QUANTUM ERROR CORRECTION WITH SILICON SPIN QUBITS

Yang LIU
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University of Basel

Purpose of reading

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Quantum error correction with silicon spin qubits

Kenta Takeda, Akito Noiri, Takashi Nakajima, Takashi Kobayashi & Seigo Tarucha

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Outline:

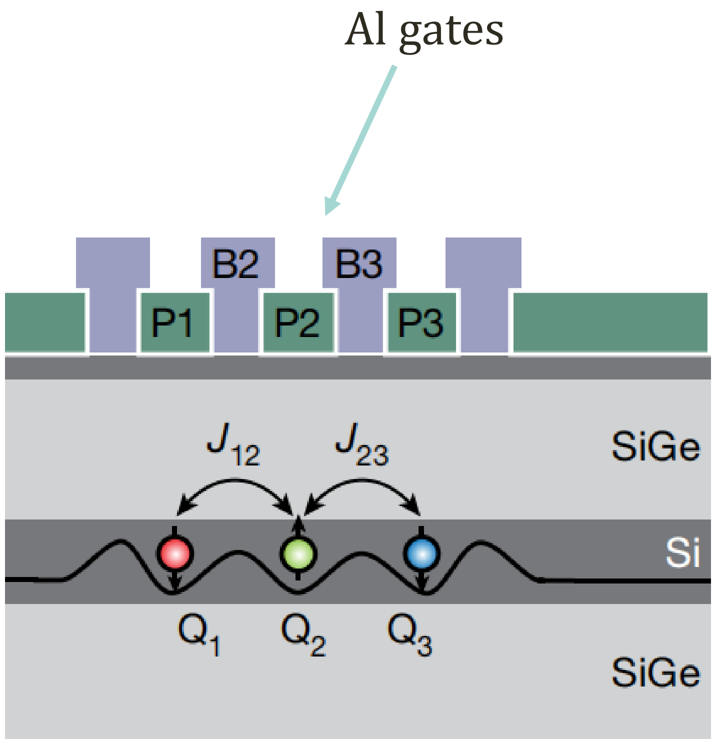
- Well tunable three-qubit Si/SiGe device
- Virtual gates and RF reflectometry
- Characterize each qubit
- Single- and double-qubit gate
- Three-qubit iToffoli gate
- Optimization and error correction

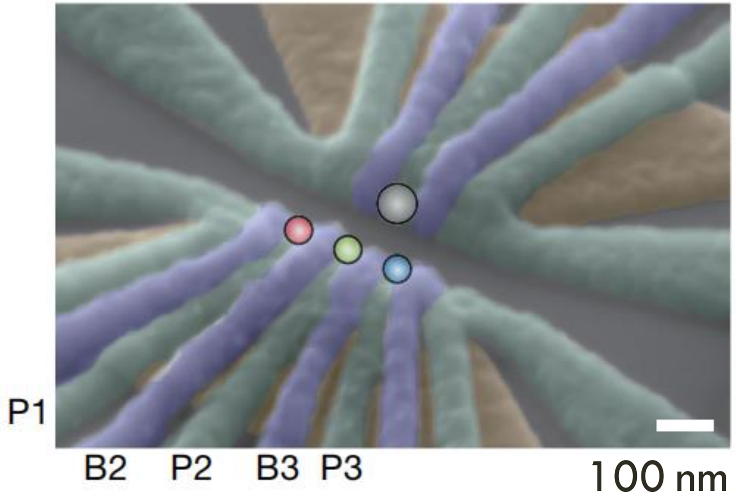
What we can do:

- Well tunable three-qubit Si/SiGe device
- Virtual gate for individual control
- RF reflectometry with varactor
- Characterize each qubit
- High fidelity single-qubit gates
- Two-qubit CZ, CPhase or SWAP gate
- Three-qubit Toffoli gate
- Optimization and error correction

