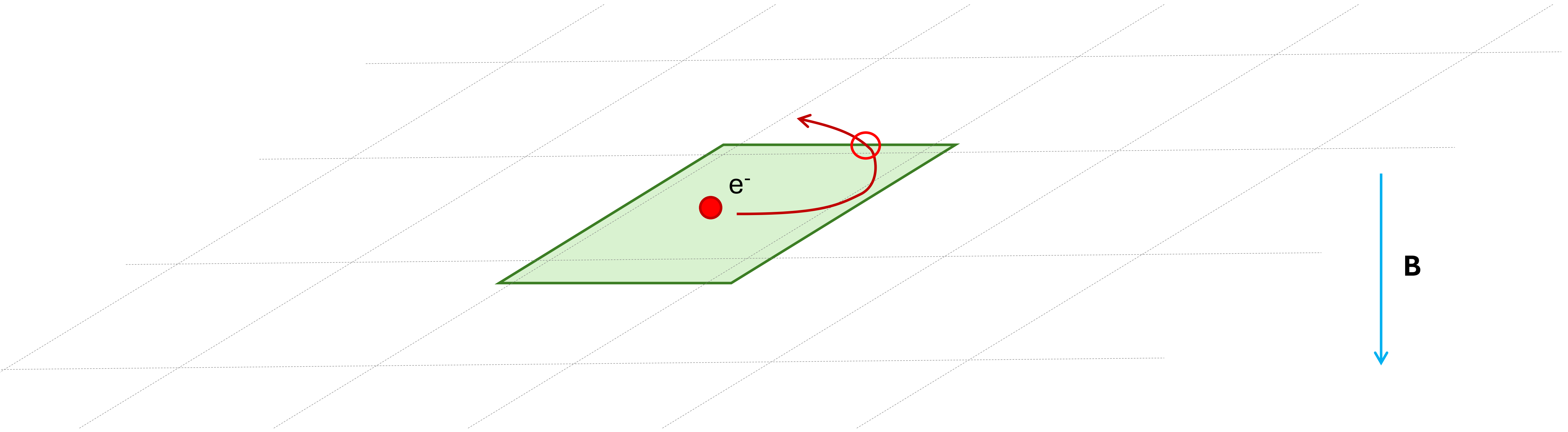

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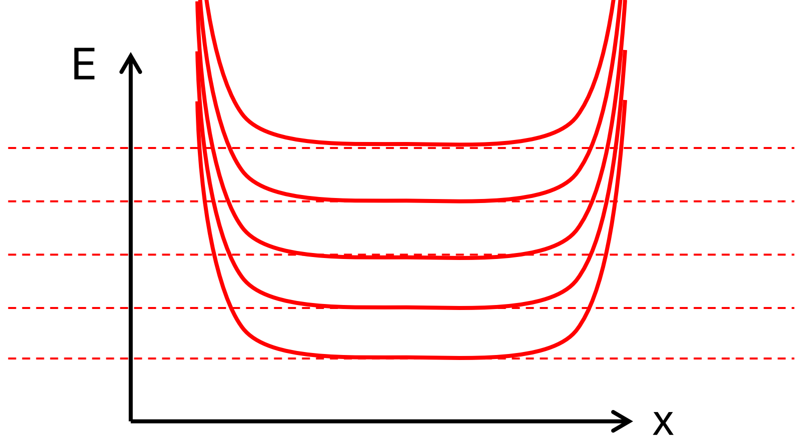

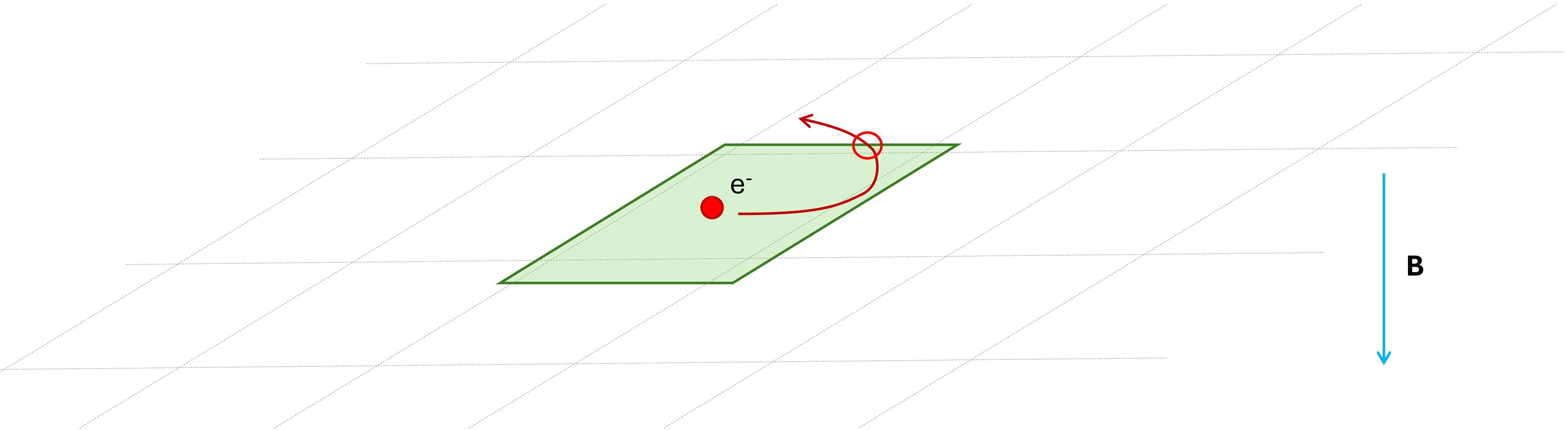

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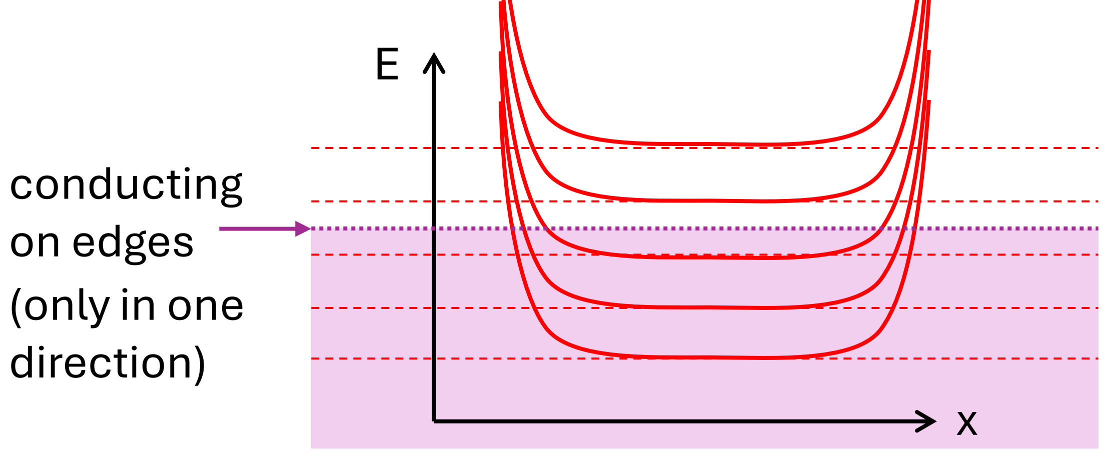

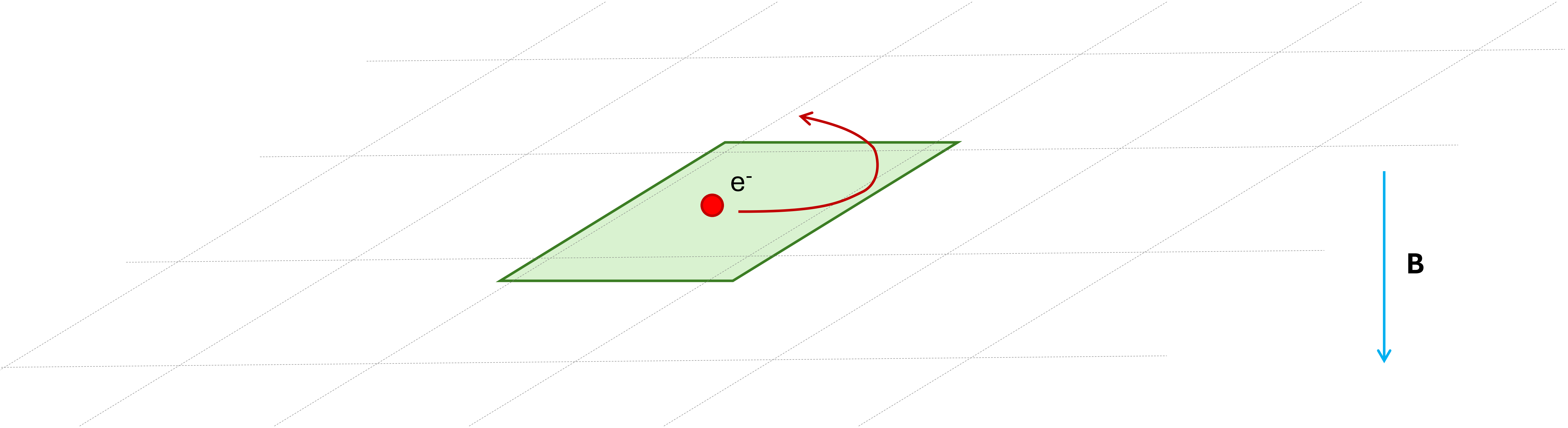

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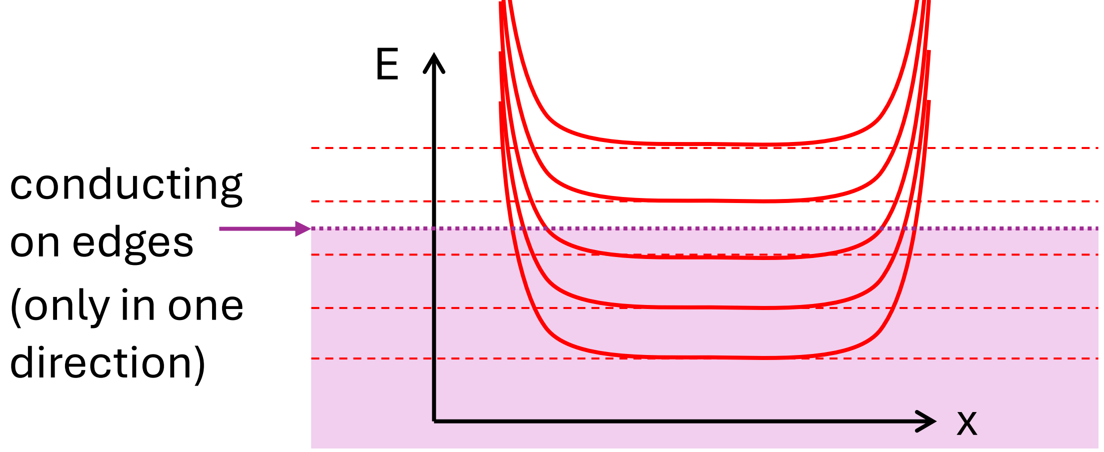
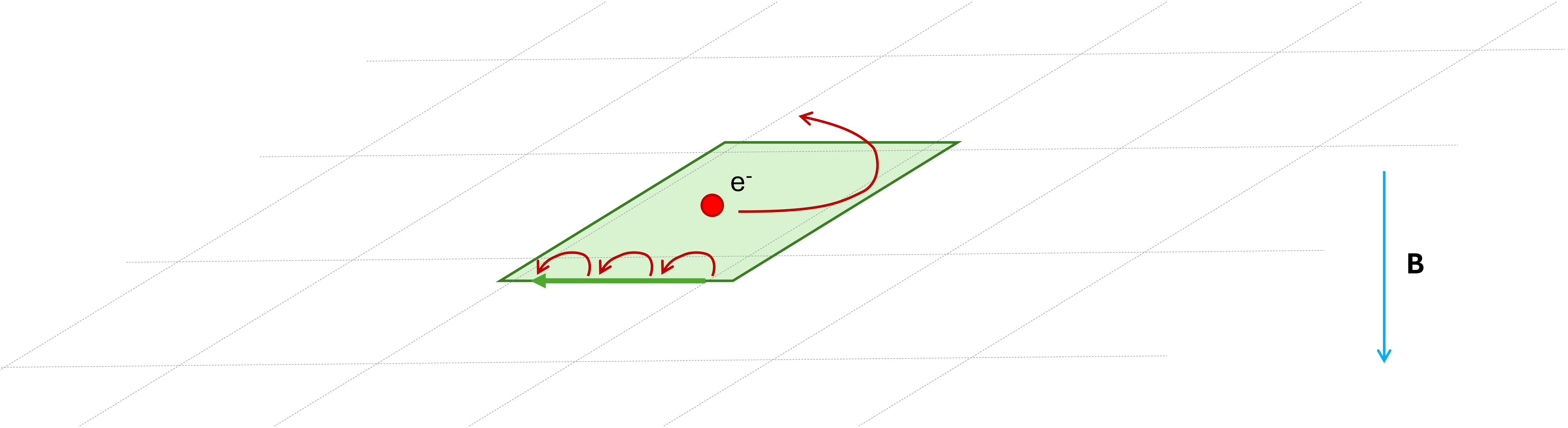
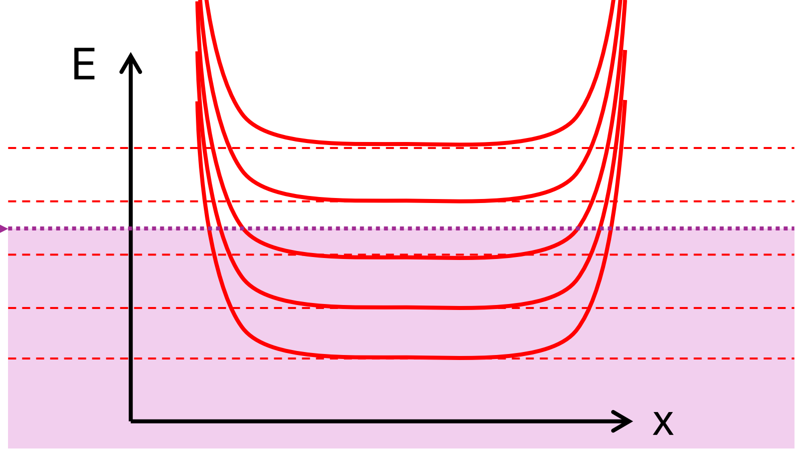
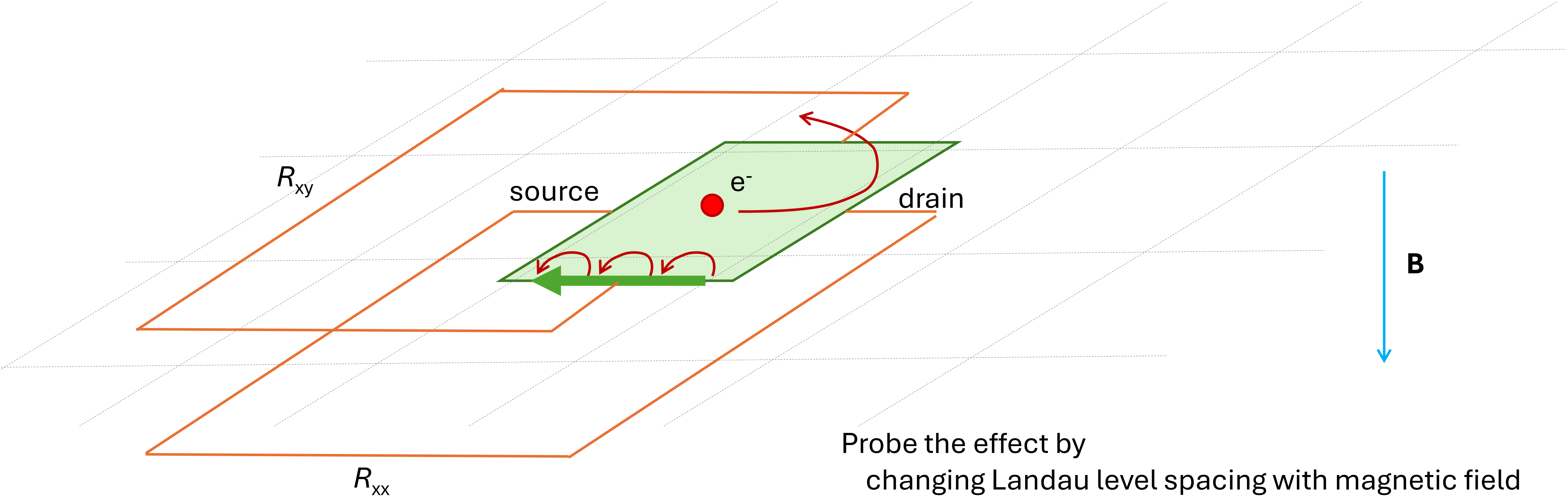




Quantum Hall effect

conducting on edges (only in one direction)



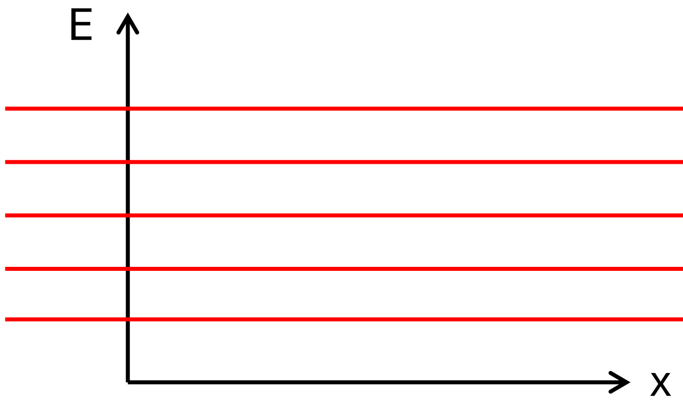


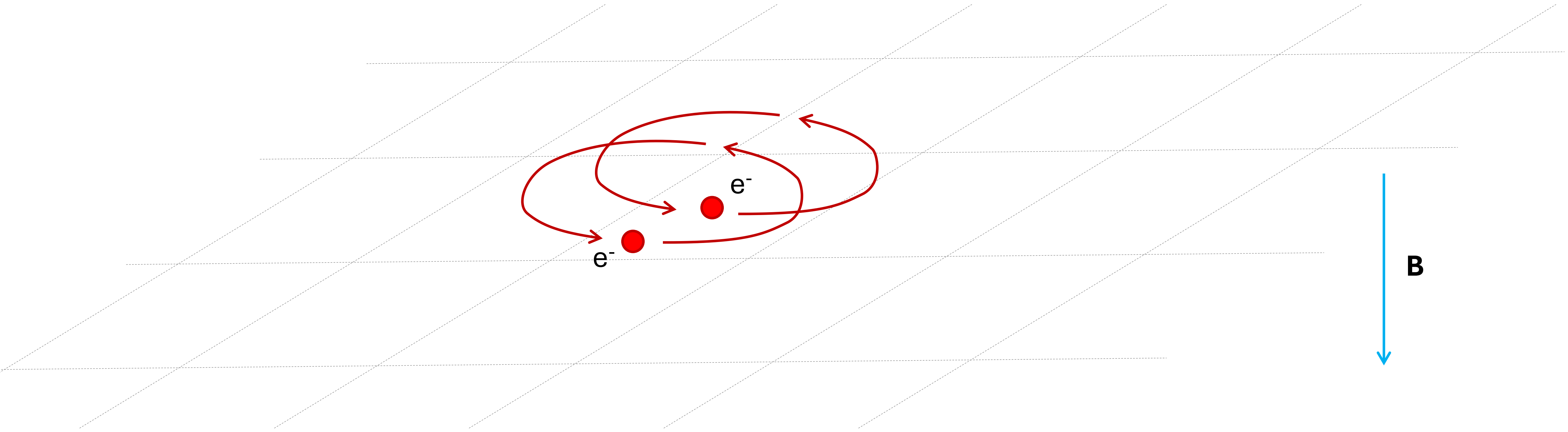
Probe the effect by

changing Landau level spacing with magnetic field

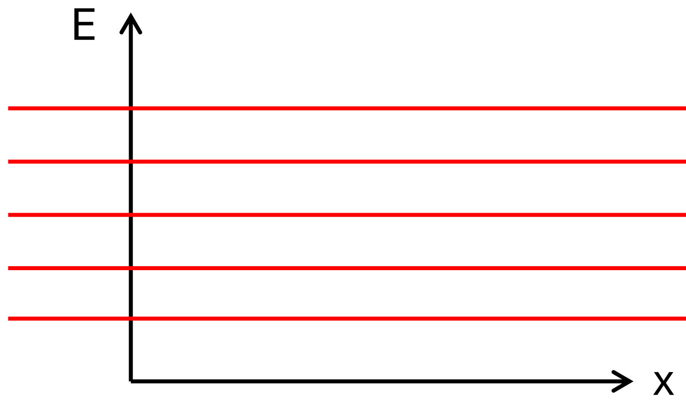
changing Fermi surface with electrochemical potential