And more…

Device

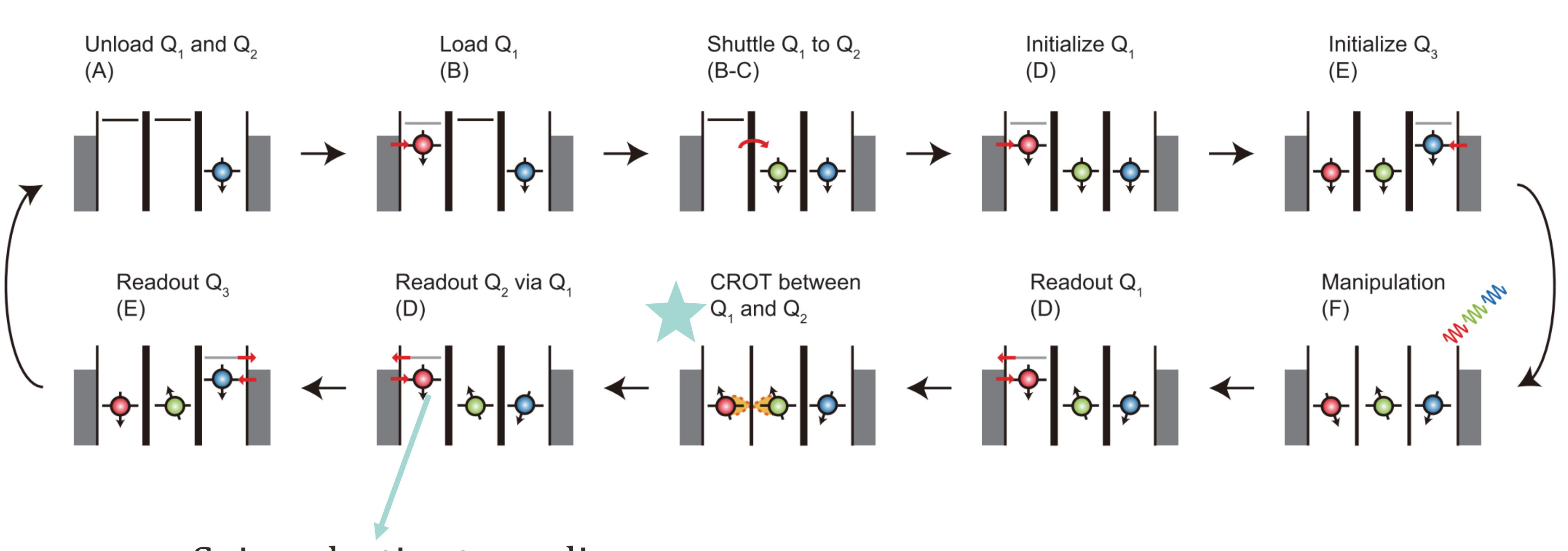




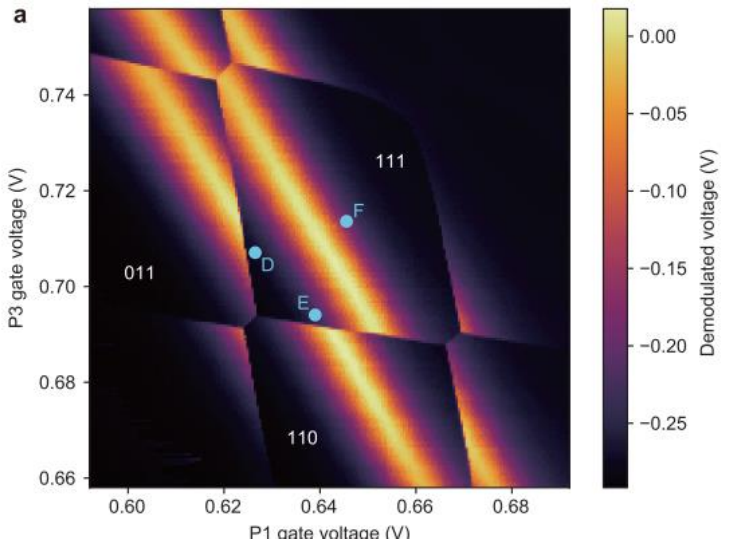
Q_2: *data qubit*
Q_1, Q_3: *ancilla qubit*

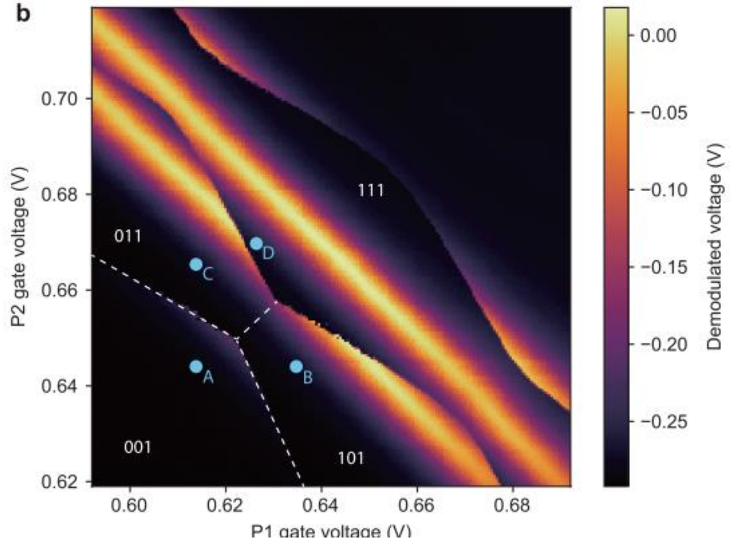
- Si/SiGe heterostructure, isotopically natural silicon
- Micro-magnet on the top of Al gates for local magnetic field gradient
- Charge sensor and RF reflectometry for sensitive and fast readout

Three-spin initialization and measurement



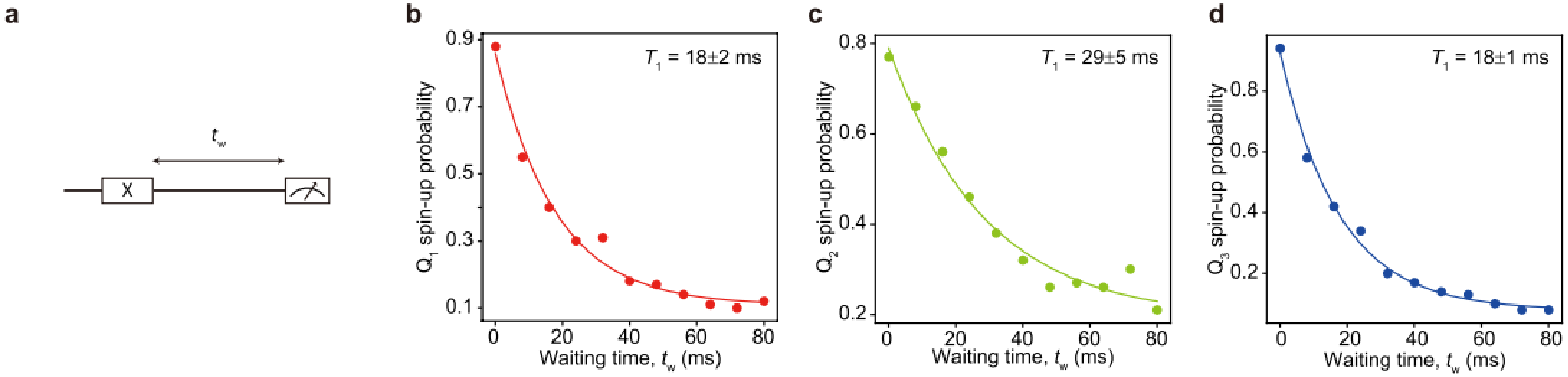
Spin-selective tunneling

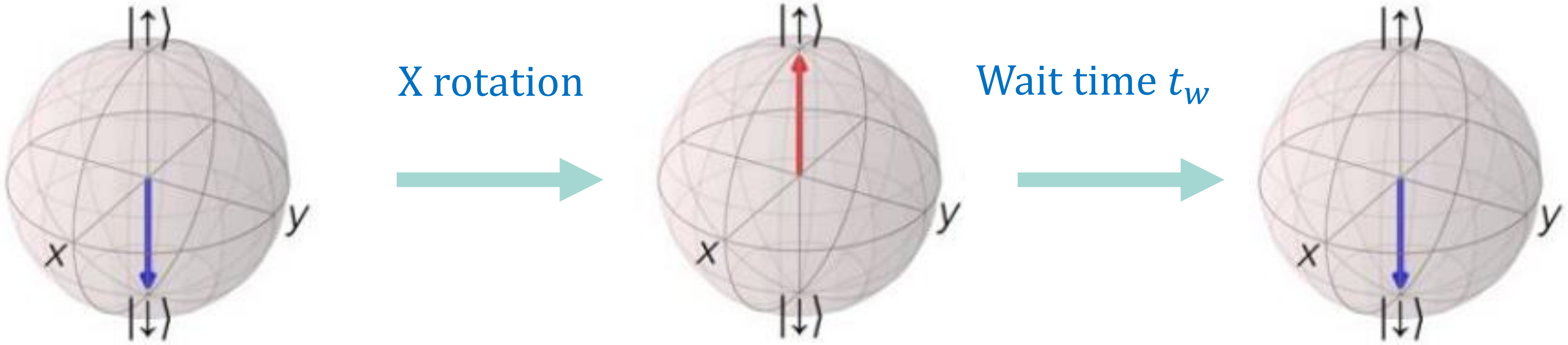




- Readout Q_1 via Q_1: turn on J_{12} (virtual B_2 gate) at the charge-symmetry point,
- Before CROT gate, $Q_1 = |\downarrow\rangle$, $Q_2 = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$,
- After CROT gate, $Q_1Q_2 = \alpha|\downarrow\uparrow\rangle + e^{i\theta}\beta|\uparrow\downarrow\rangle$ (flip Q_1 when $Q_2 = |\downarrow\rangle$)