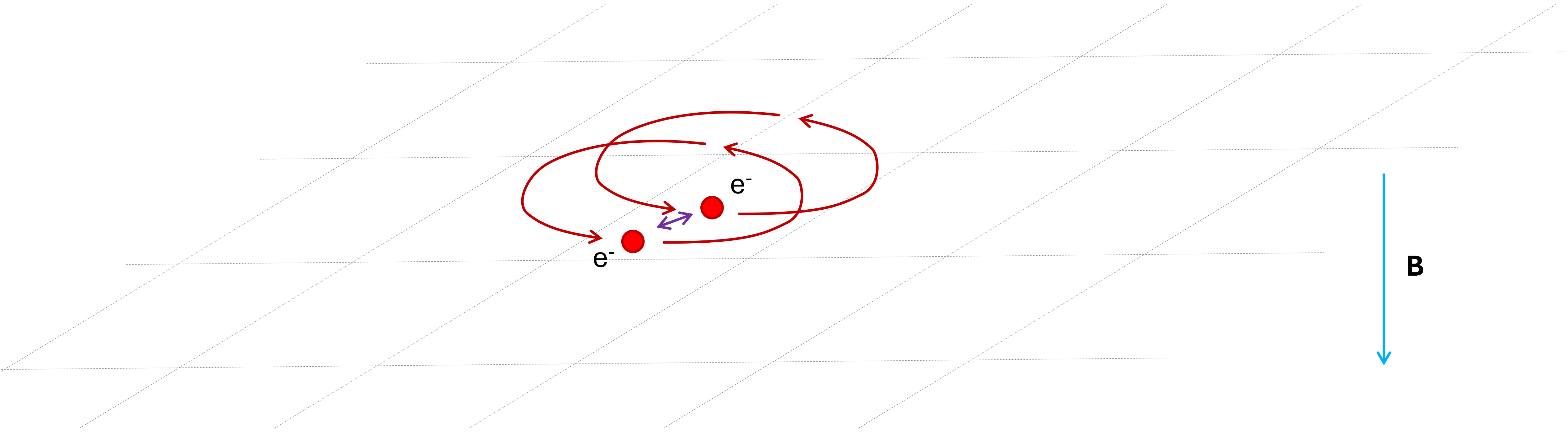
Fractional Quantum Hall effect





Fractional Quantum Hall effect





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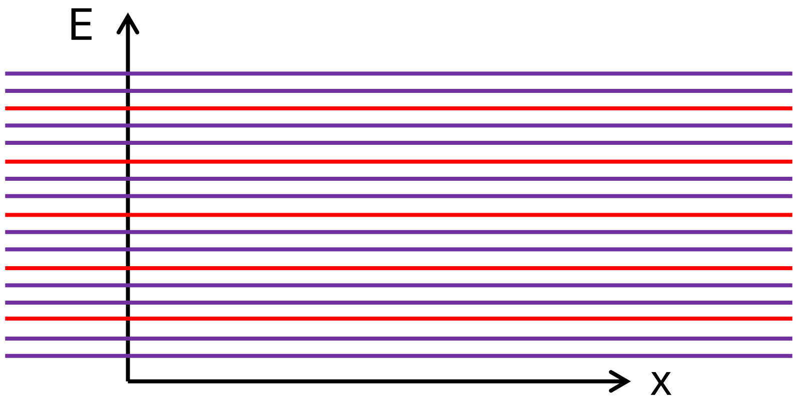

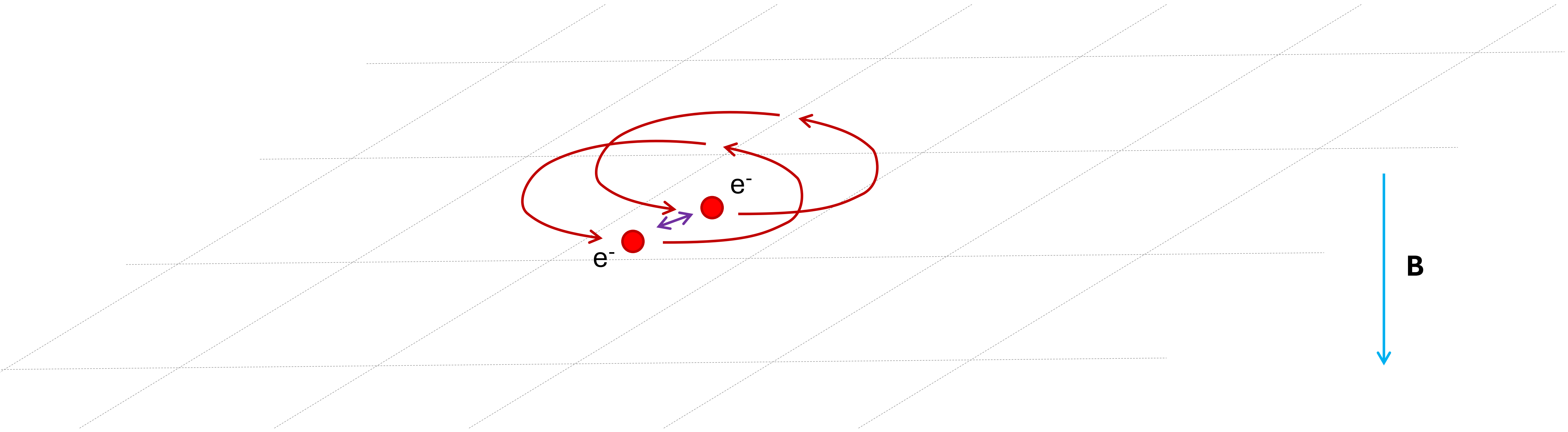

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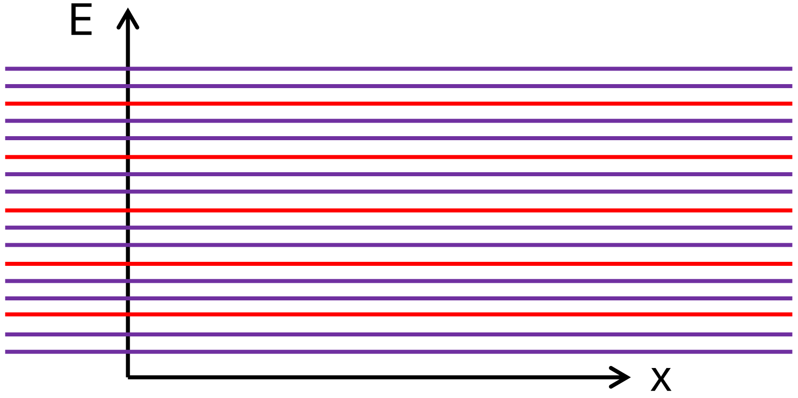

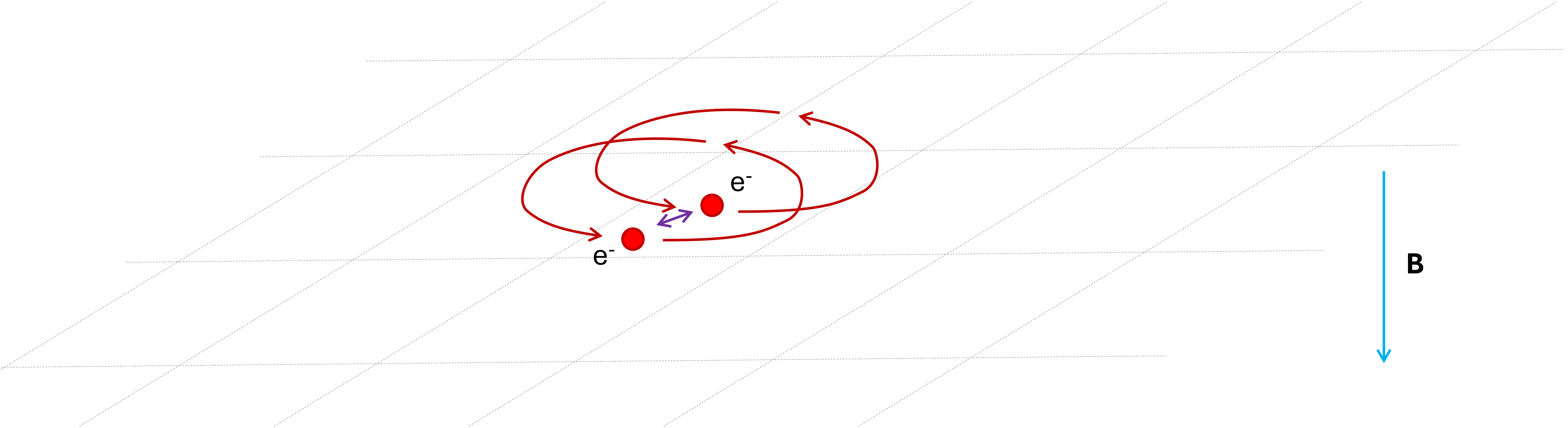

Laughlin states
Composite fermions

'Integer quantum Hall effect of composite fermions'

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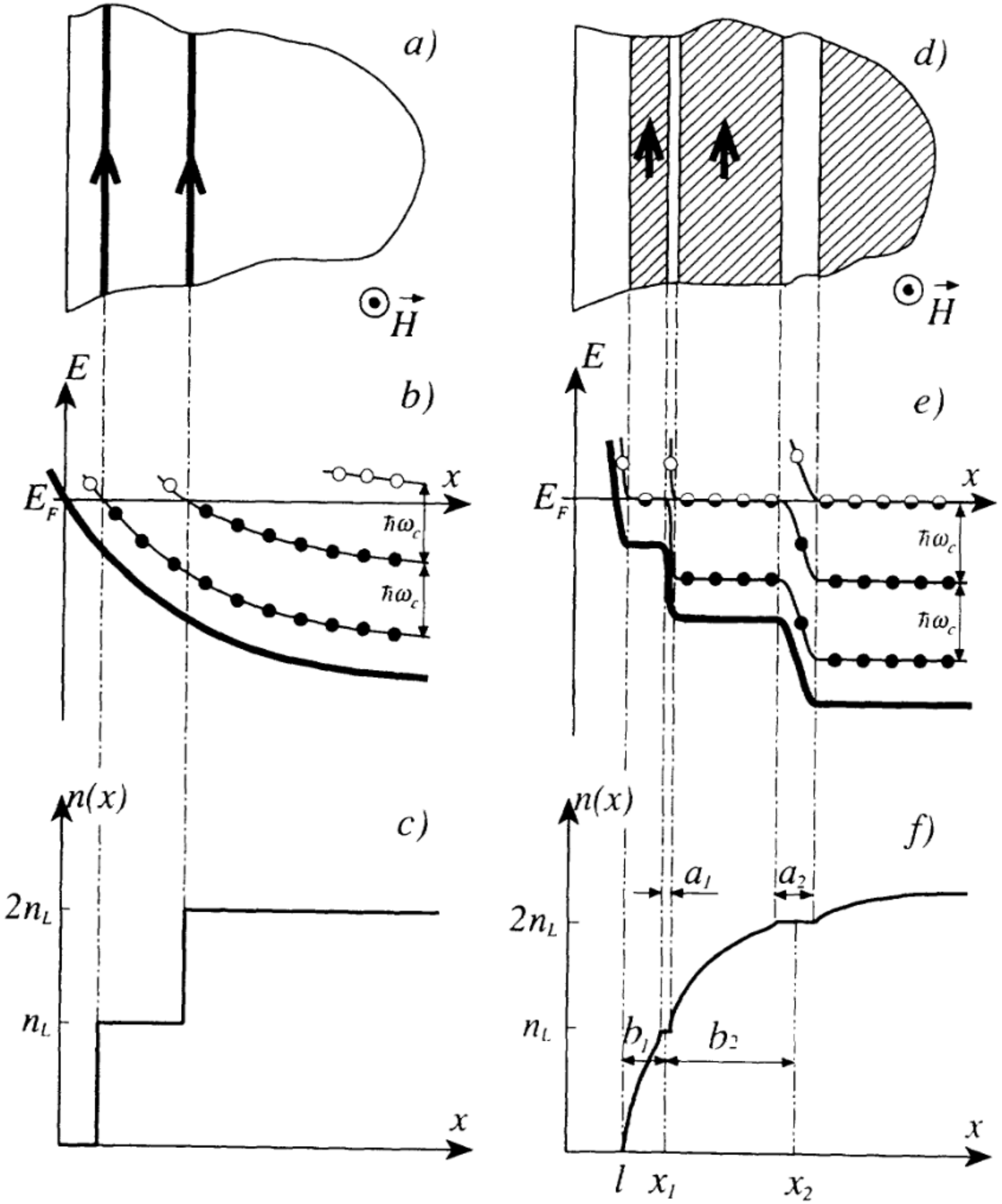
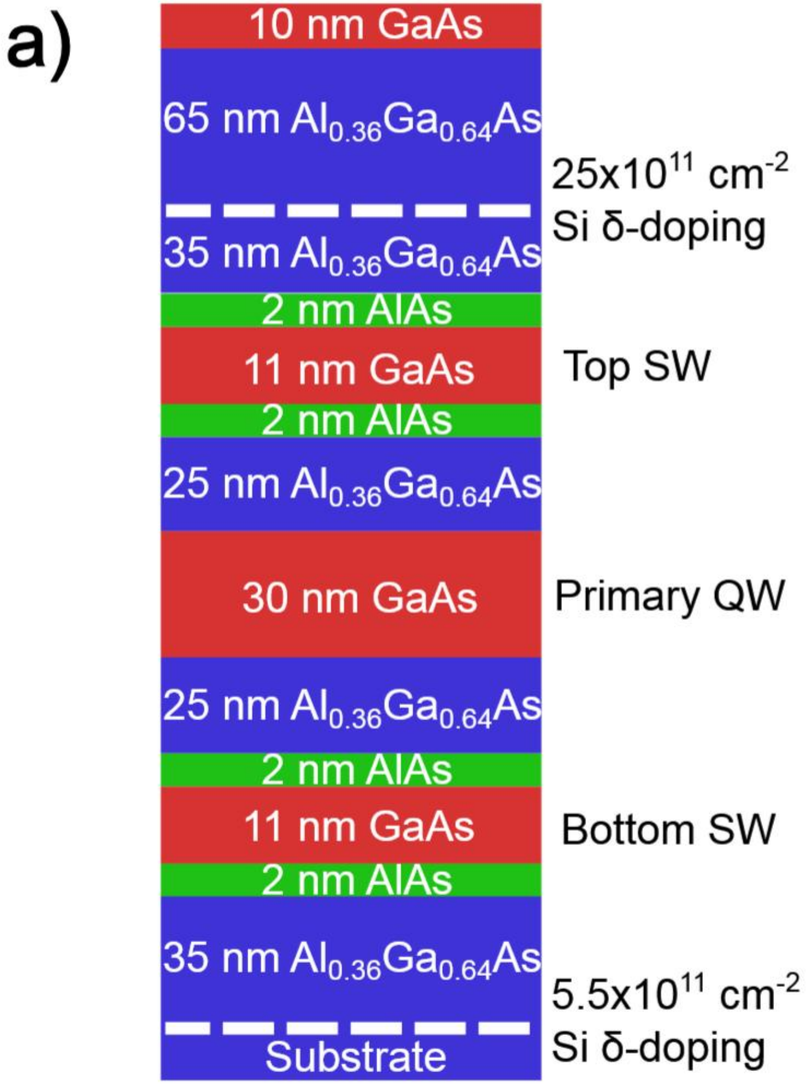
Importance of abrupt confinement: edge state reconstruction



D. B. Chklovskii, B. I. Shklovskii, and L. I. Glazman, Electrostatics of edge channels, Phys. Rev. B **46**, 4026 (1992).

GaAs heterostructure



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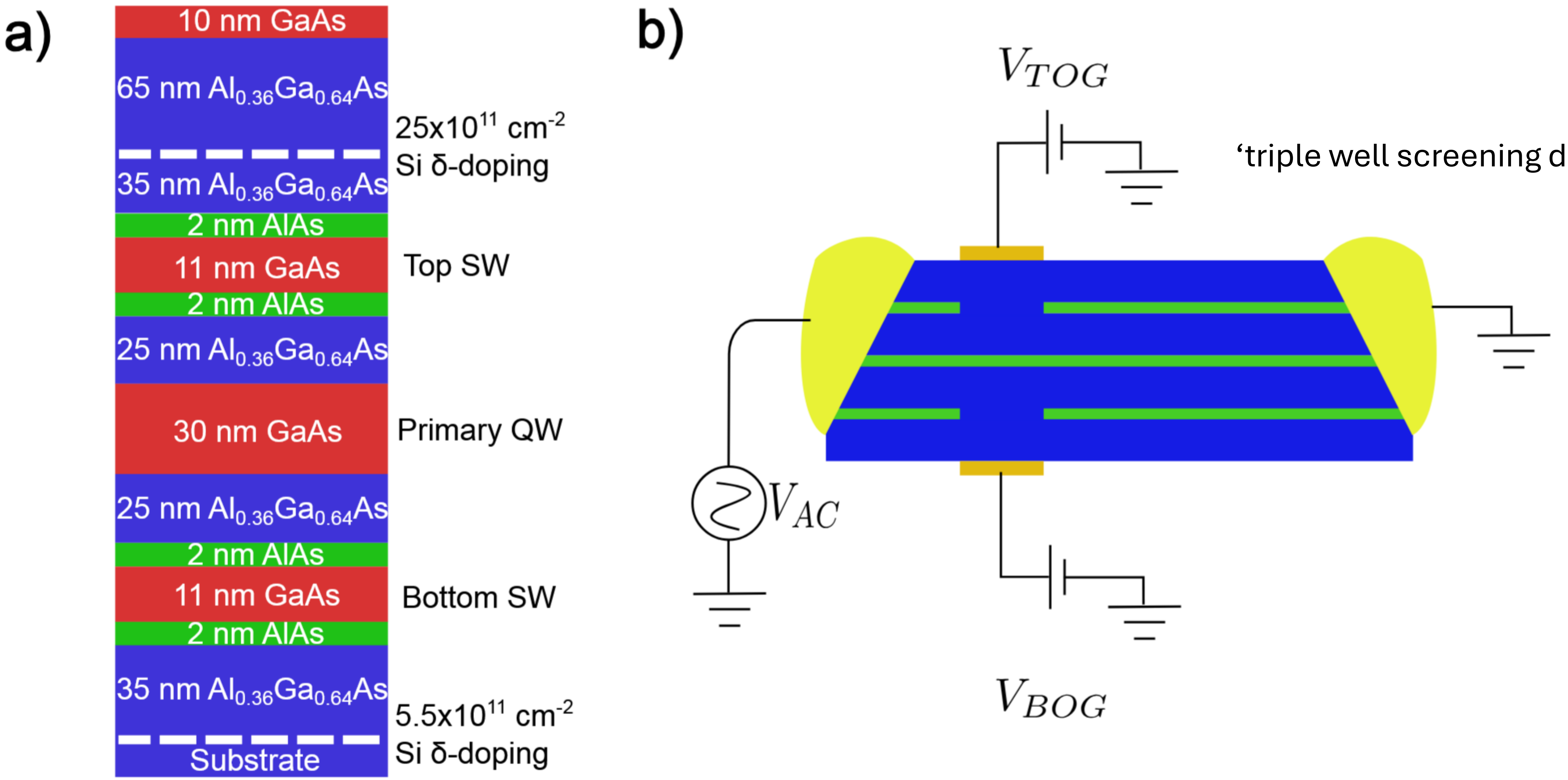

‘triple well screening design’

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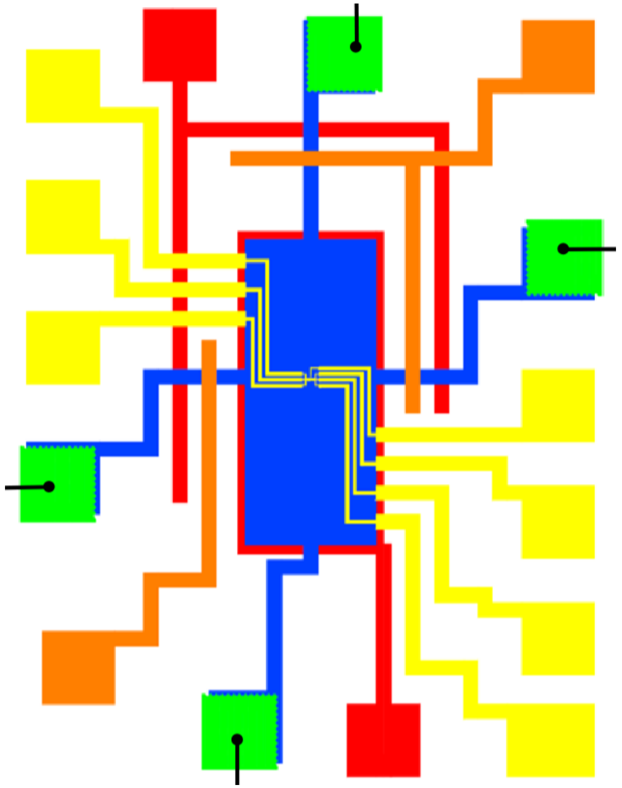
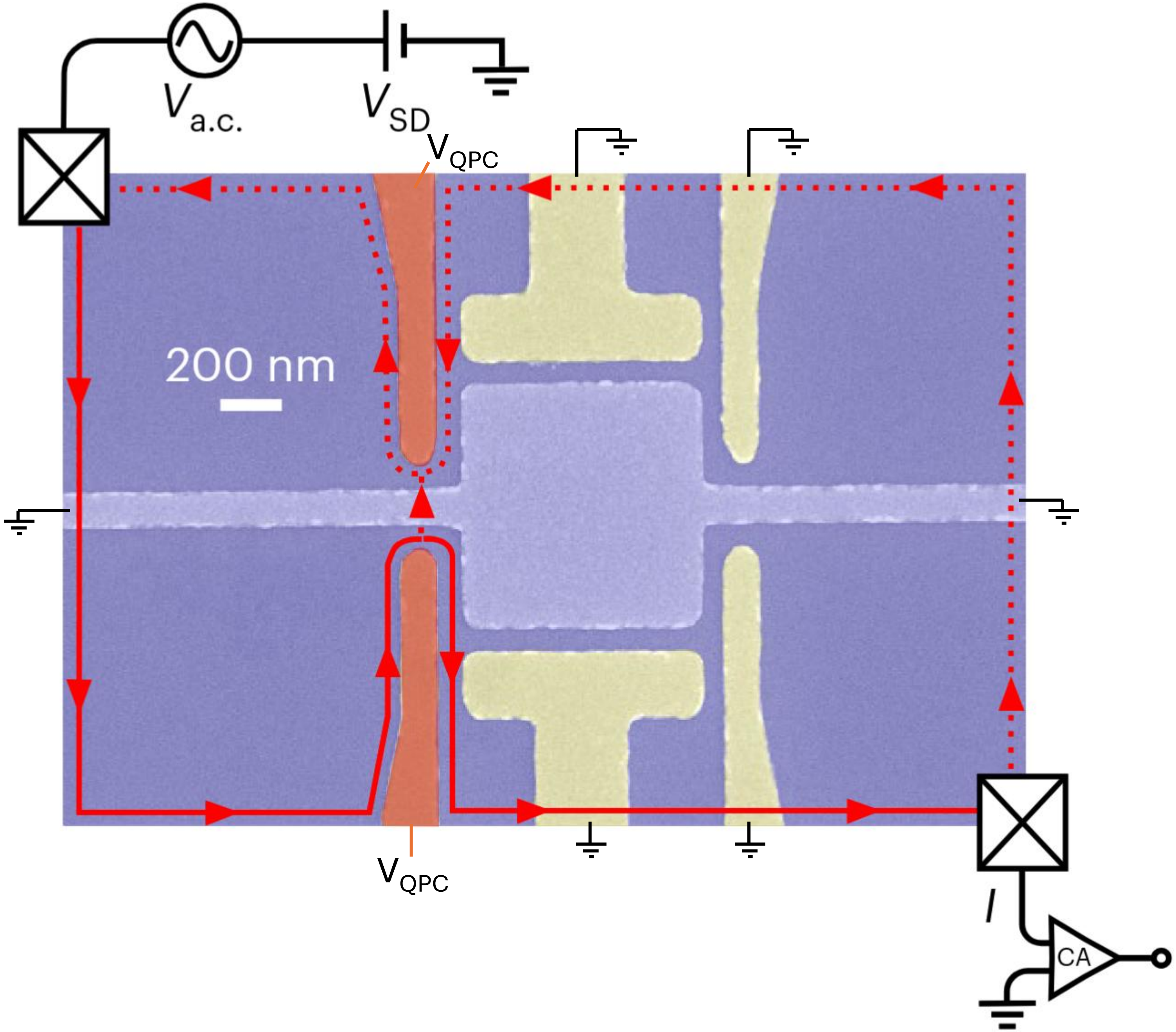
Device