Relaxation time T_1

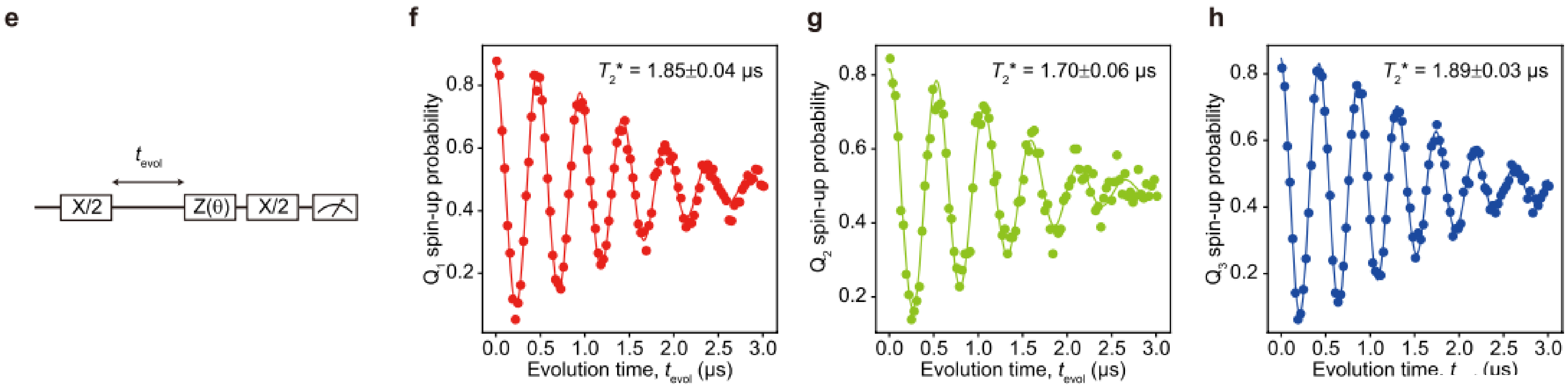


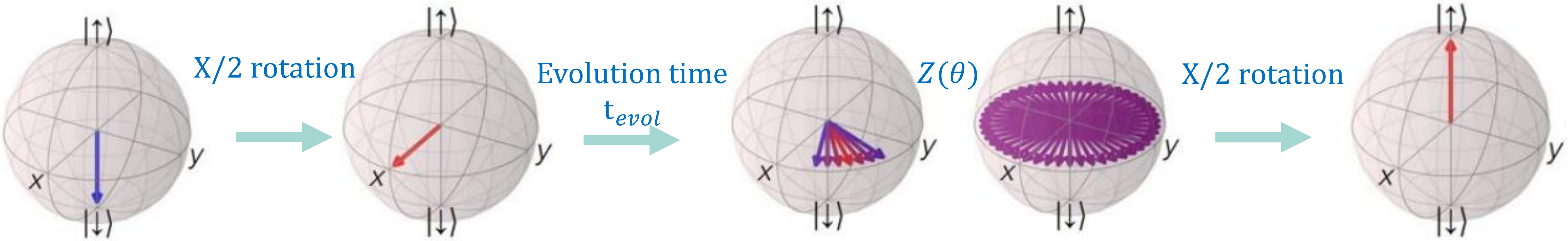


$$P_{up} = P_0 \exp\left(-\frac{t_w}{T_1}\right)$$

T_1: the time qubit relaxes from the excited state to the ground state

Dephasing time T_2^*





This schematic shows Y/2 rotation :)

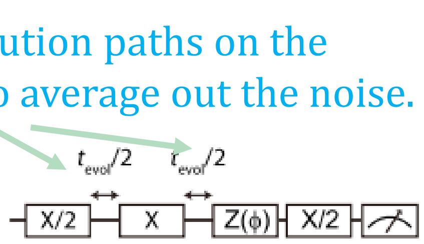
$$P(t_{evol}) = A exp\left(-\left(\frac{t_{evol}}{T_2^*}\right)^2\right)\cos(2\pi\delta f t_{evol} + \phi) + B$$