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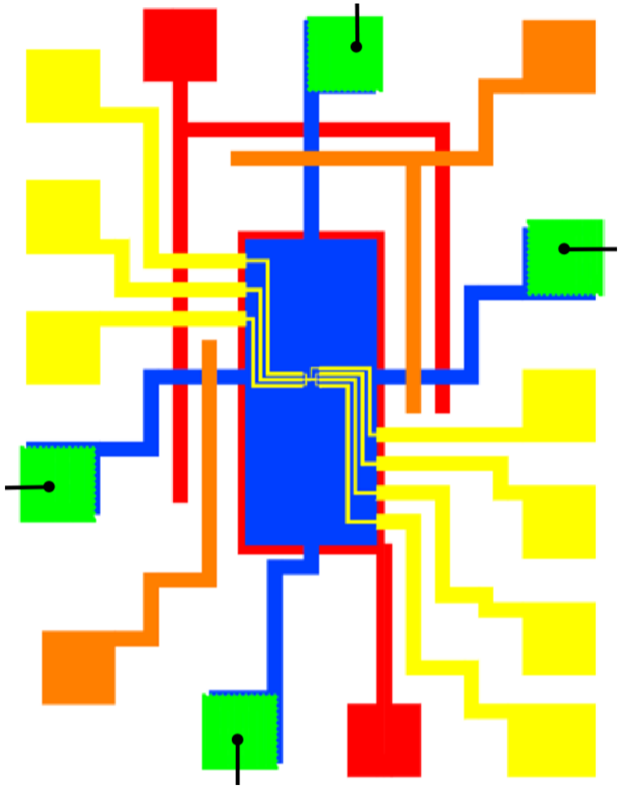
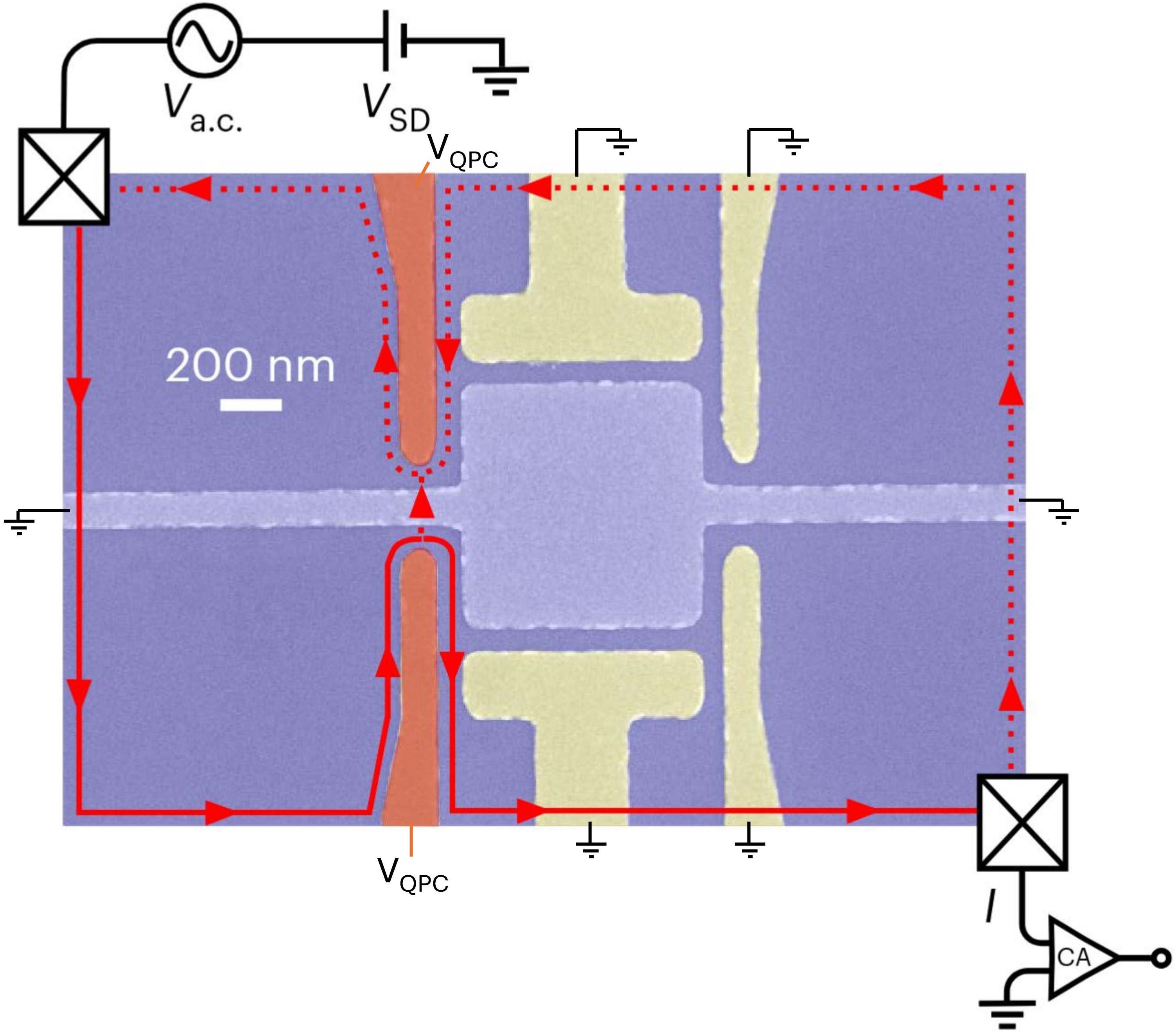
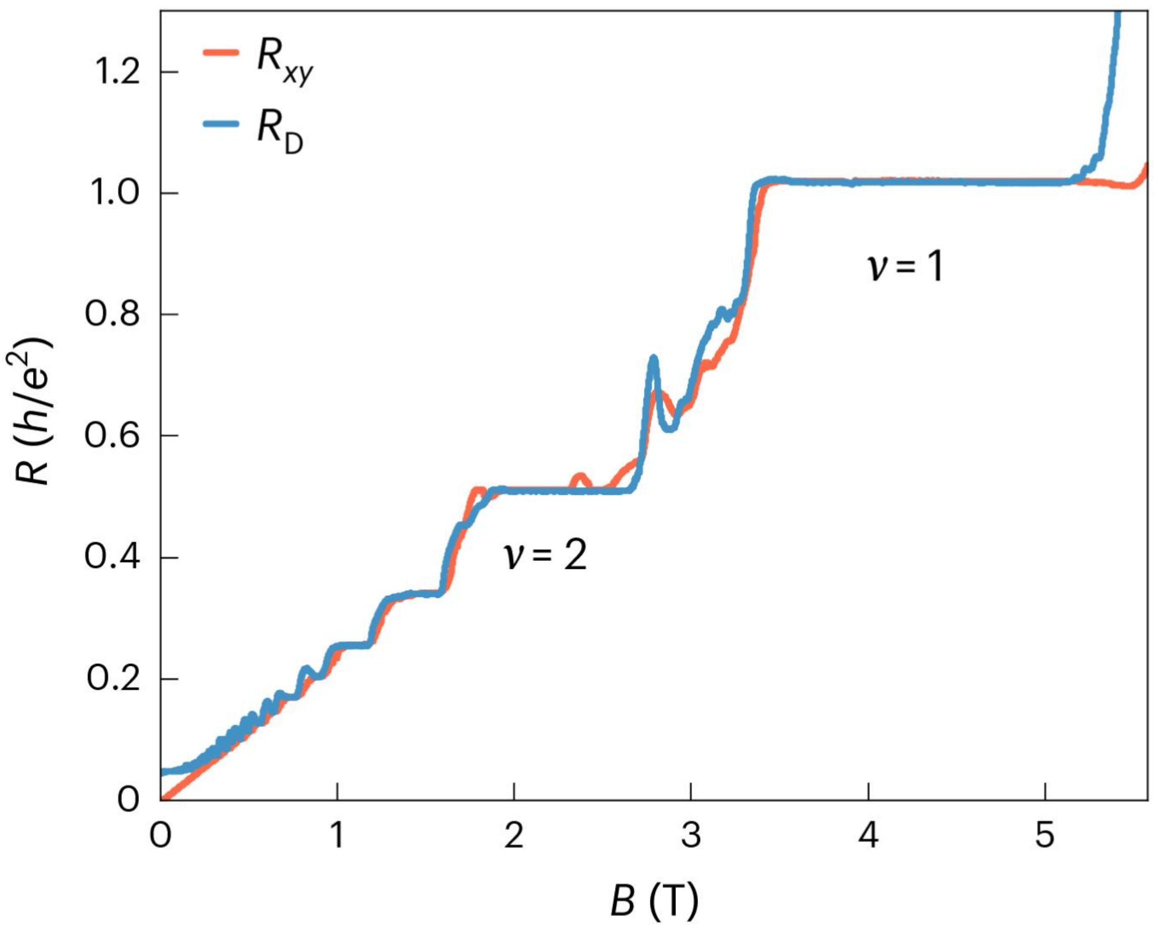




Similar devices used previously in:

- J. Nakamura *et al.*, Aharonov–Bohm interference of fractional quantum Hall edge modes, Nat. Phys. **15**, 563 (2019).
- J. Nakamura *et al.*, Direct observation of anyonic braiding statistics, Nat. Phys. **16**, 931 (2020).

Quantum Hall effect in the bulk and across the device



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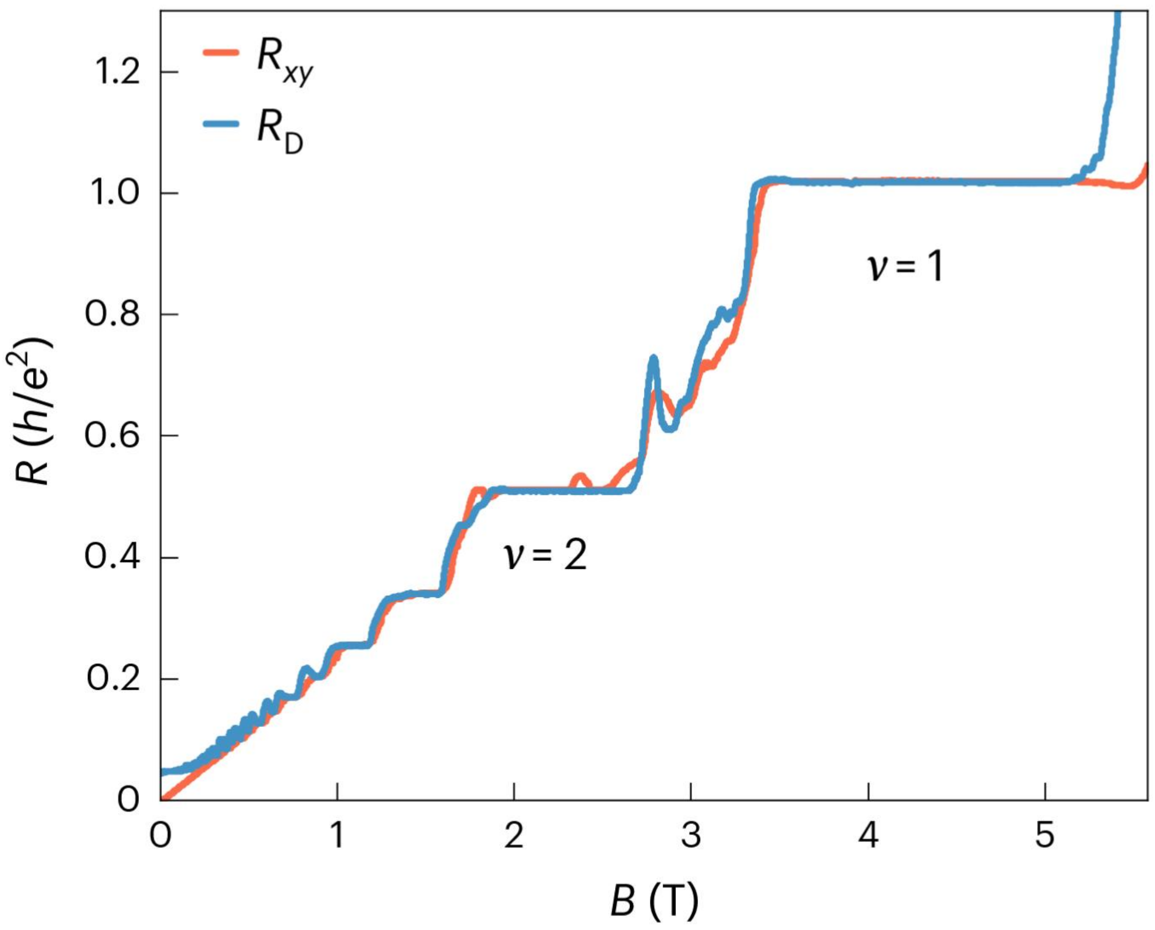

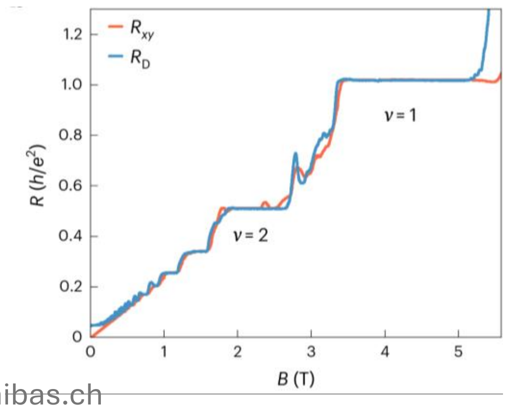

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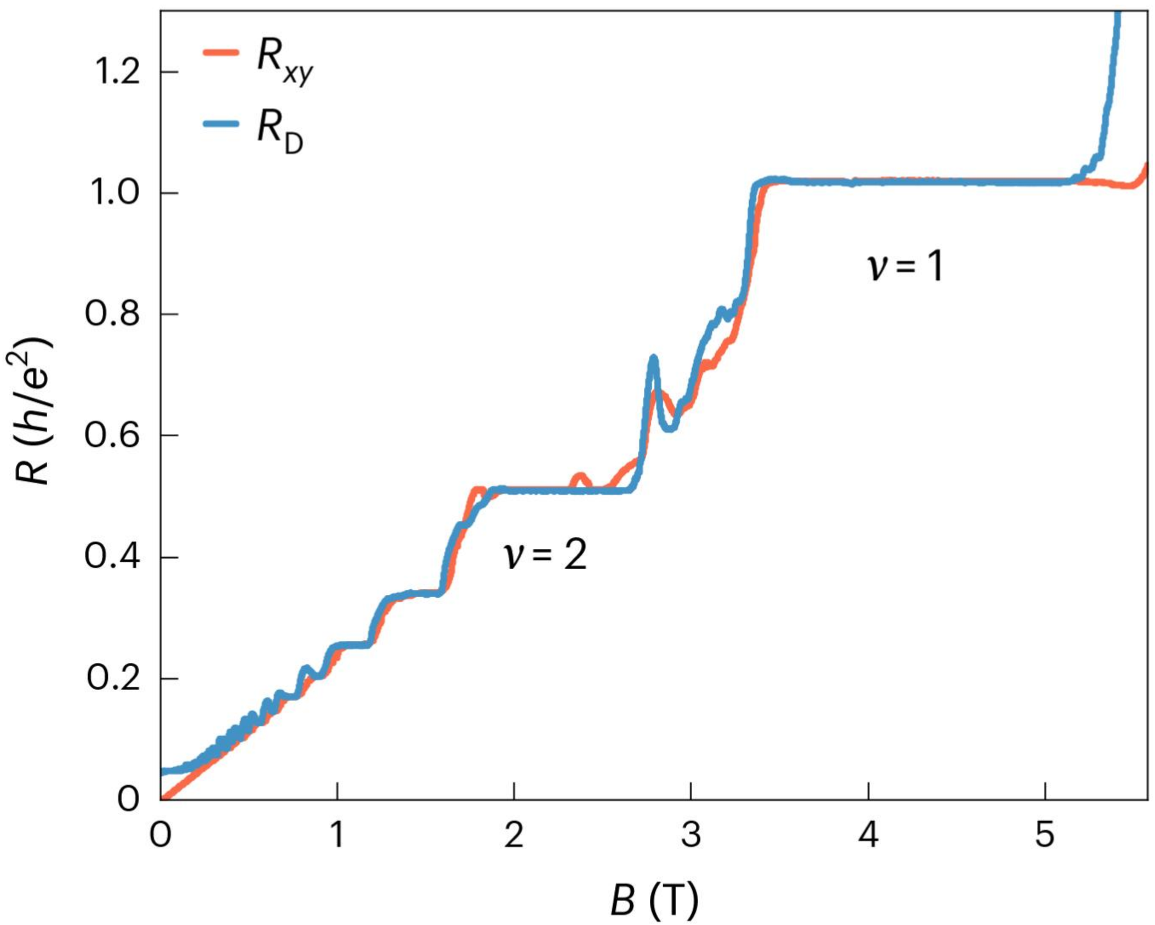

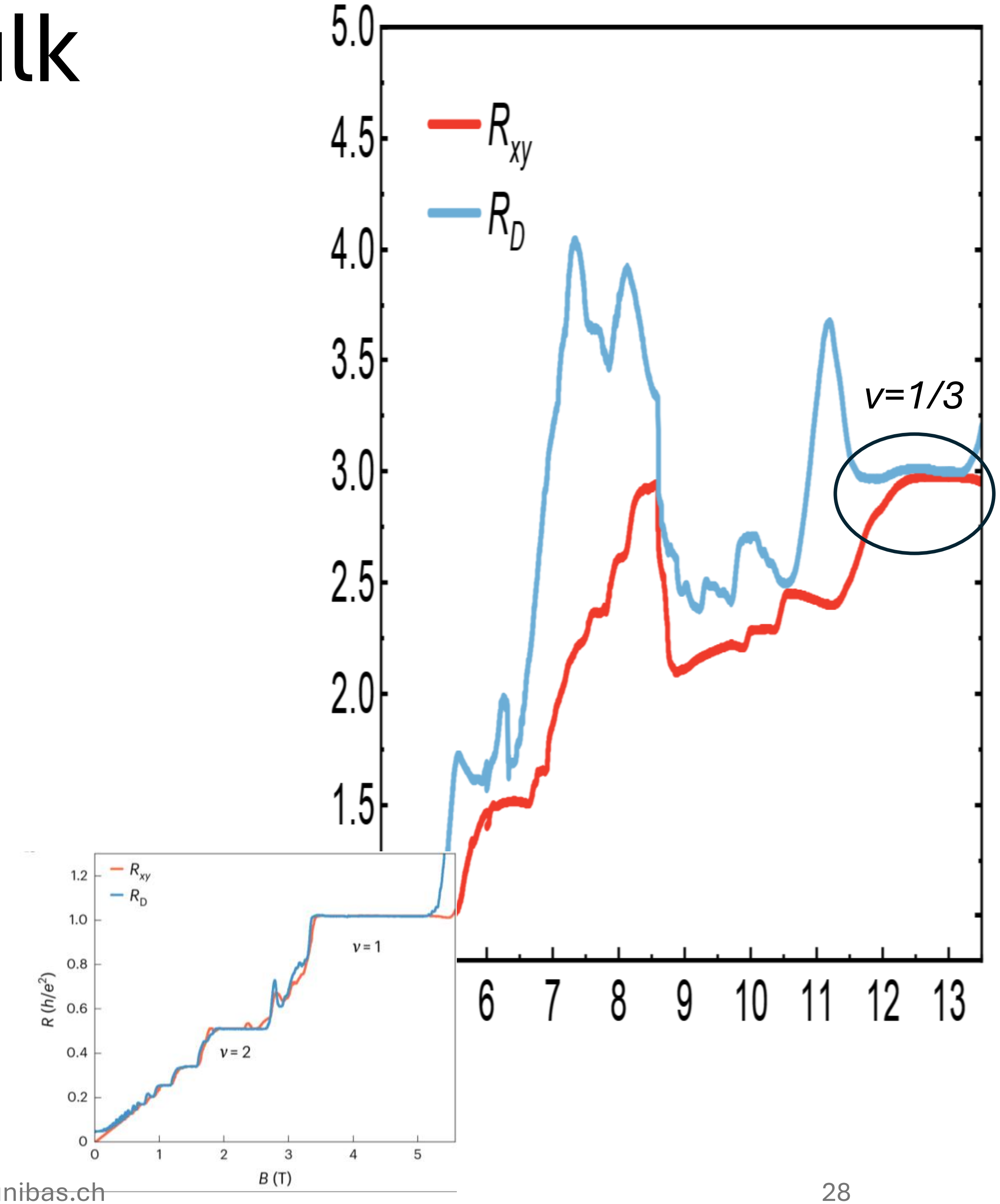

Tunnelling of chiral Luttinger liquids

(for small coupling of counter-propagating modes)