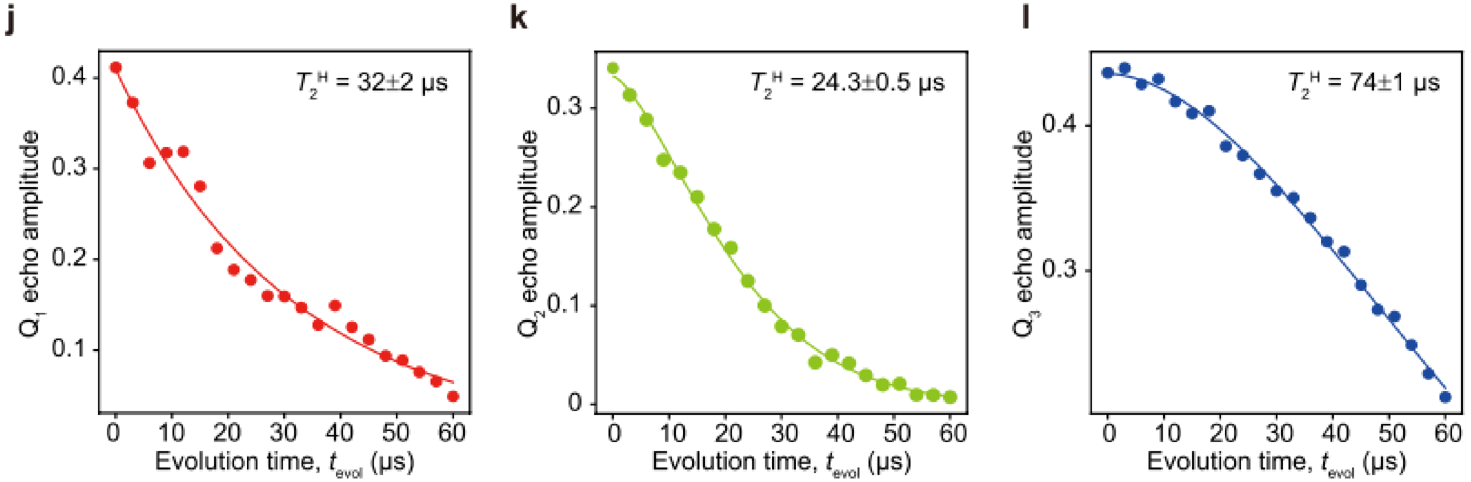
δf: oscillation frequency

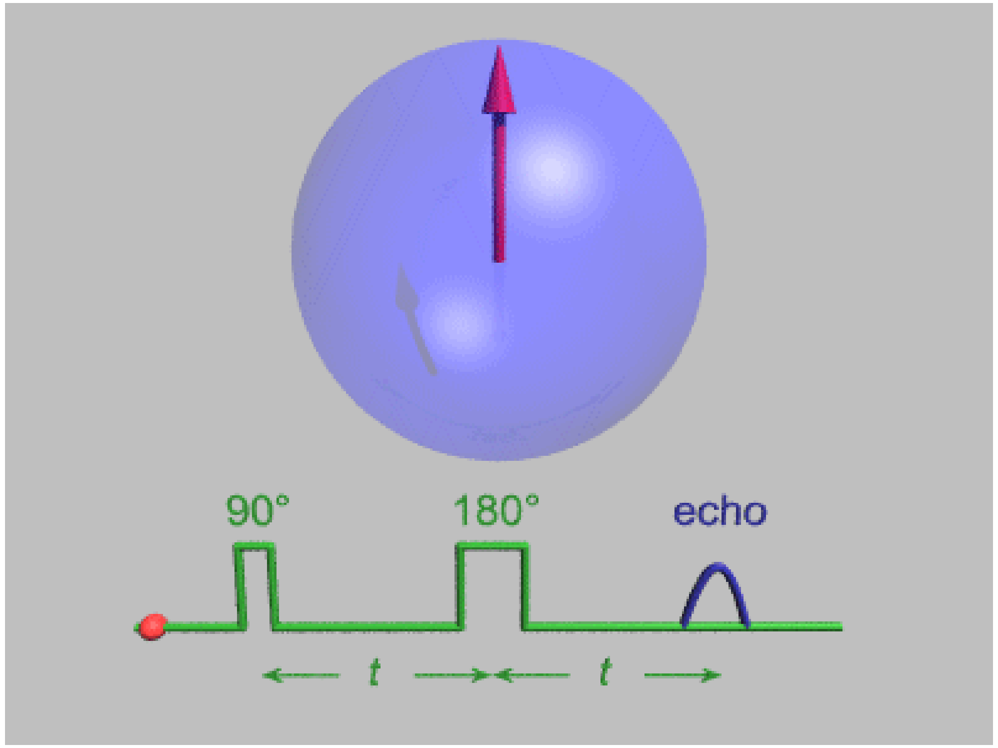
T_2^* : how long the phase of a qubit stays coherent

Hahn Echo T_2^{Hahn}

Two opposite evolution paths on the equatorial plane to average out the noise.







$$P(t_{evol}) = V exp(-\left(\frac{t_{evol}}{T_2^H}\right)^{\gamma})$$

V: visibility

γ: exponent

T_2^H: echo time

Single-qubit rotation: two axes rotation

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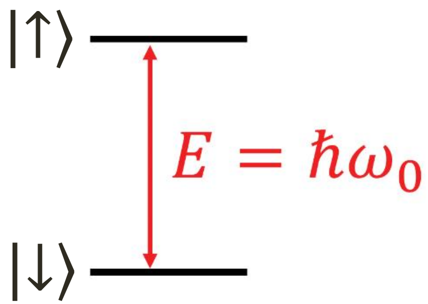
This paper

$$X(\pi) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \Longleftrightarrow \quad Y(\pi) = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad\qquad Z(\pi) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



$\Omega(\theta) = \Omega(t)\cos(\omega_0 t - \gamma)$
$\Omega(t)$: drive strength
ω_0: drive frequency
γ: drive phase

$\gamma = 0$, X rotation; $\gamma = \frac{\pi}{2}$, Y rotation

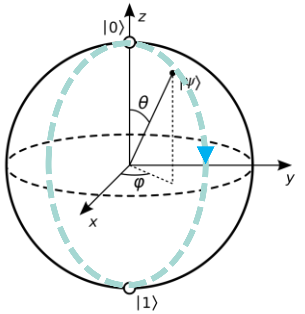
Virtual Z gate: rotate the axes rather than the qubit states