Tunnelling conductance

$$G_t = \frac{e^2}{h}\left(\frac{2\pi T}{T_0}\right)^{2g-2} f_g\left(e^* \frac{V_{SD}}{k_B T}\right)$$

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Parameters of topological order of Abelian FQHE liquids in the Laughlin sequence:

$e^* = \nu e$

$g =$ ν (fractional quasiparticles)
$1/\nu$ (electrons

θ_a (anyonic statistical angle) independent

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$$B(z_1, z_2) = \frac{\Gamma(z_1)\Gamma(z_2)}{\Gamma(z_1 + z_2)}$$

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Tunnelling of chiral Luttinger liquids

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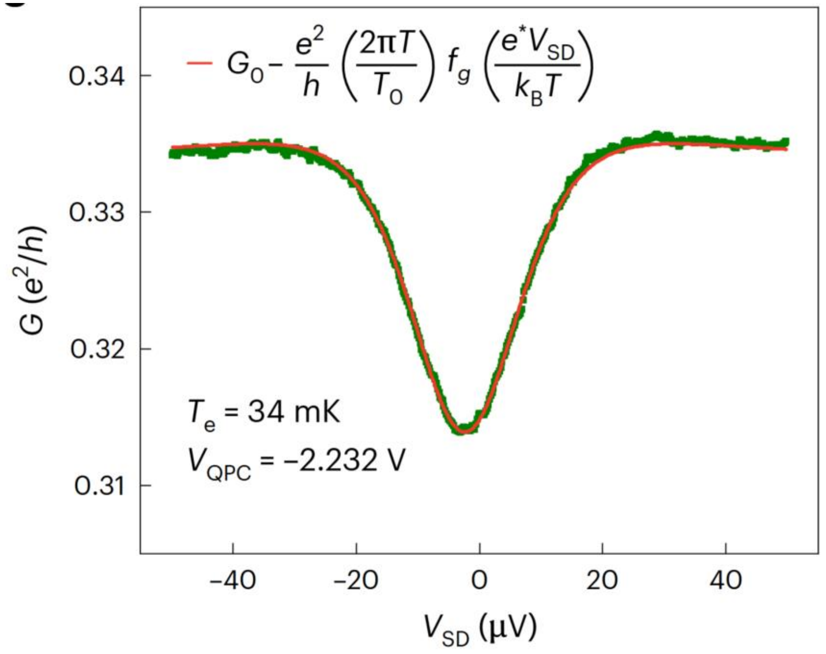
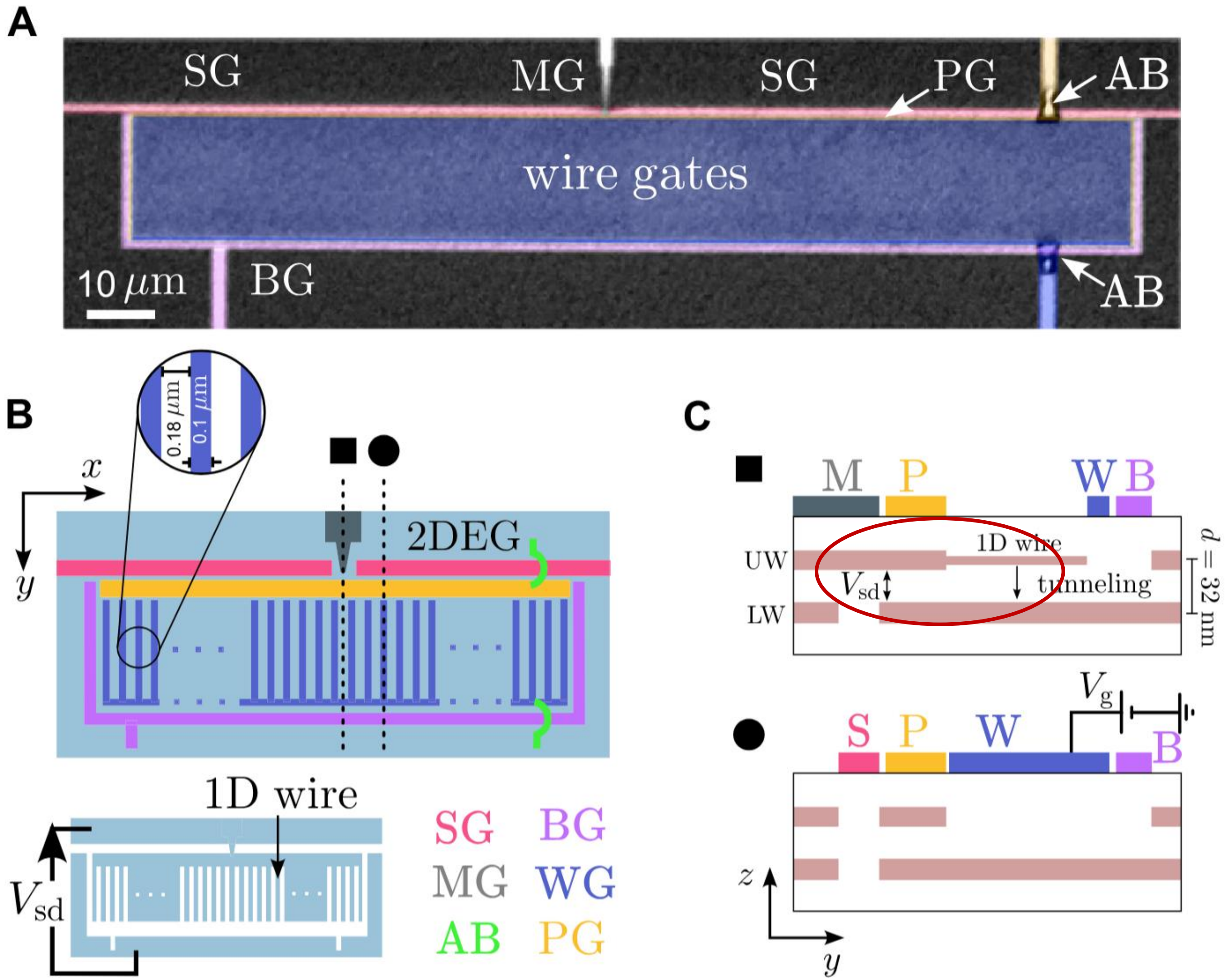


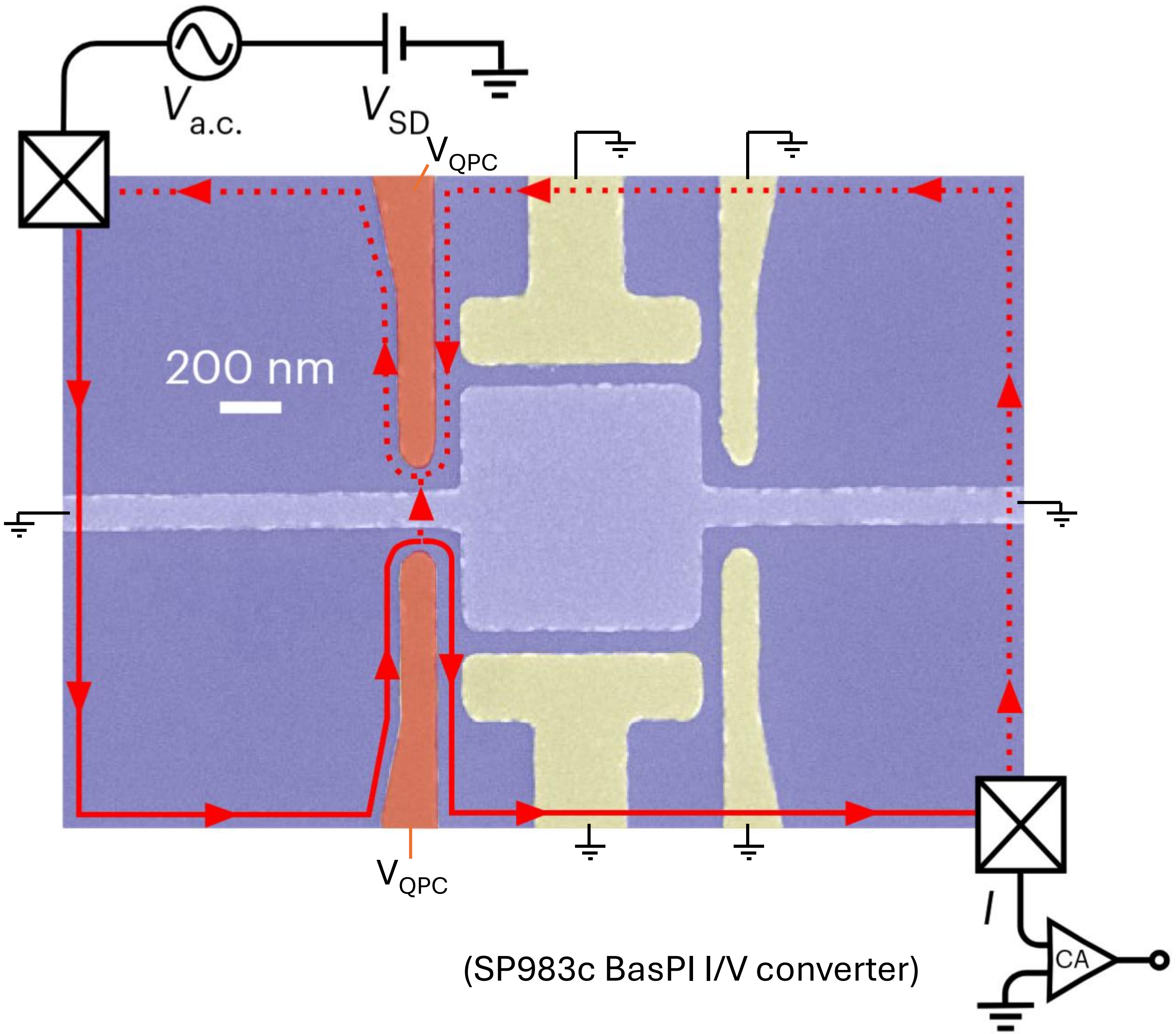
Recent related experiment in the group



$$G(V_{sd},T)=AT^{\alpha}\cosh\left(\frac{eV_{sd}}{2k_BT}\right)\left|\Gamma\left(\frac{1+\alpha}{2}+\frac{ieV_{sd}}{2\pi k_BT}\right)\right|^2$$

H. Weldeyesus, P. M. T. Vianez, O. Sharifi Sedeh, et al., Dominant end-tunneling effect in two distinct Luttinger liquids coexisting in one quantum wire, Nat Commun **16**, 6997 (2025).

Measuring tunnelling in the device



(SP983c BasPI I/V converter)

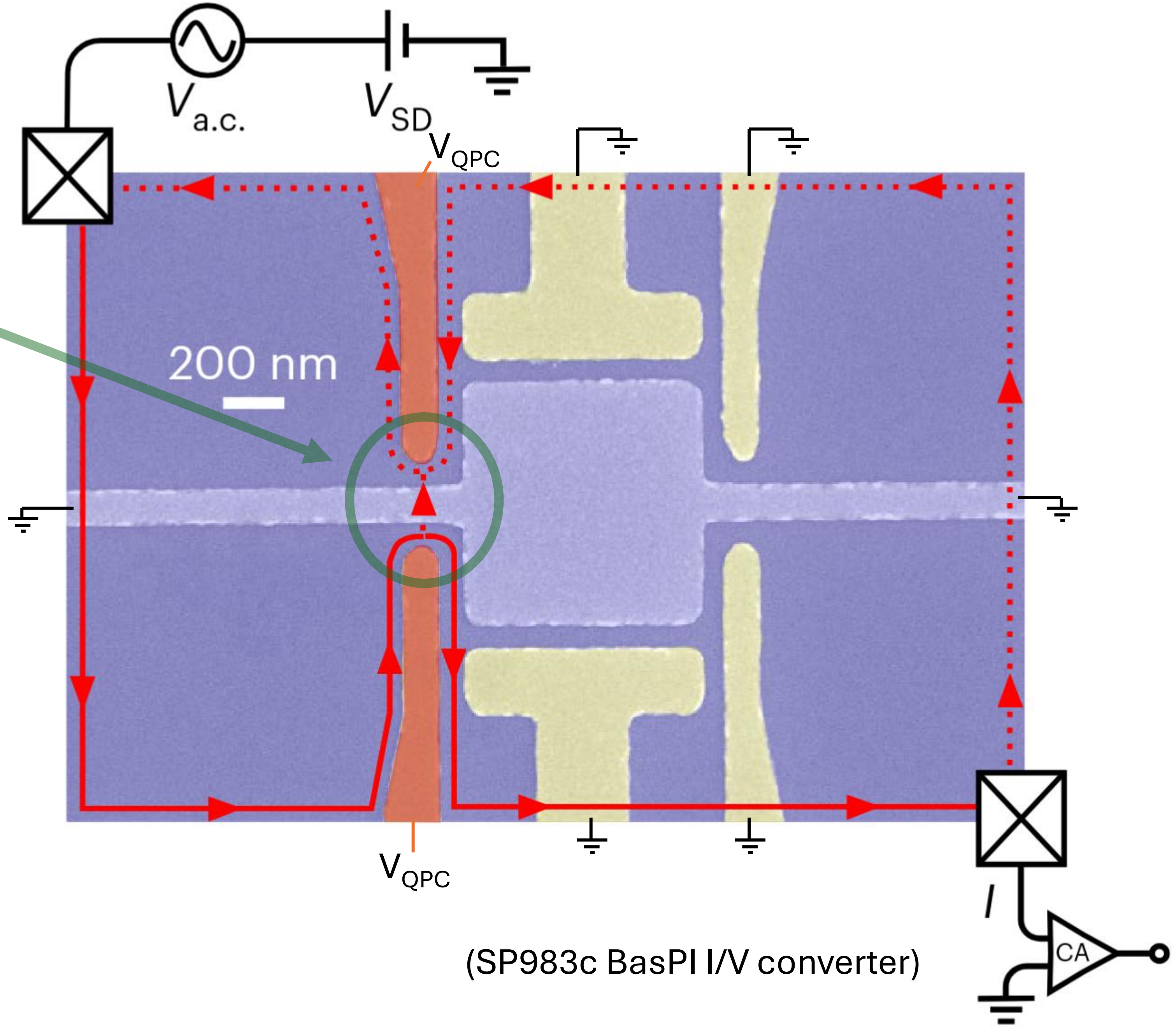
Measuring tunnelling in the device

*‘Universal **anyon tunnelling in a chiral Luttinger liquid**’*

Luttinger liquid: 1D conductor with non-negligible electron-electron interactions

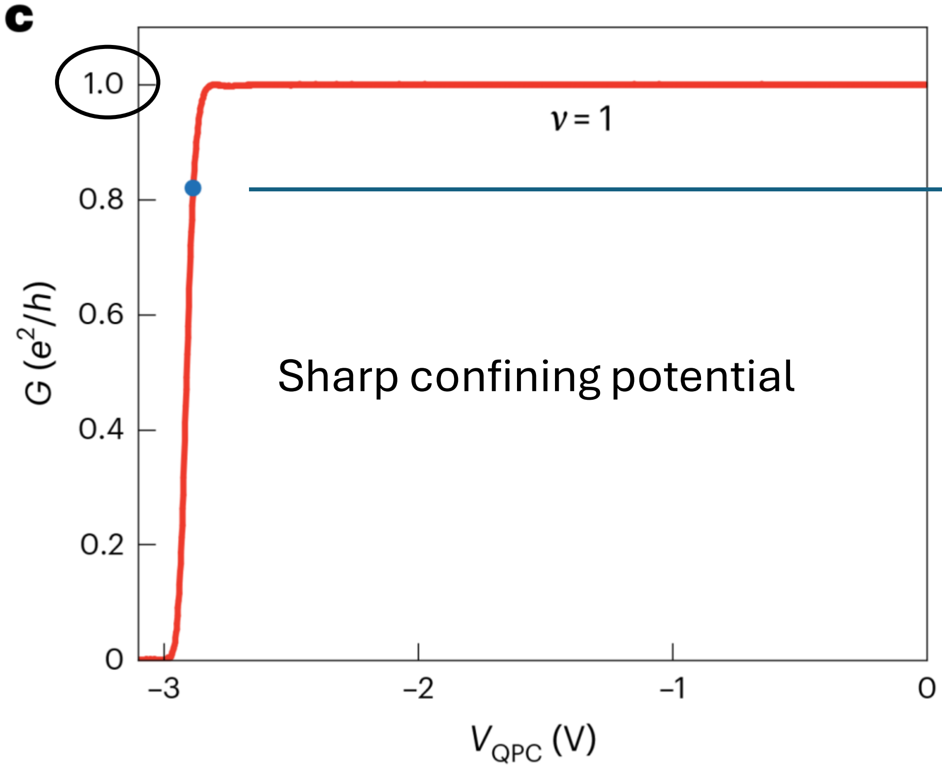
Chiral Luttinger liquid: unidirectional conductor, here arising from a GaAs 2DEG in magnetic field from the quantum Hall effect

Anyons: quasiparticles with complex-valued exchange rules, generalising fermions and bosons, here arising from the fractional quantum Hall effect at 1/3 filling factor

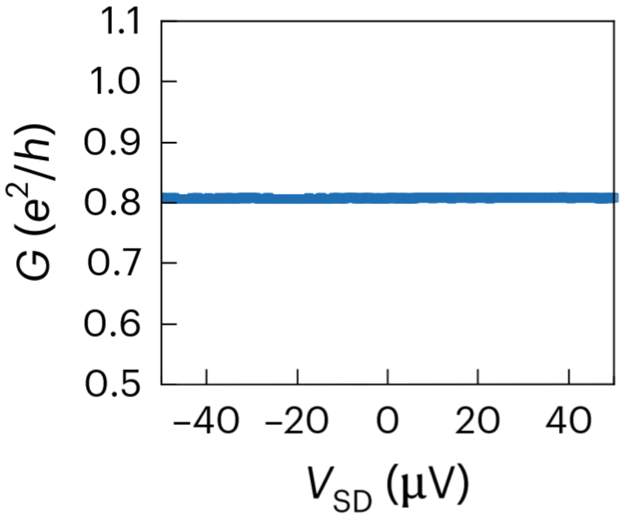


(SP983c BasPI I/V converter)

At filling factor v = 1 (B = 4.20T)



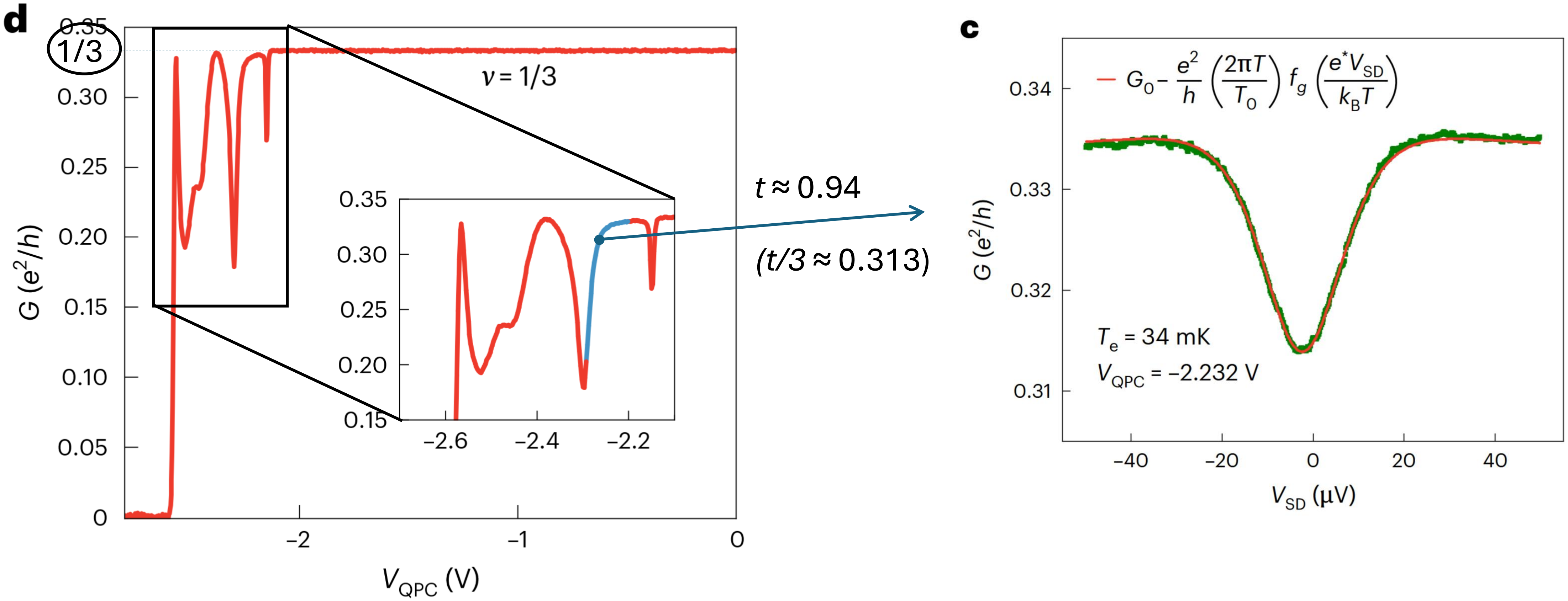
$t \approx 0.8$



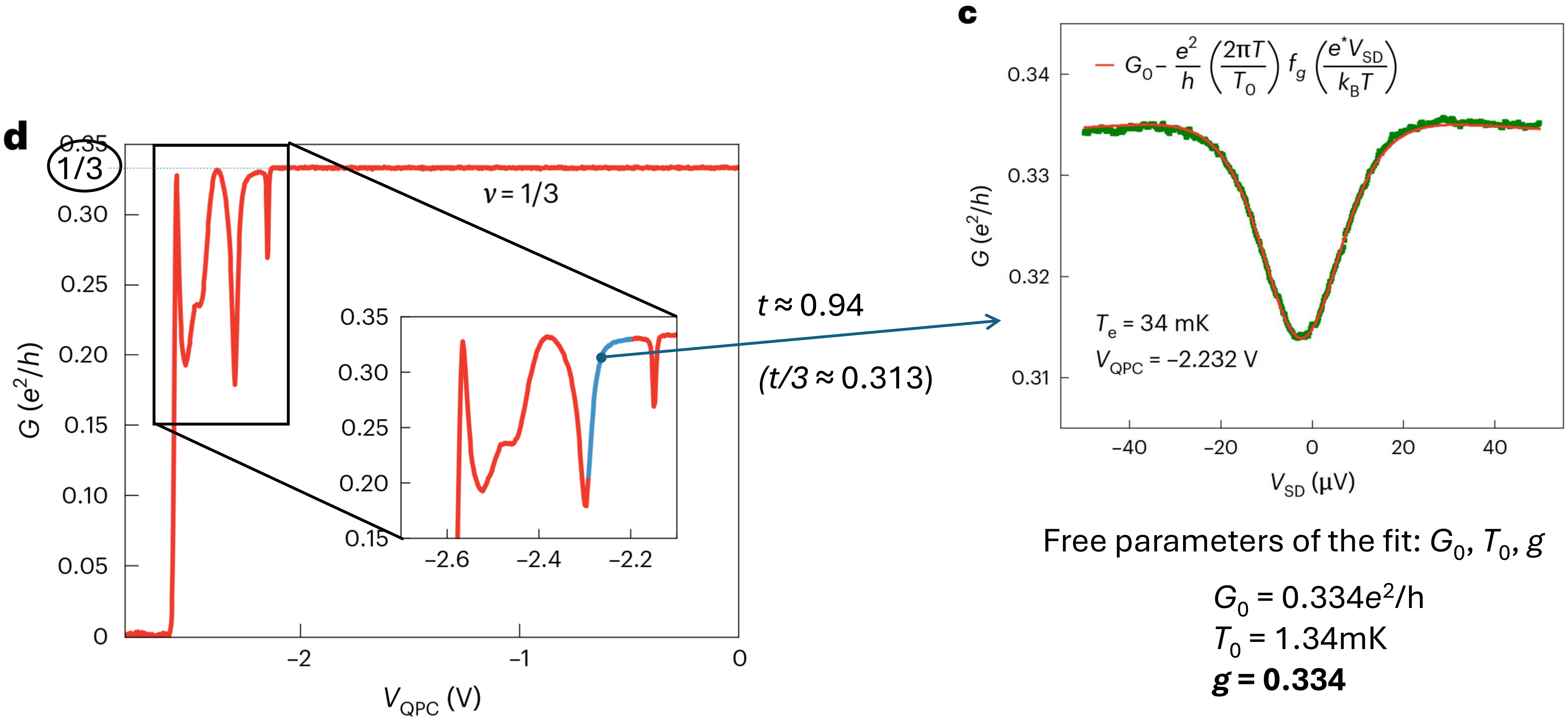
Constant $G(V_{SD})$ from 'Fermi liquid theory'

tune QPC to get weak tunnelling → measure differential conductance vs. voltage bias curve

At filling factor $\nu = 1/3$ (B = 12.83T)



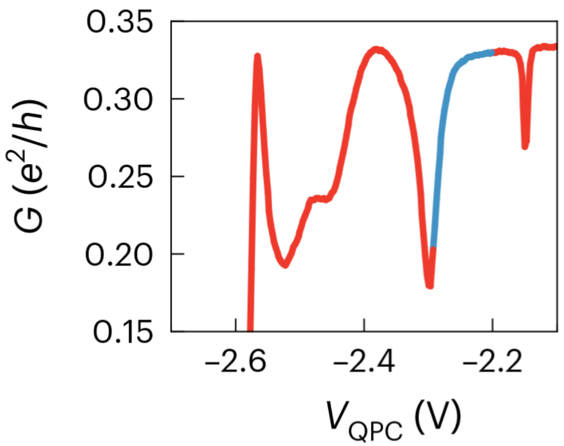
At filling factor $v = 1/3$ (B = 12.83T)



Free parameters of the fit: G_0, T_0, g

$G_0 = 0.334e^2/h$
$T_0 = 1.34$mK
$g = 0.334$

More bias curves a function of V_{QPC} (i.e. changing t)



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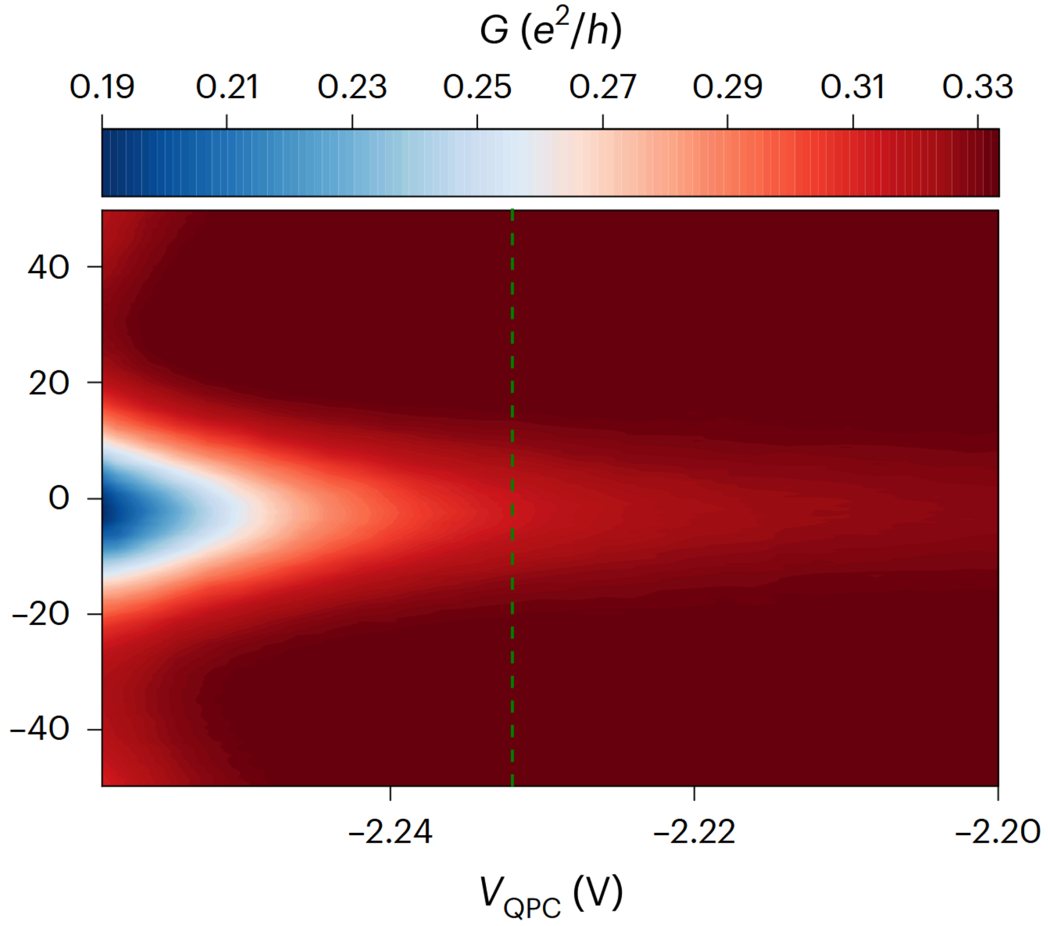
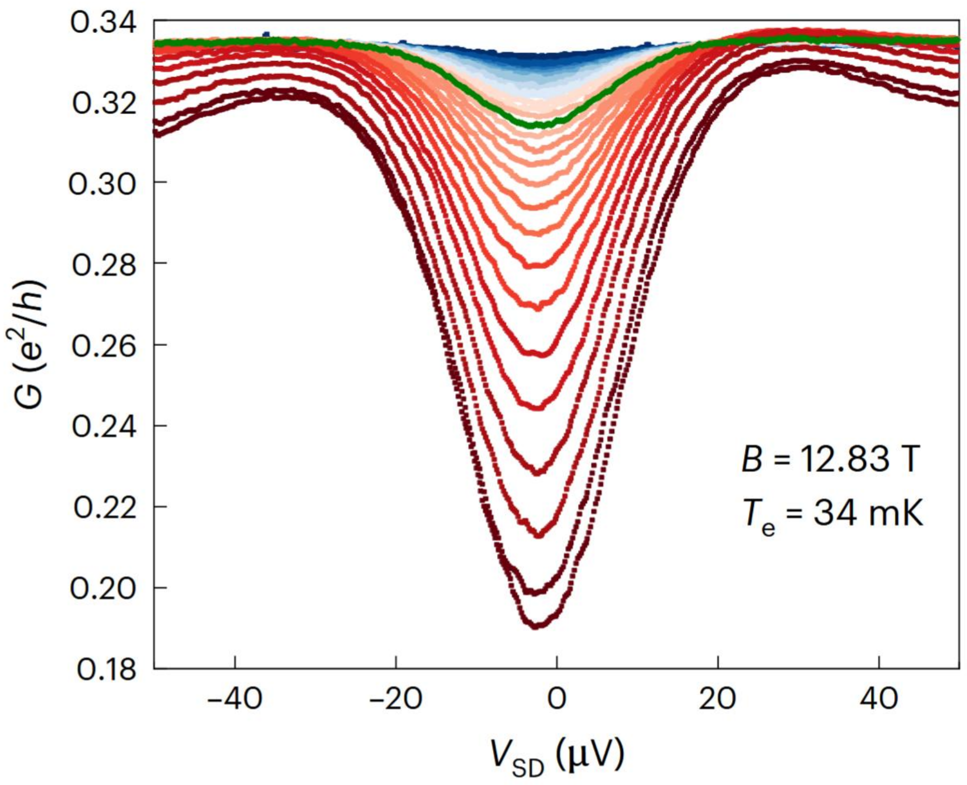
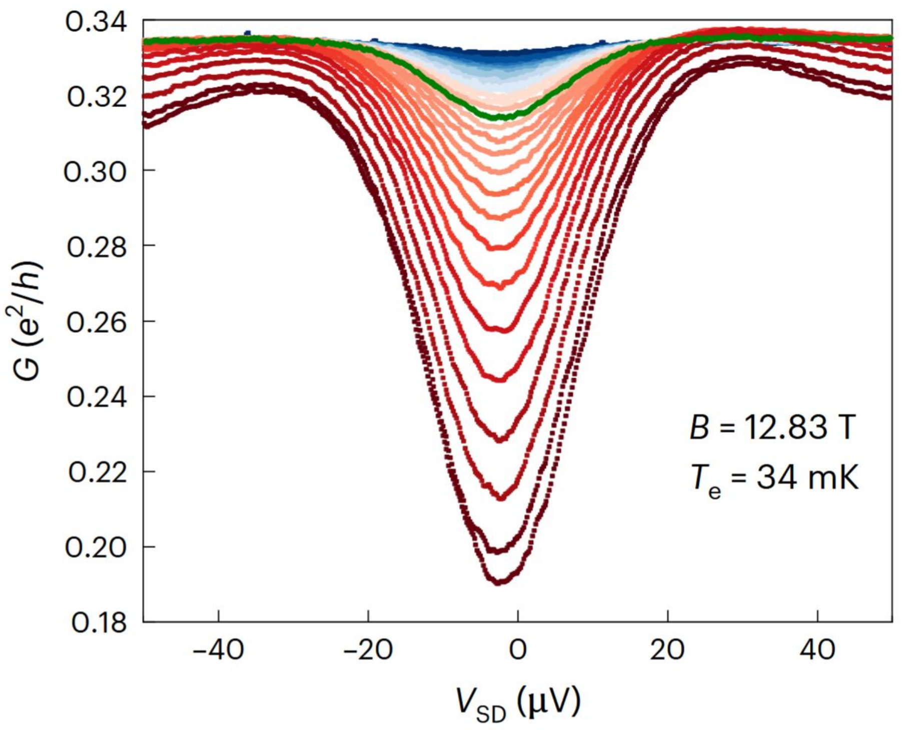
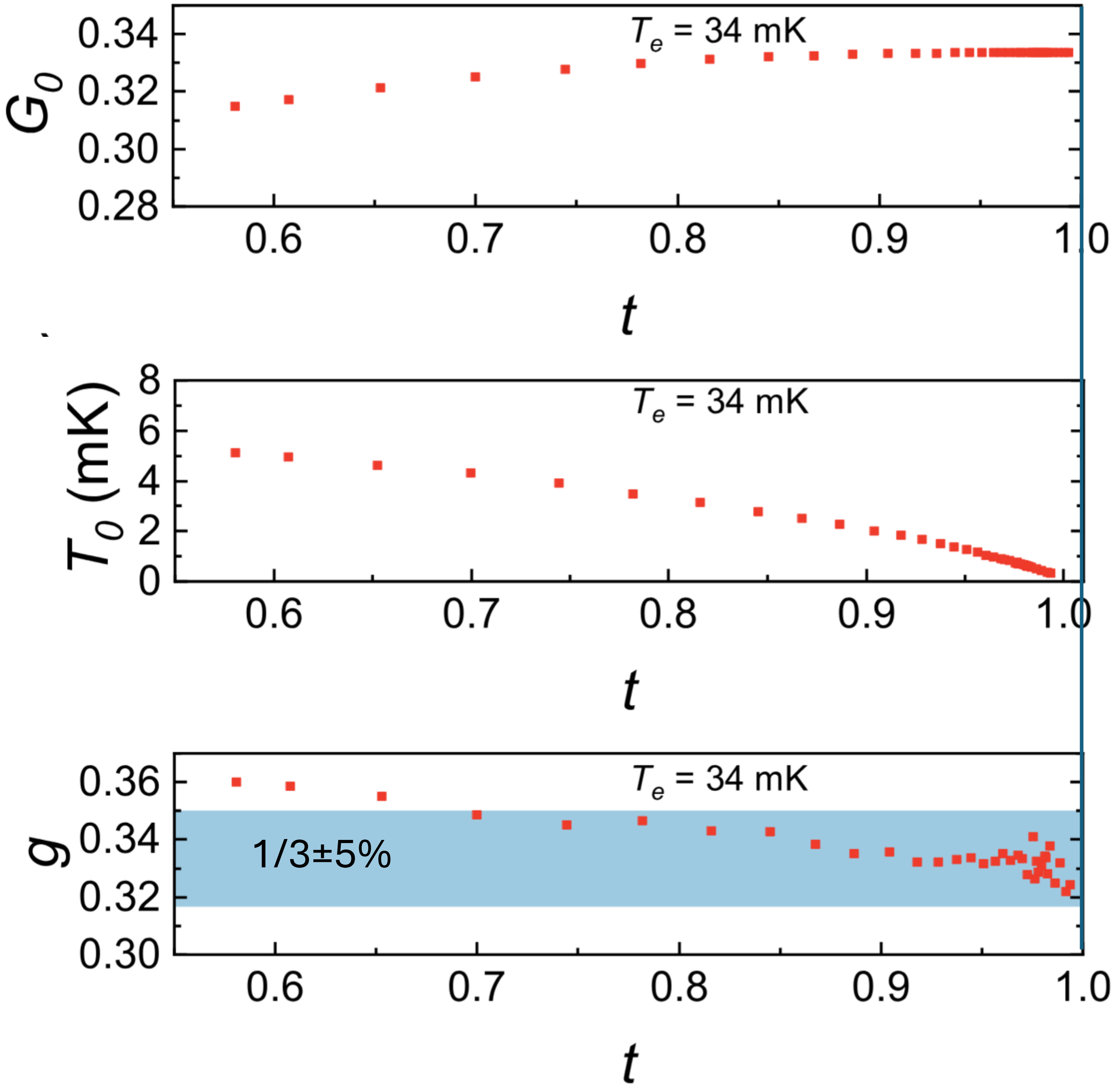