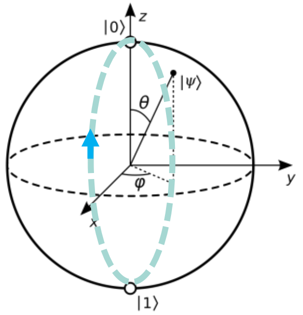
Hamiltonian in qubit rotating frame:

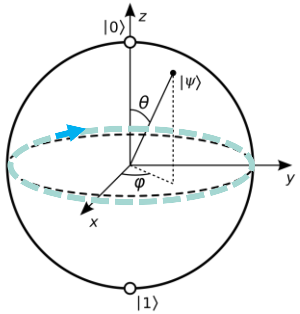
$$H = \hbar\left(\sum_n \frac{\Omega(t)}{2}[\cos(\gamma)\,\sigma_x + \sin(\gamma)\sigma_y]\right)$$

Unitarty transformation

$$U = e^{-\frac{i\Omega T}{2}[\cos(\gamma)\sigma_x + \sin(\gamma)\sigma_y]}$$







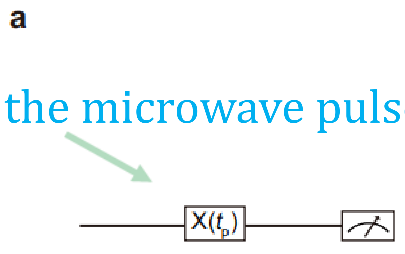
- In this device, resonant frequency ~ 20GHz
- Phase rotation is virtually implemented by shifting the reference phase of the I/Q modulation waveform.

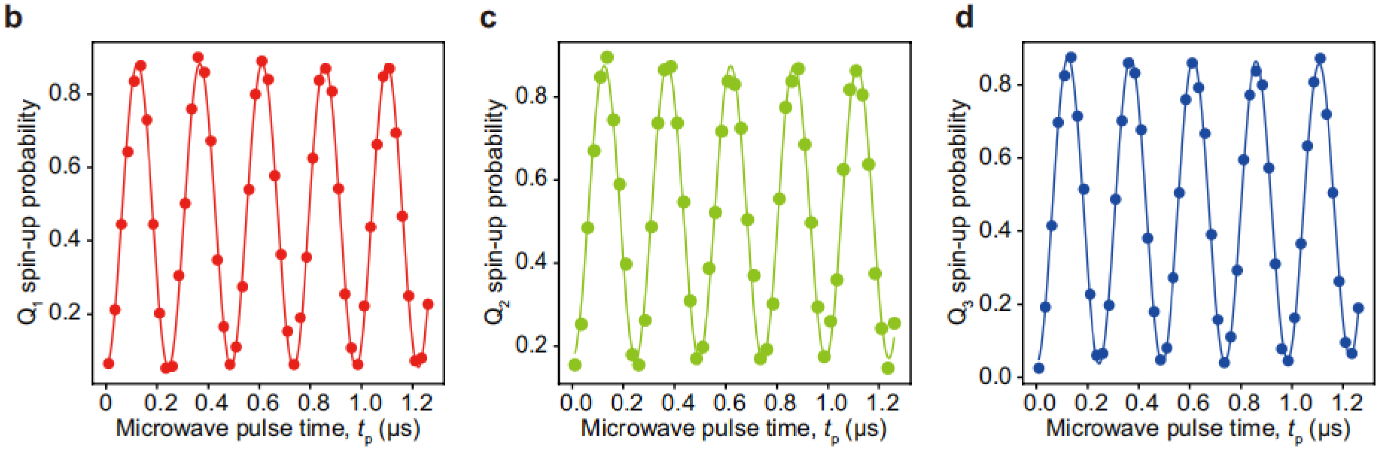
T. F. Watson *et al*, Nature 555, 633–637 (2018)

McKay *et al*, Phys. Rev. A 96, 022330 (2017)

Rabi oscillation and gate fidelity

t_p: duration of the microwave pulse





Rabi Oscillation: $P(t_p) = A\sin(\omega_{rabi}t + \phi)$

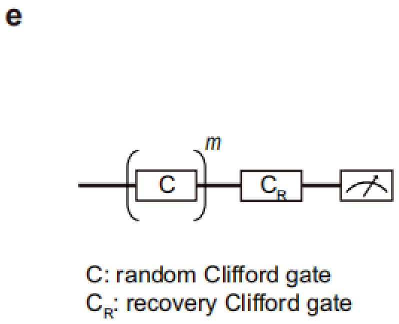
ω_{rabi}: the rotate angular frequency around X axis