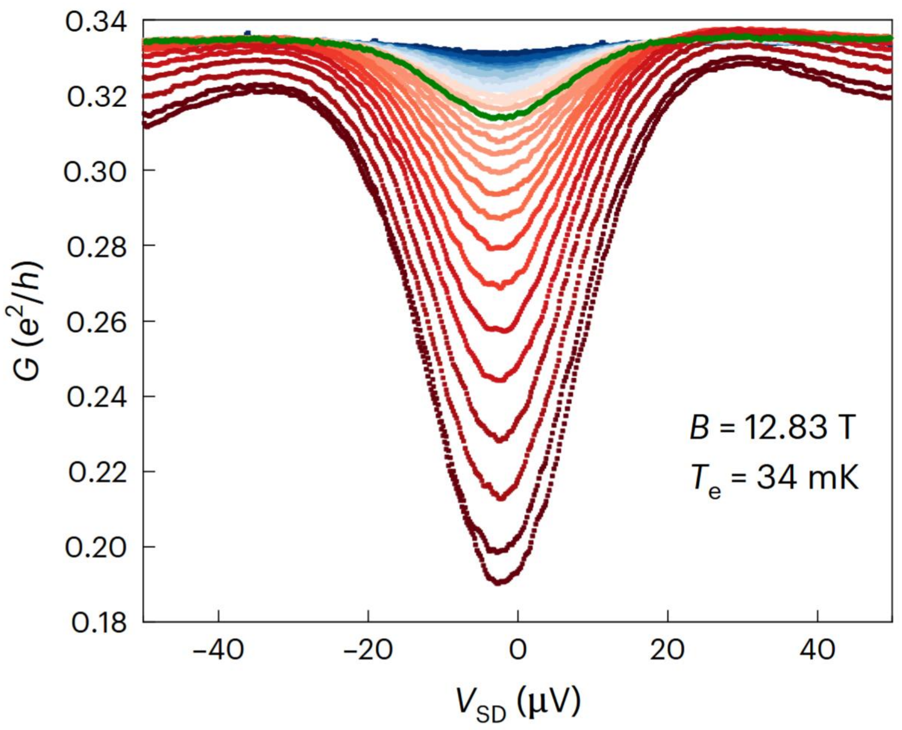


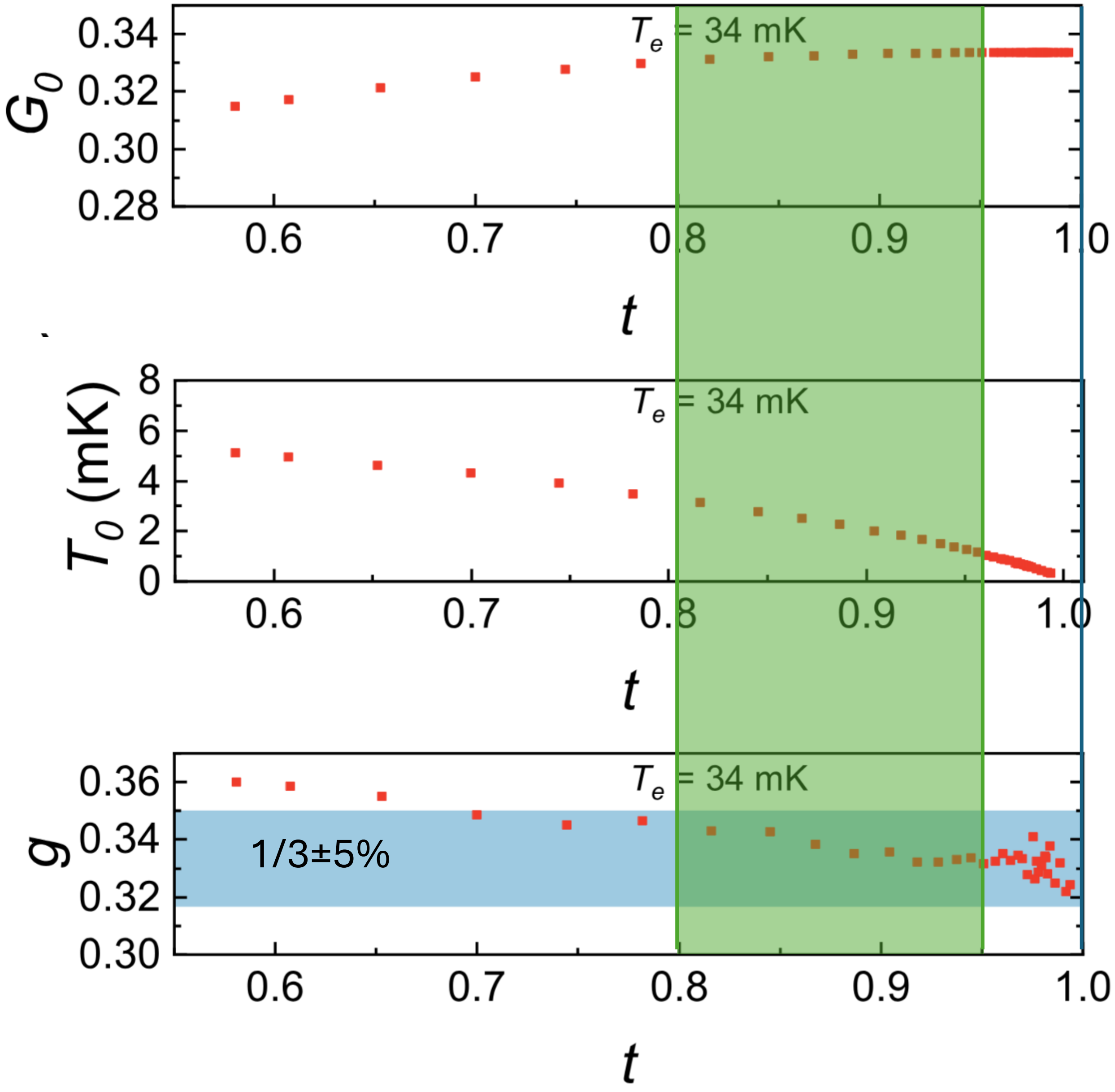
Fitting 35 curves
~0.5 < t < 1



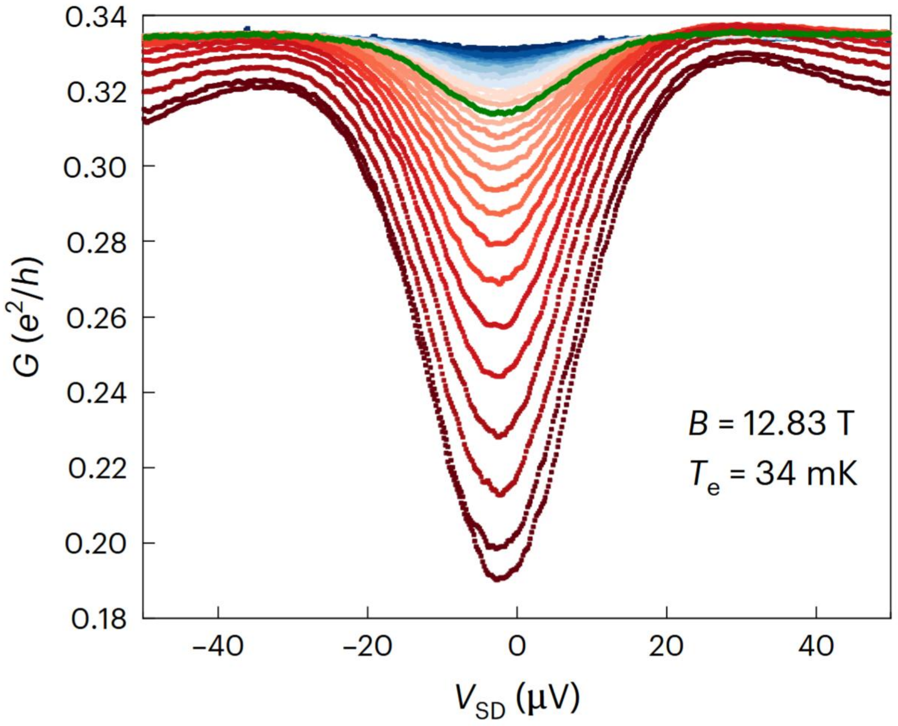


Fitting 35 curves ~0.5 < *t* < 1



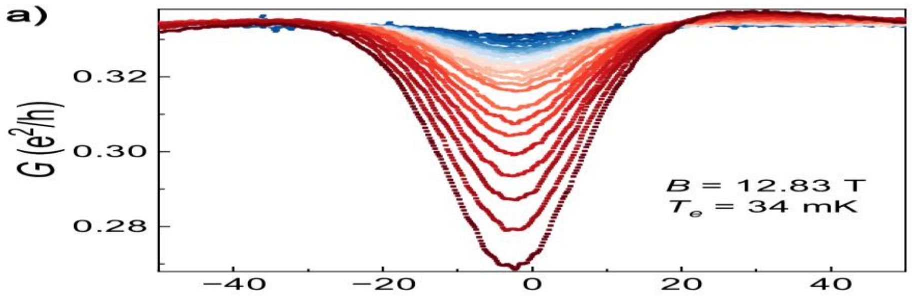


Fitting for different t

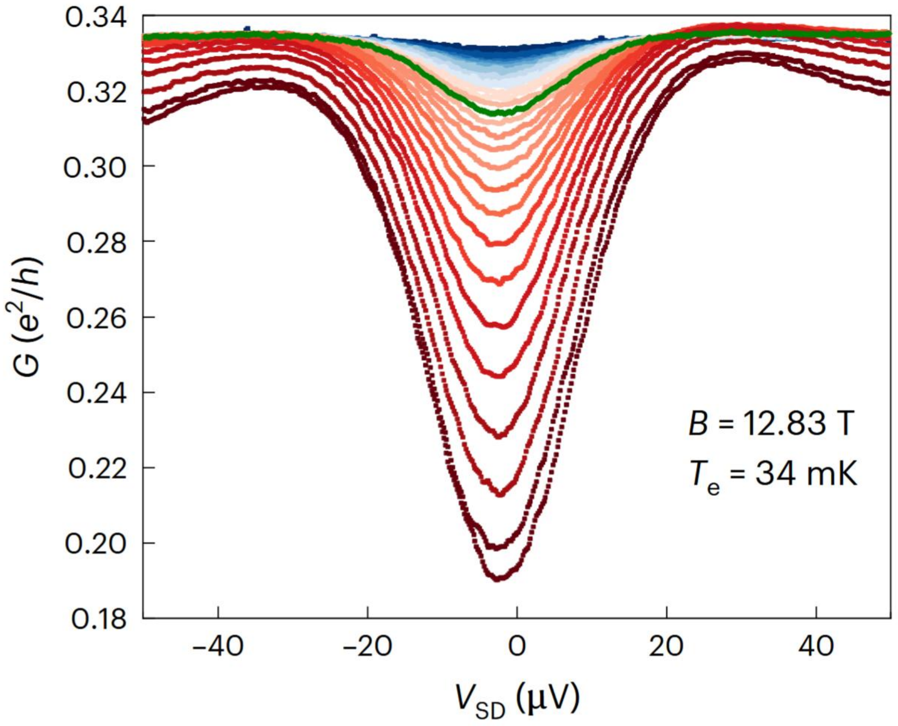


0.8 < t < 0.995

(29/35 curves)



Fitting for different t



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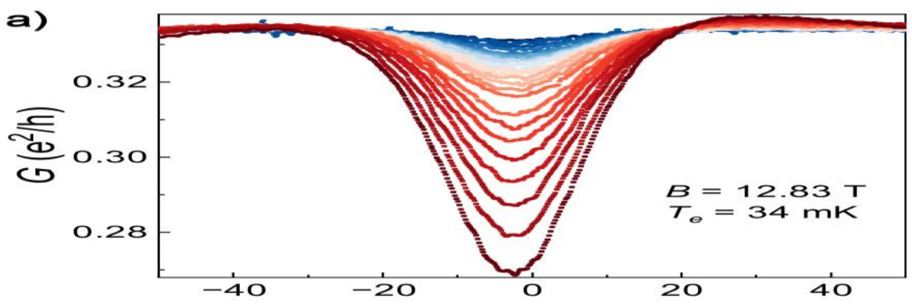
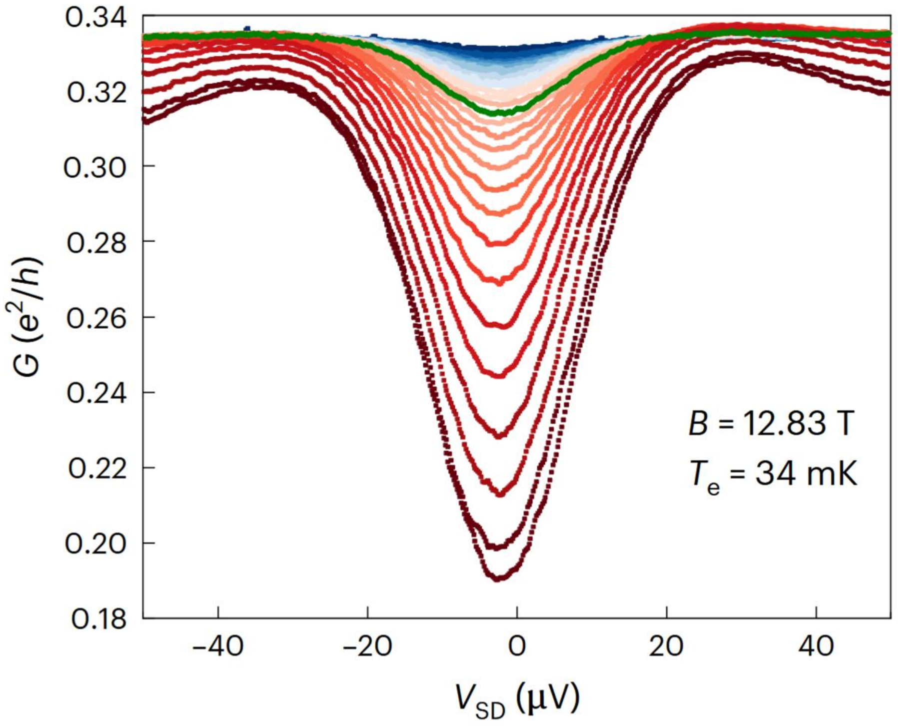


$G_0 = 0.333 \pm 0.001 e^2/h$

$T_0 = f(t)$

$\boldsymbol{g = 0.333 \pm 0.005}$

Fitting for different t



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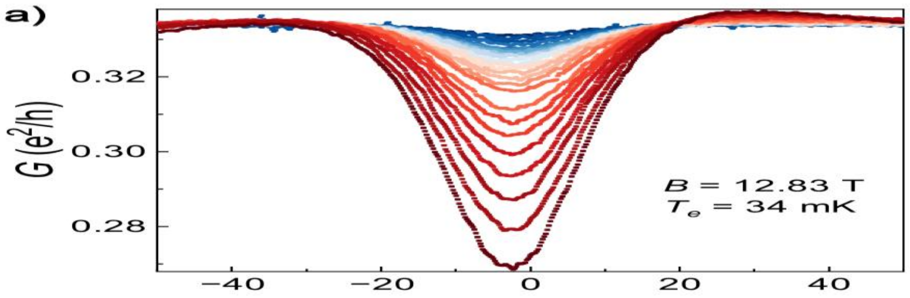
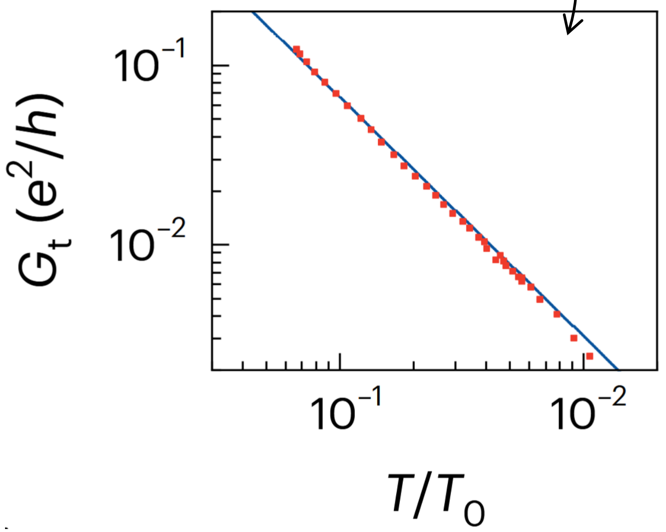


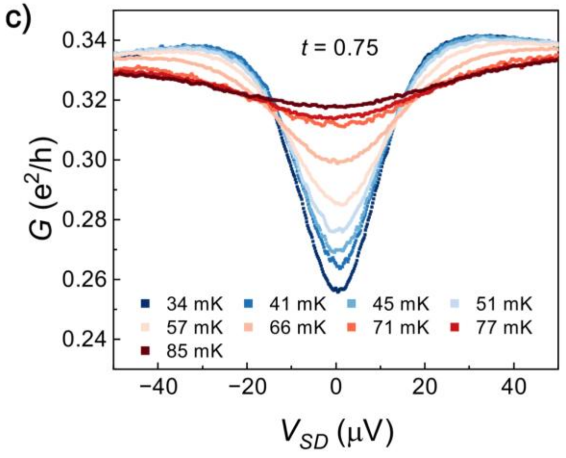
$G_0 = 0.333 \pm 0.001 e^2/h$

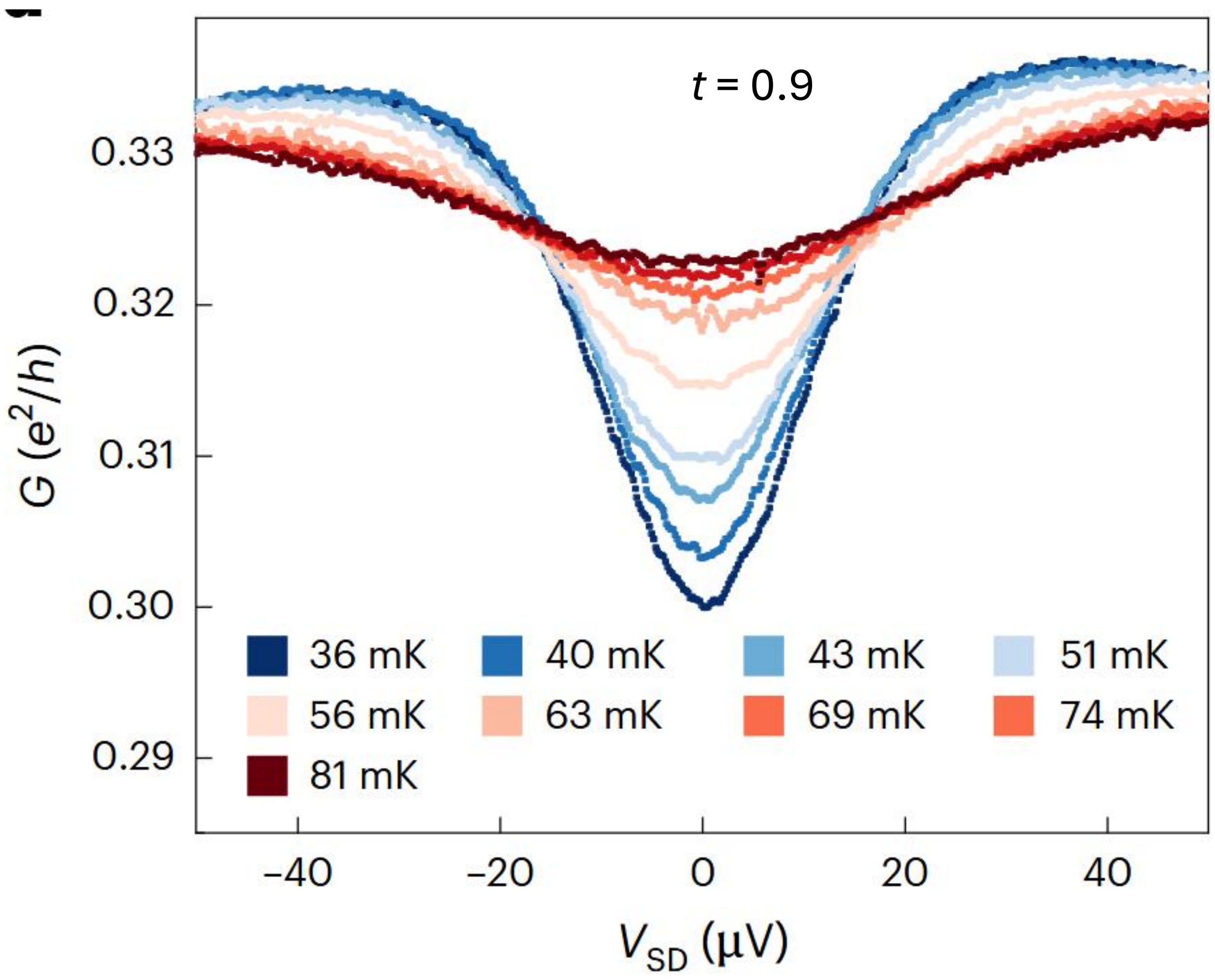
$T_0 = f(t)$ (gives power law for G_t)

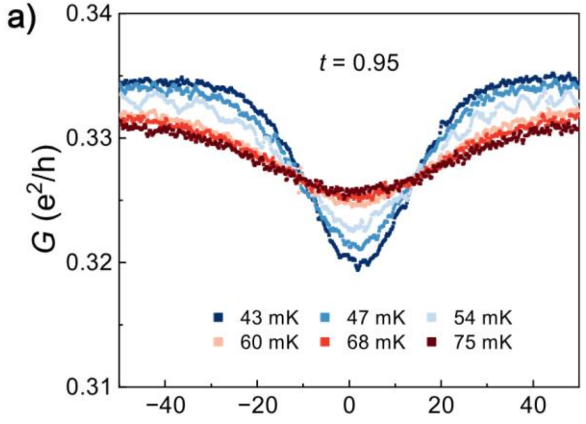
***g* = 0.333±0.005**

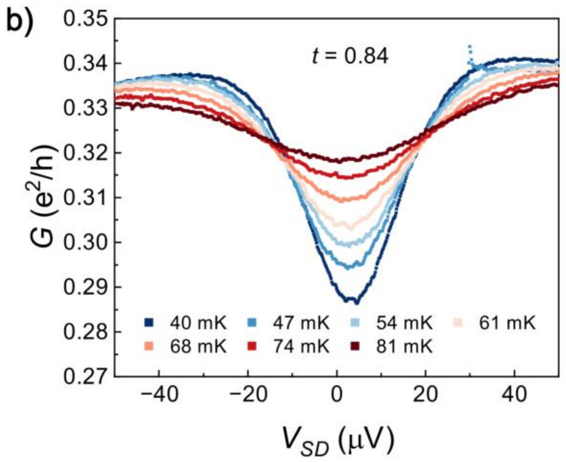


Sweeping temperature as well









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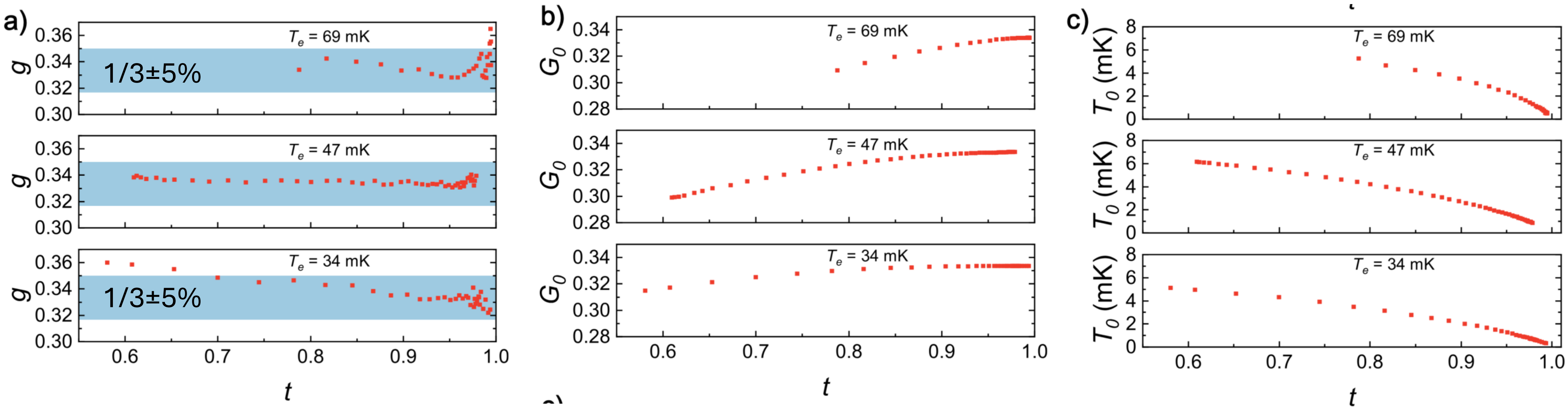
Trends of differential conductance fits at different temperatures



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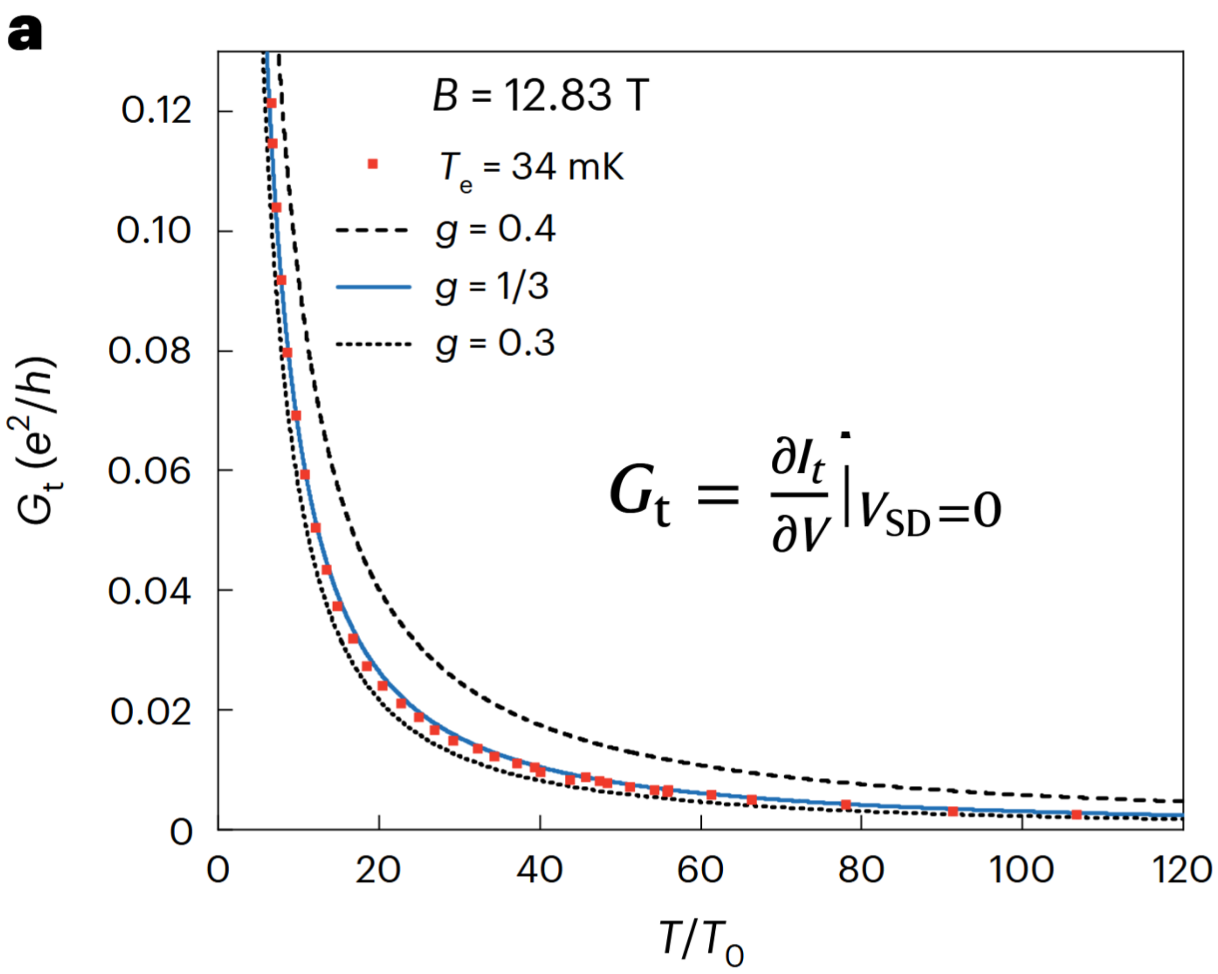
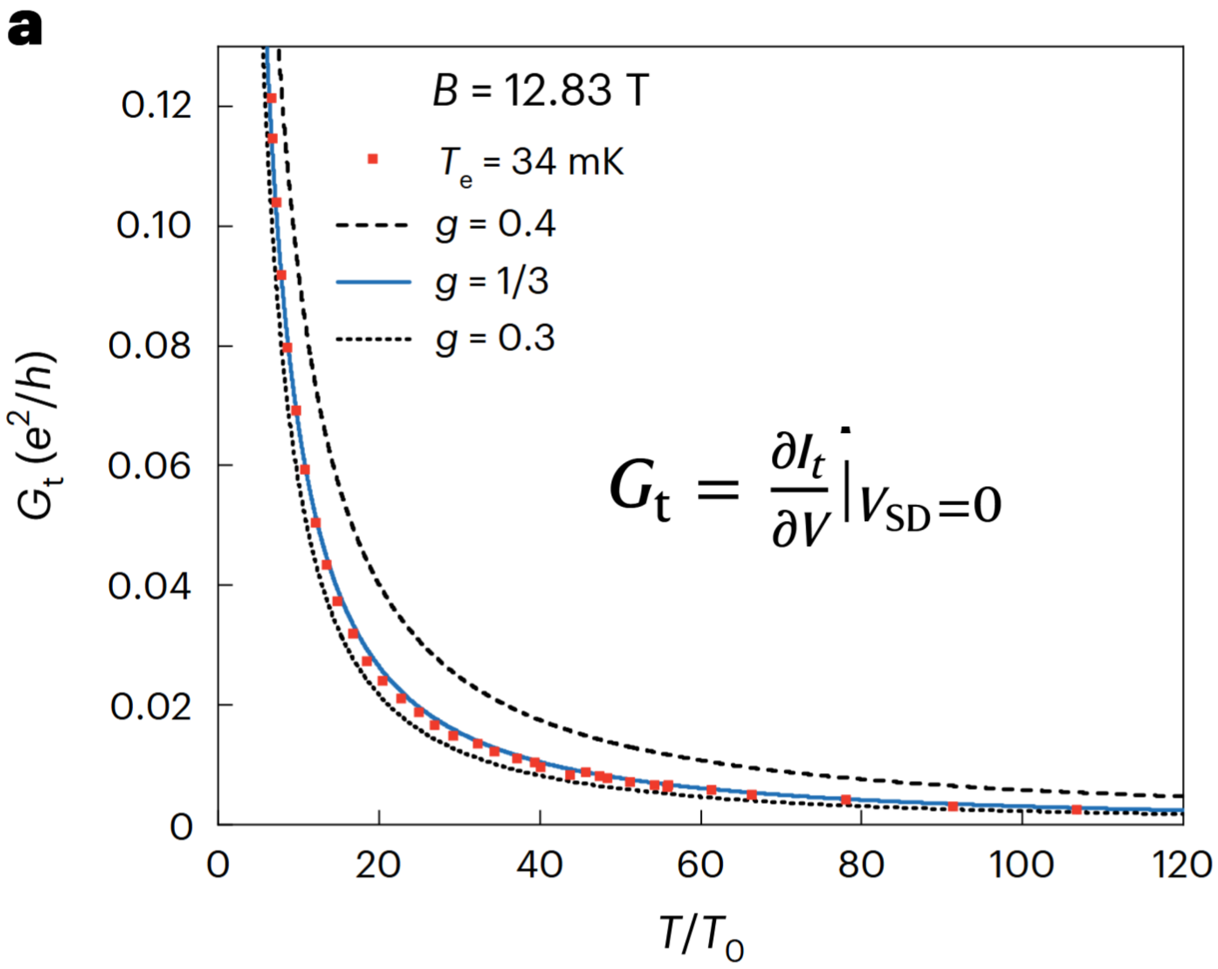
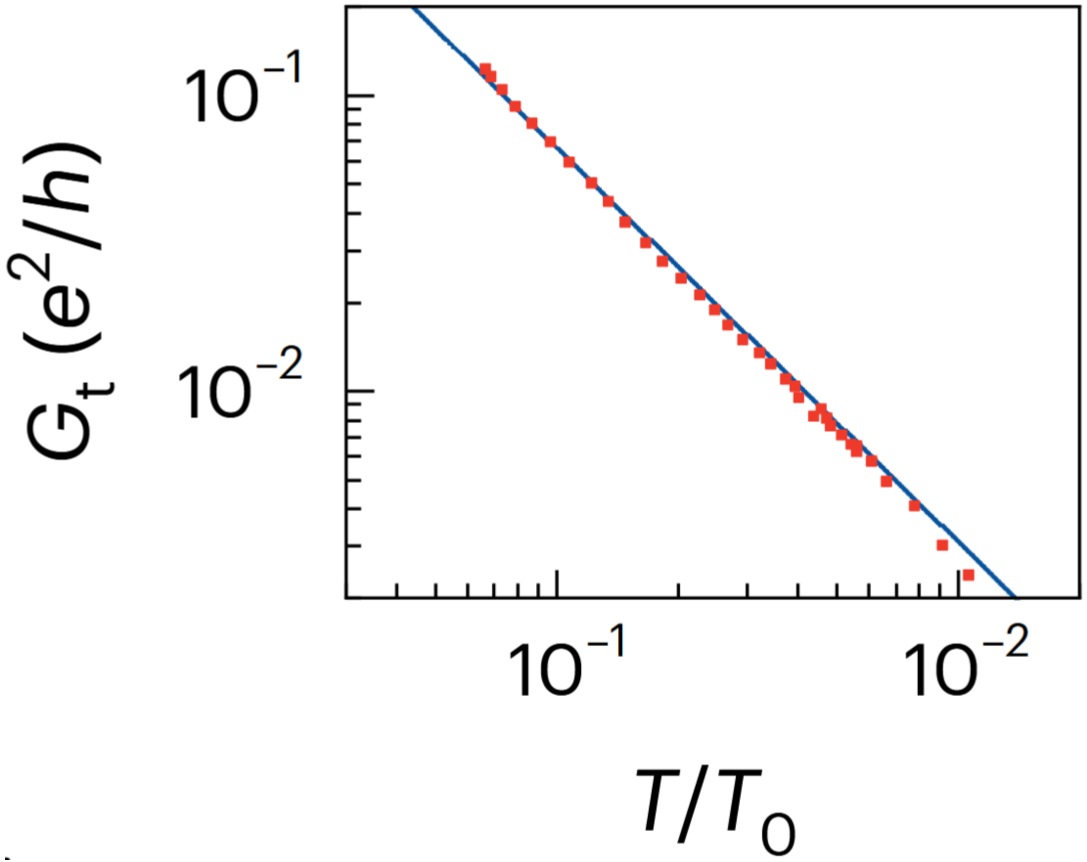
Investigating zero bias differential conductance



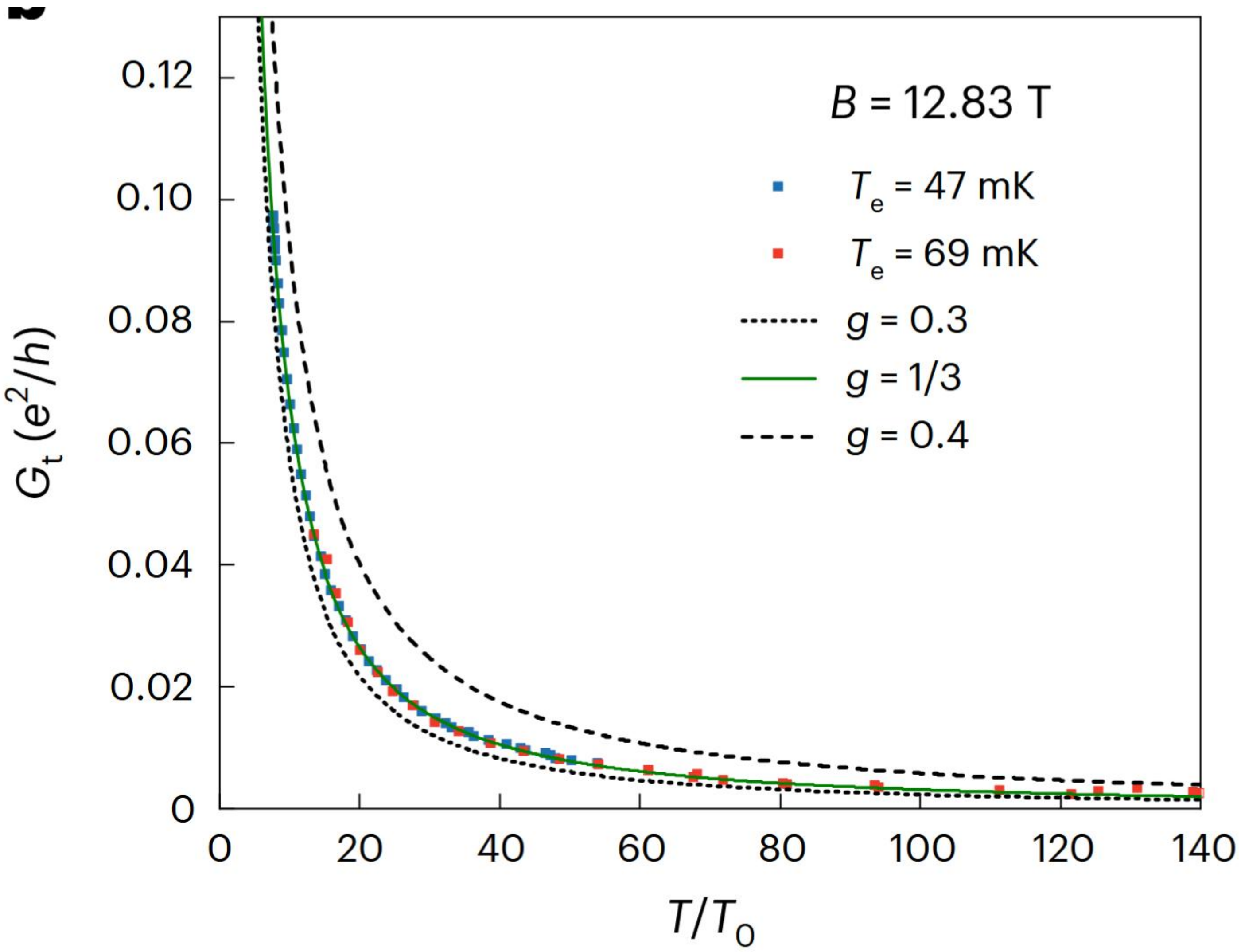
Investigating zero bias differential conductance



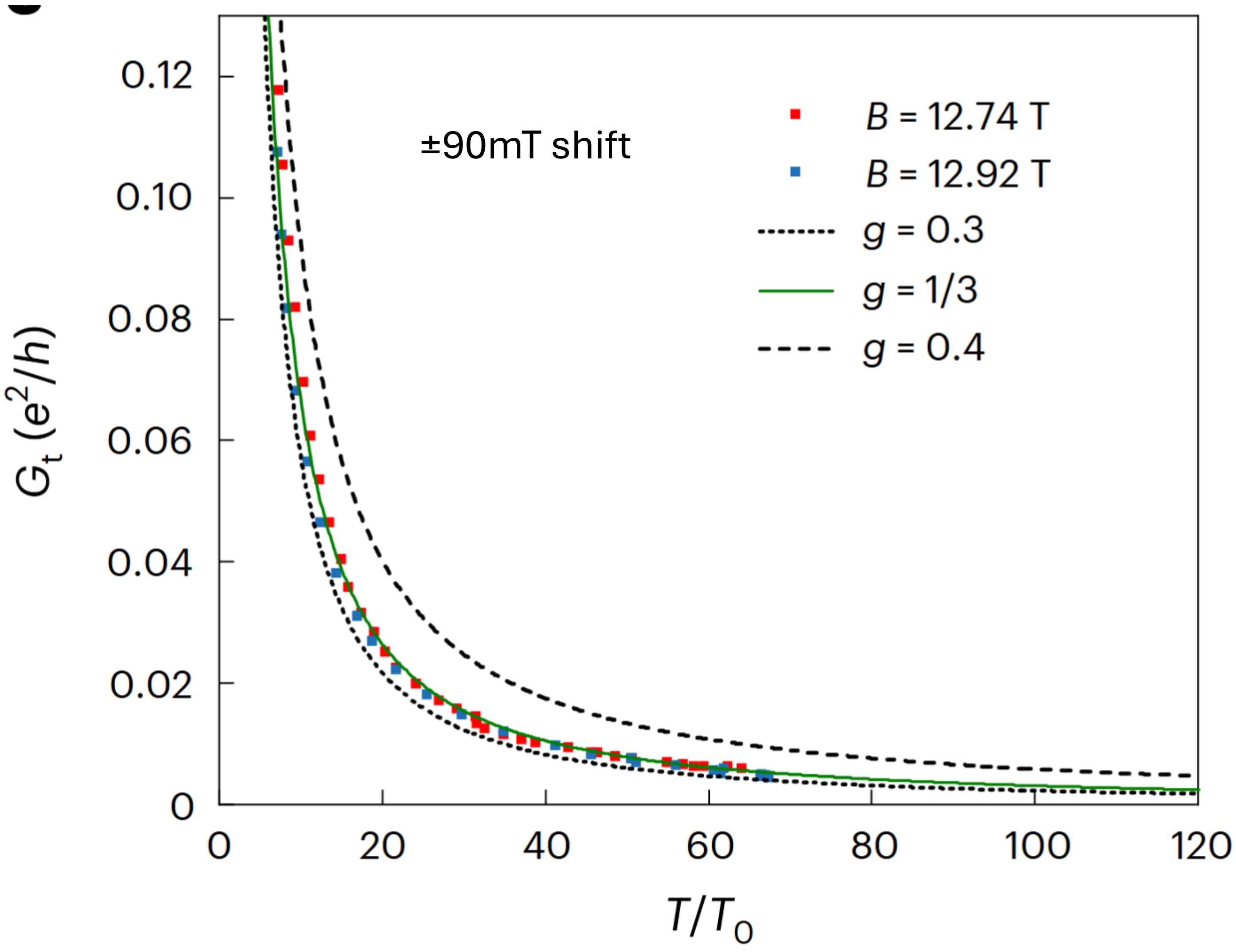
Same in log-log scale:



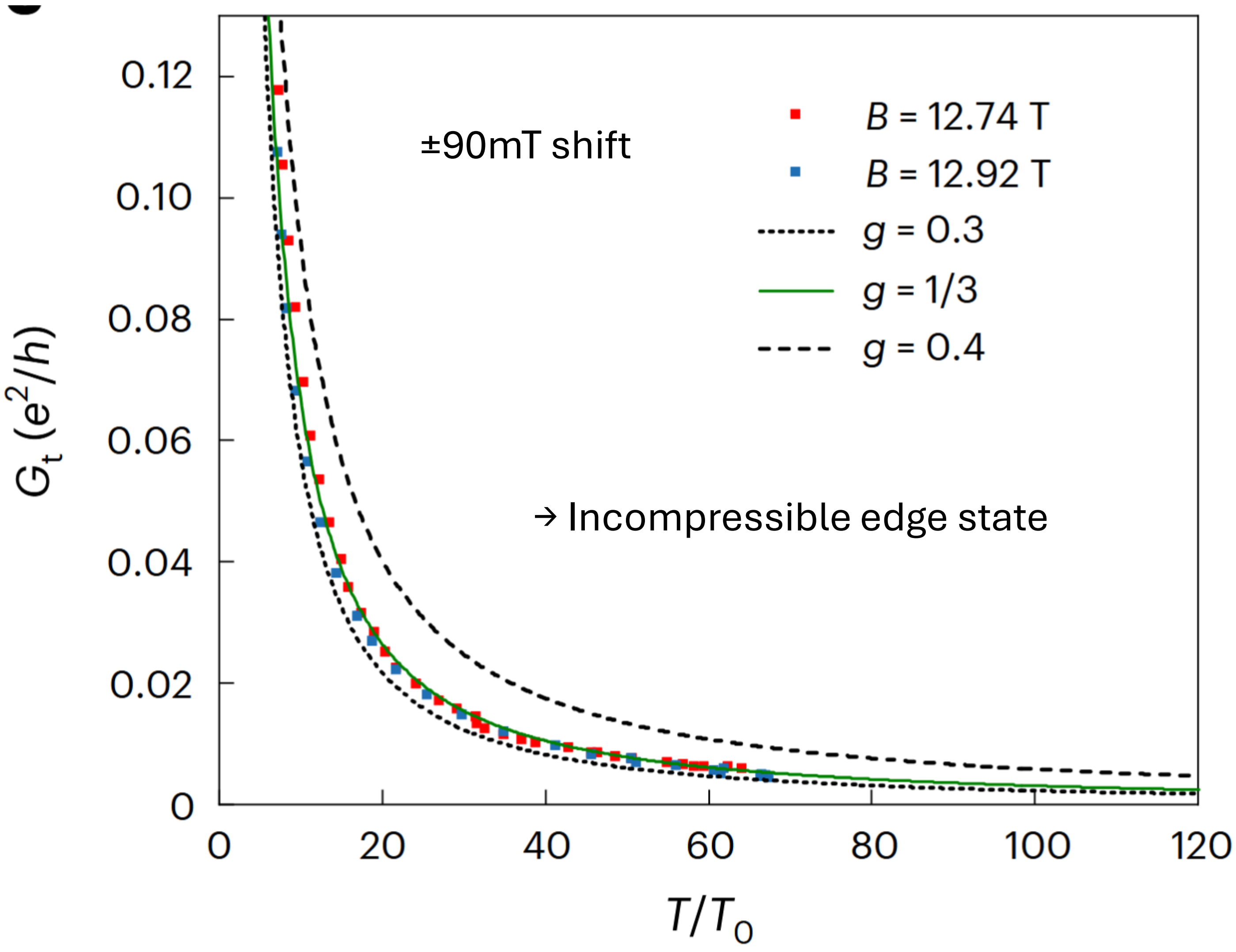
Investigating zero bias differential conductance: effect of temperature



Investigating zero bias differential conductance: effect of magnetic field



Investigating zero bias differential conductance: effect of magnetic field



Universal scaling behaviour at non-zero bias

$$G_{\mathrm{t}} = \frac{e^2}{h}\left(\frac{2\pi T}{T_0}\right)^{2g-2} f_g\left(e^* \frac{V_{\mathrm{SD}}}{k_{\mathrm{B}}T}\right)$$

Universal scaling behaviour at non-zero bias

$$G_t = \frac{e^2}{h}\left(\frac{2\pi T}{T_0}\right)^{2g-2} f_g\left(e^* \frac{V_{SD}}{k_B T}\right)$$

$$\tilde{G} = \frac{h}{e^2}\left(\frac{T_0}{2\pi T}\right)^{2g-2}(G_0 - G) \longrightarrow \tilde{G} = f_g\left(e^* \frac{V_{SD}}{k_B T}\right)$$