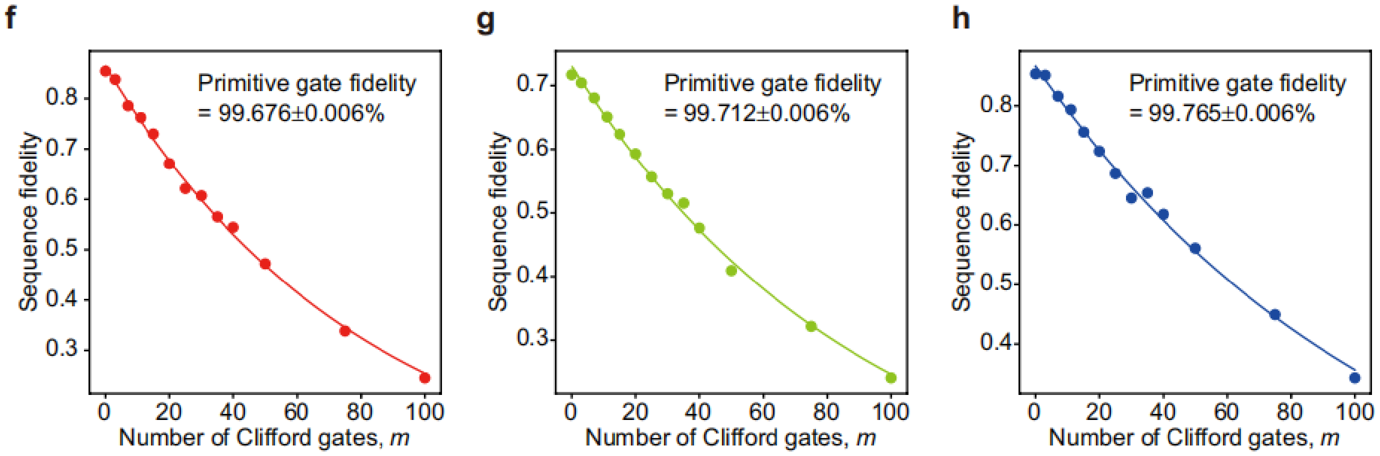
$\omega_{rabi} \sim 4MHz$ in this device