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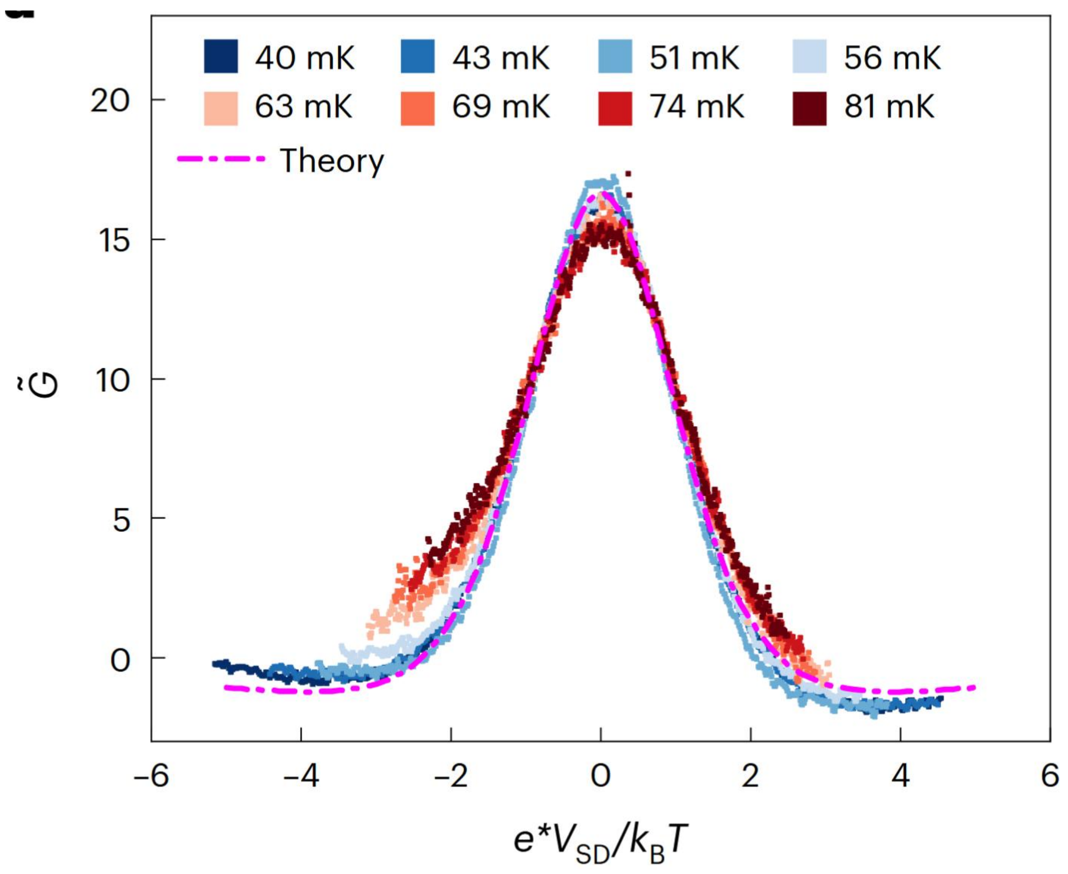

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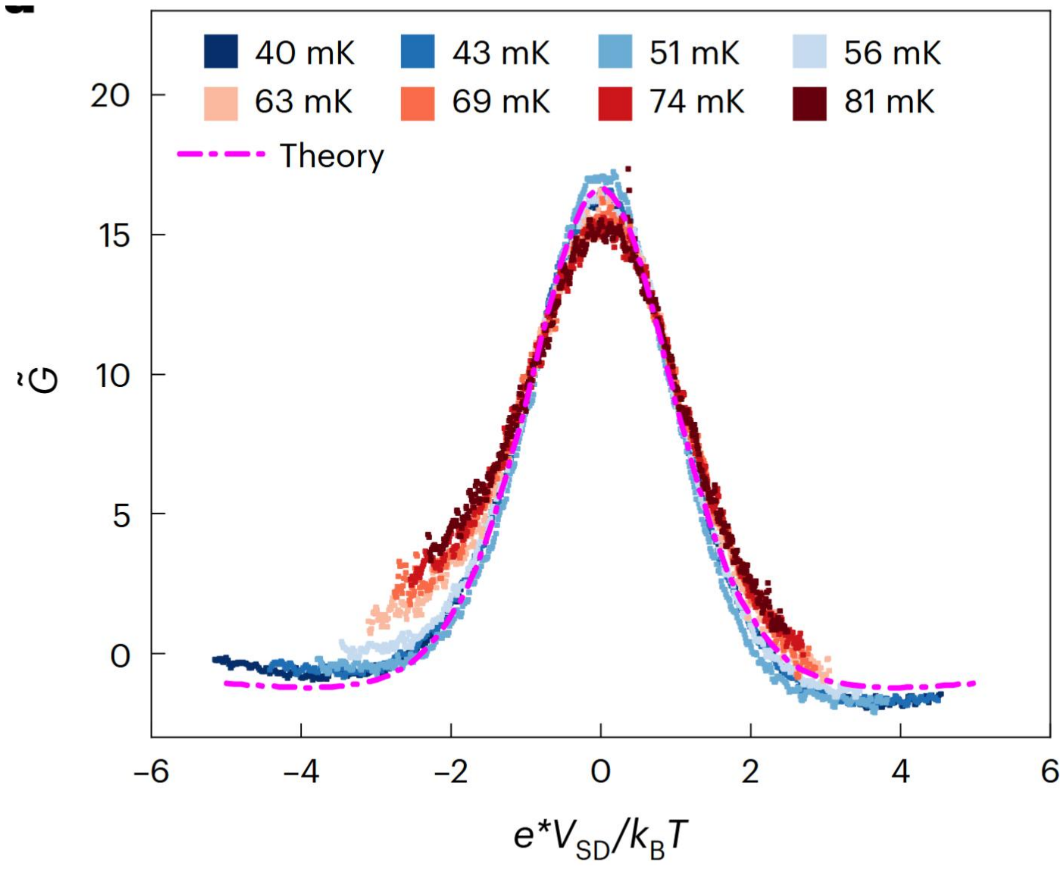

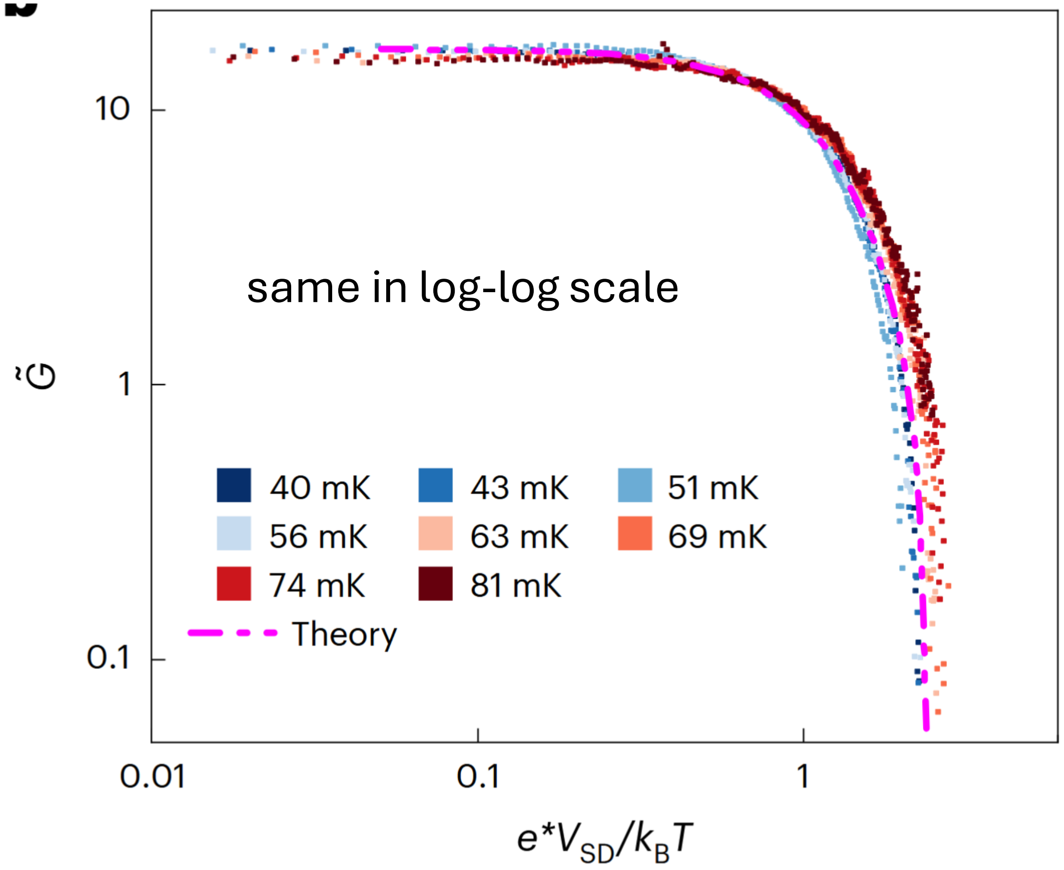

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Universal scaling confirmed for the first time for anyons in a chiral Luttinger liquid

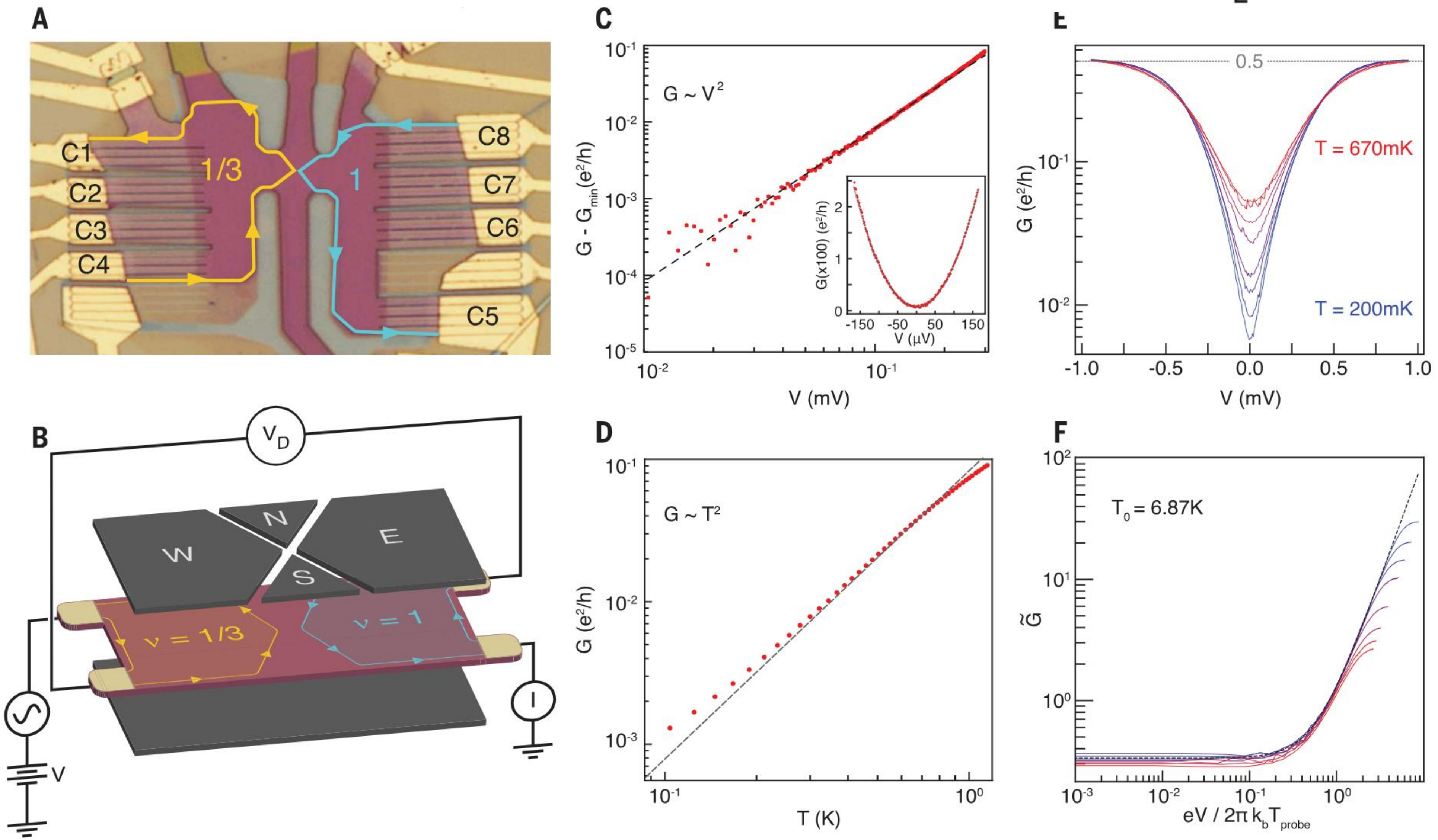
- Previous experiments showed ‘qualitative inconsistency’ with the theory, for example ‘minimum in tunnelling conductance at low bias and lowest temperatures’

- One paper showed tunnelling between $v = 1$ and $v = 1/3$ states in graphene: L. A. Cohen, et al., Universal chiral Luttinger liquid behavior in a graphene fractional quantum Hall point contact, Science **382**, 542 (2023).

- Here quantitative agreement with expected power law for tunnelling at filling factor 1/3, robust across different temperatures, magnetic field, and voltage bias

Similar tunnelling in graphene

$$G(V,T) = \frac{e^2}{2h}\left(\frac{2\pi T}{T_0}\right)^2\left[\frac{1}{3} + \left(\frac{eV}{2\pi k_b T}\right)^2 + \ldots\right]$$



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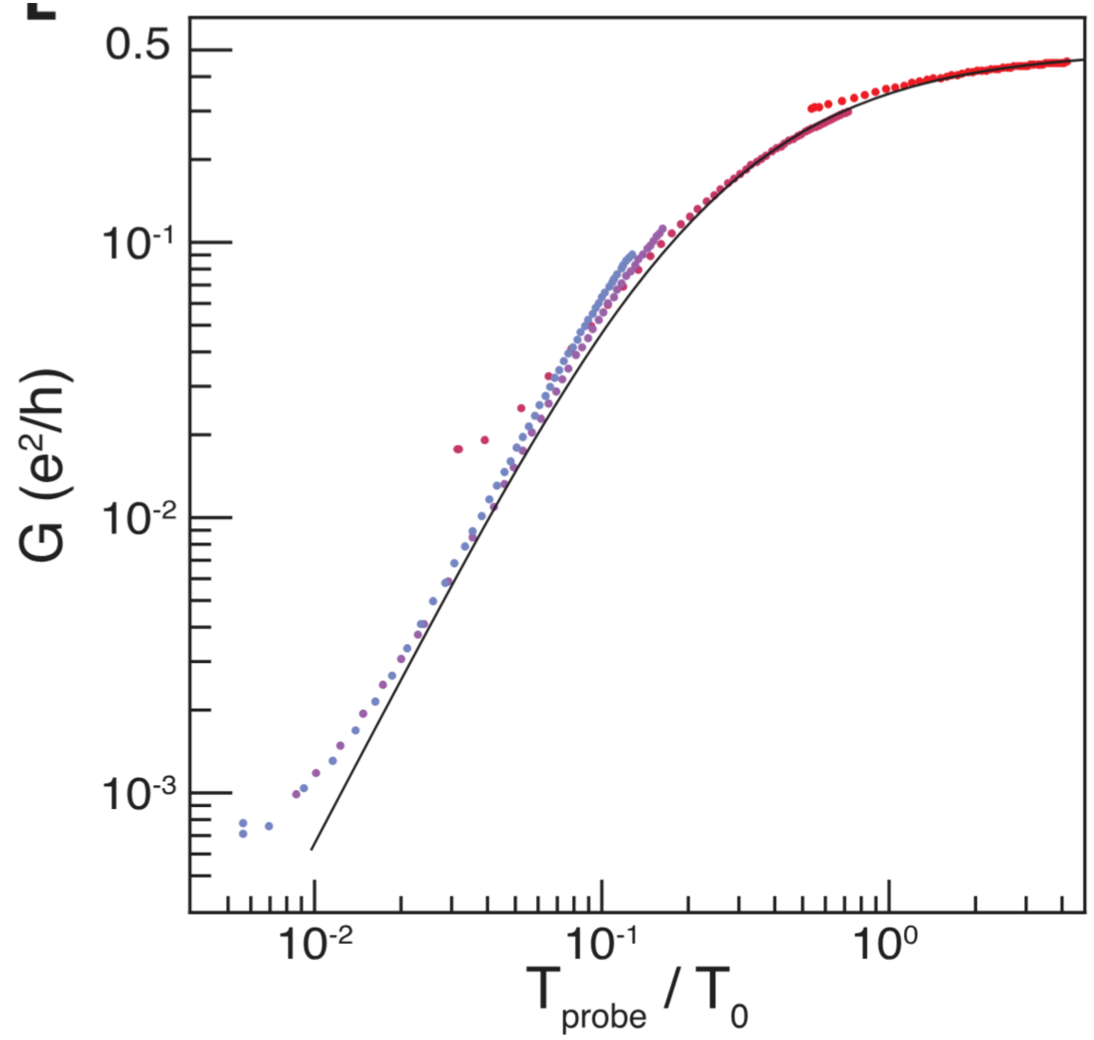
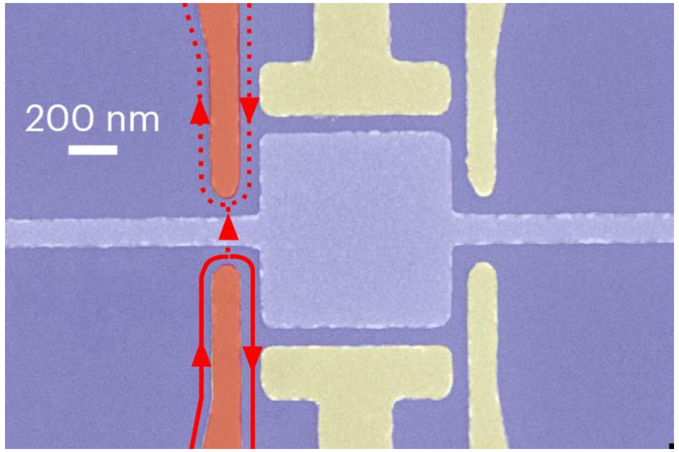


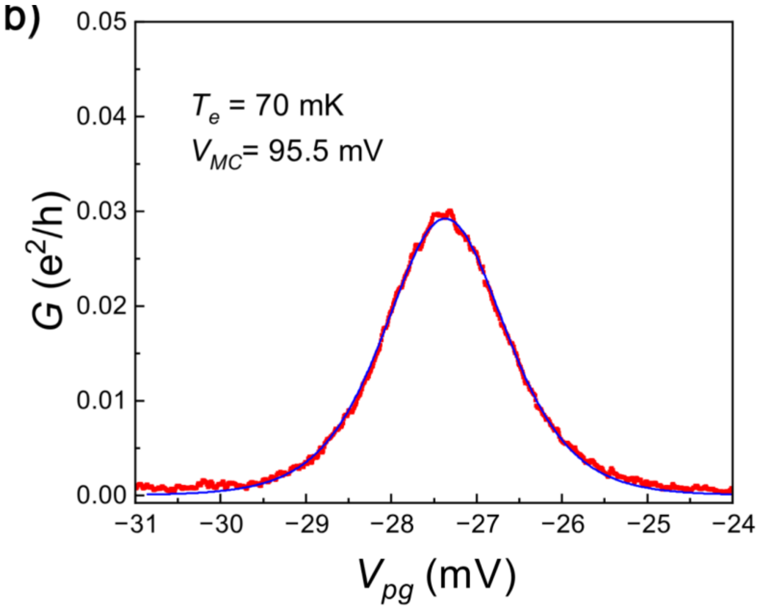
Coulomb blockade thermometry

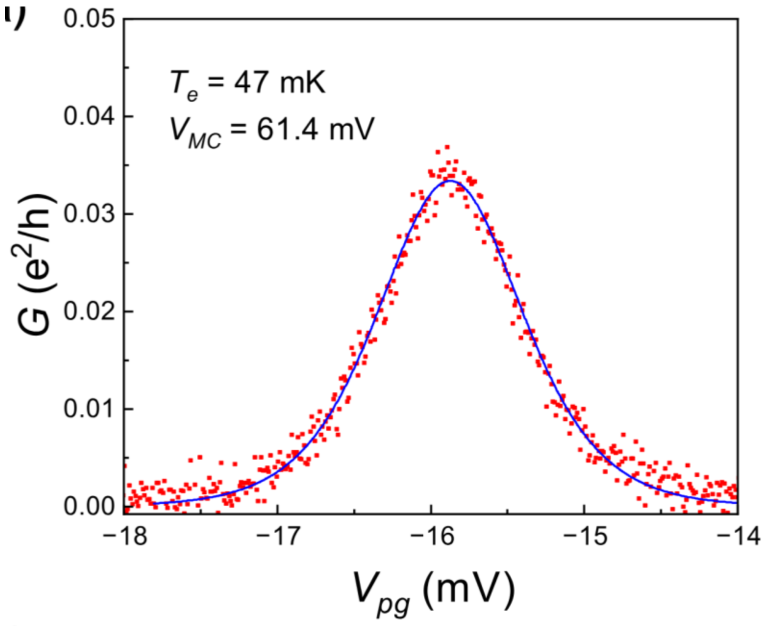


$1\mu m^2$ Fabry-Pérot interferometer
Charging energy $e^2/C \approx 91\mu eV$
Lever arm $\alpha \approx 0.016$

fit

$$G \propto cosh^{-2}(\alpha V_{pg}/2k_b T)$$





34mK?

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Electron temperatures used in calculations

- Base electron temperature of electrons of 34mK was measured via Coulomb blockade thermometry

- Higher temperatures were taken to be the mixing chamber temperature, with calibration beforehand showing they were the same above 40mK

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Conclusions

- Good agreement with theoretical predictions for tunnelling of anyons in a chiral Luttinger liquid (in the weak backscattering limit, i.e. *t* close to unity here)

- As for previous papers with similar devices, the ancillary screening wells seem to be an important ingredient, probably ‘creating a sharper confining potential at the edges of the FQHE liquid’, preventing edge reconstruction

- ‘Classification of the topological order responsible for the bulk quantization of the FQHE liquids can be fully characterized by a sequence of measurements with a Fabry-Pérot interferometer’