Benchmark fidelity: $F(m) = Vp^m$

p: depolarizing parameter

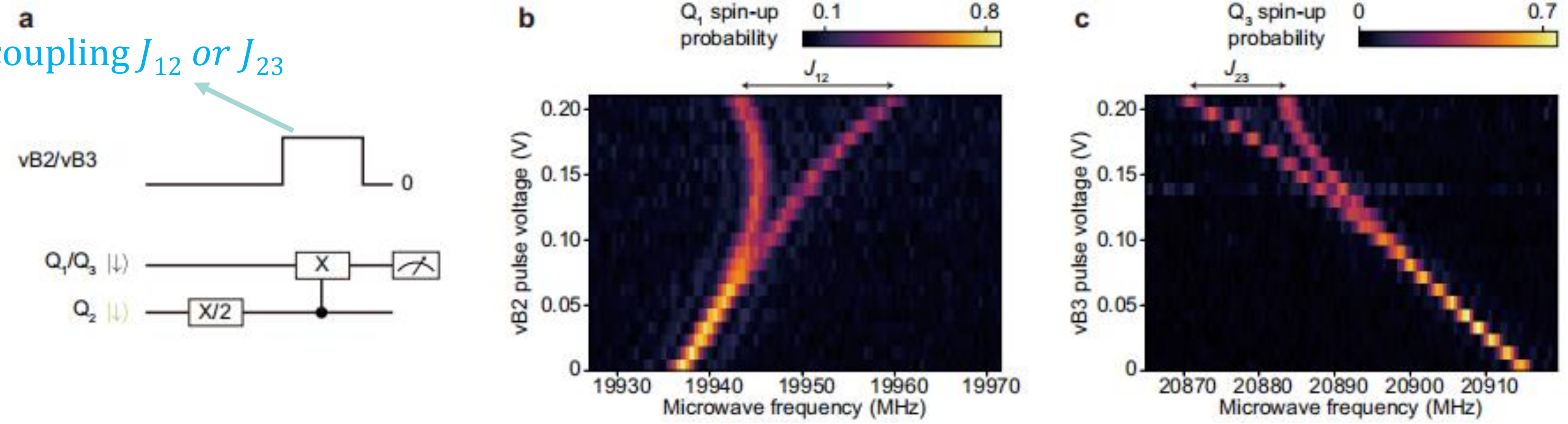
V: visibility





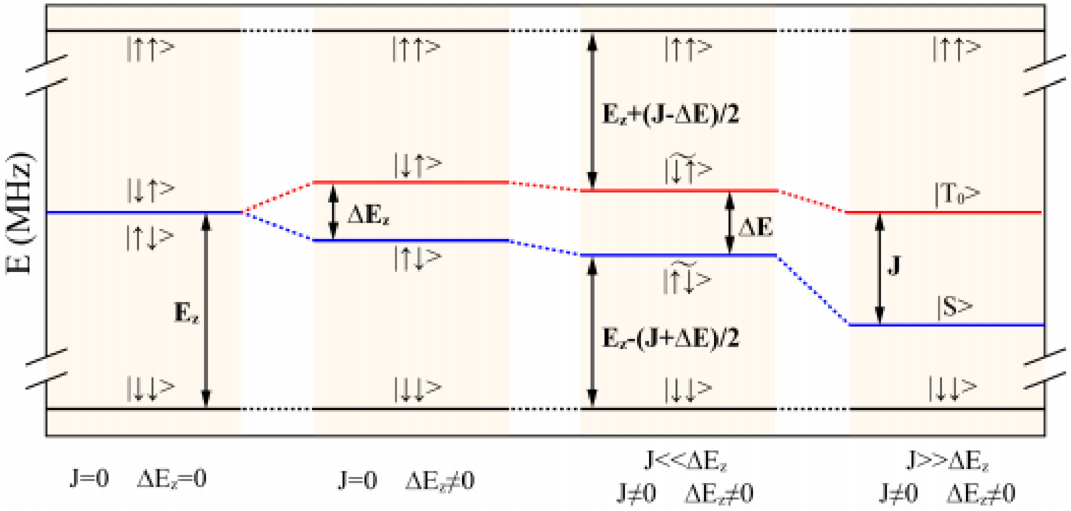
Two-qubit couplings

Turn on the coupling J_{12} or J_{23}



$\frac{X}{2}$ *gate on* Q_2 ⟶ *CROT gate on* Q_1Q_2

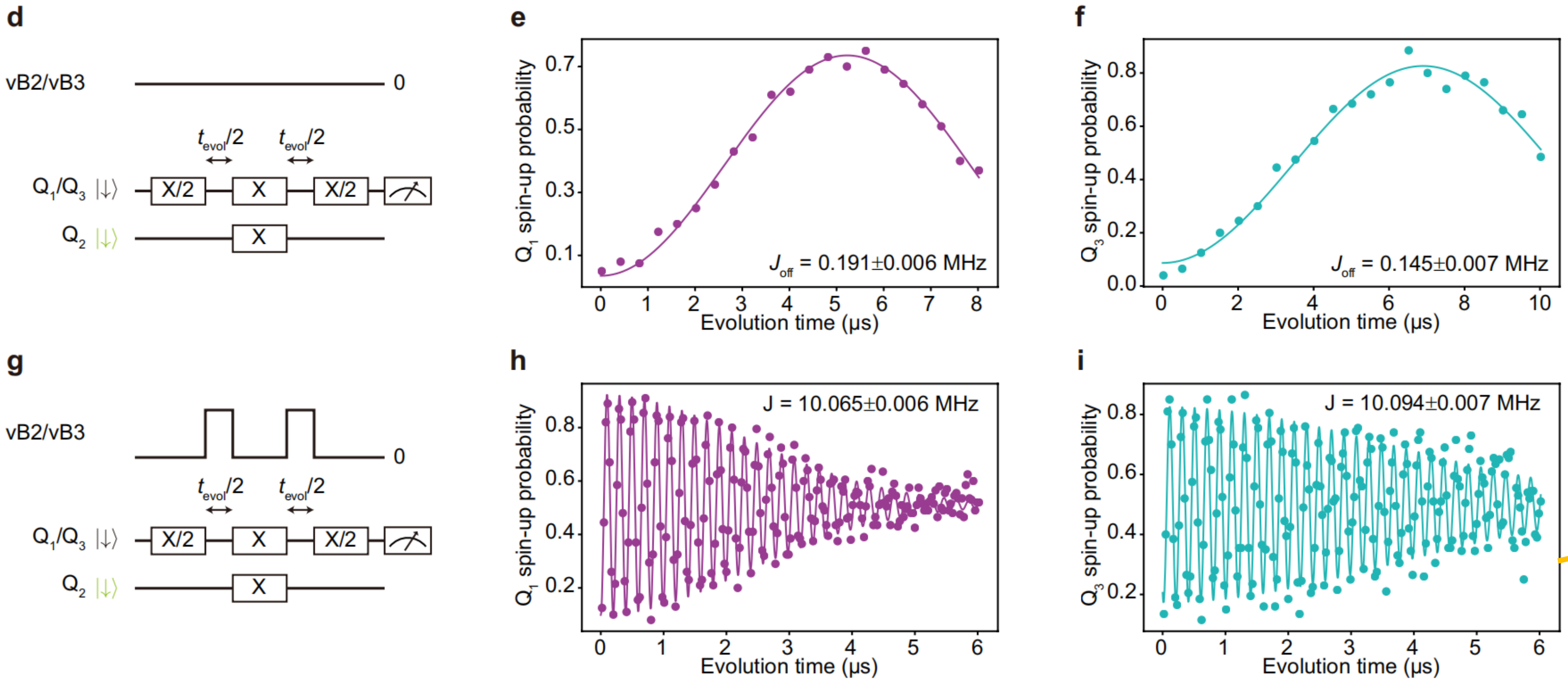
$$Q_1Q_2 = |\downarrow\downarrow\rangle \longrightarrow Q_1Q_2 = \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle + |\downarrow\uparrow\rangle) \longrightarrow Q_1Q_2 = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

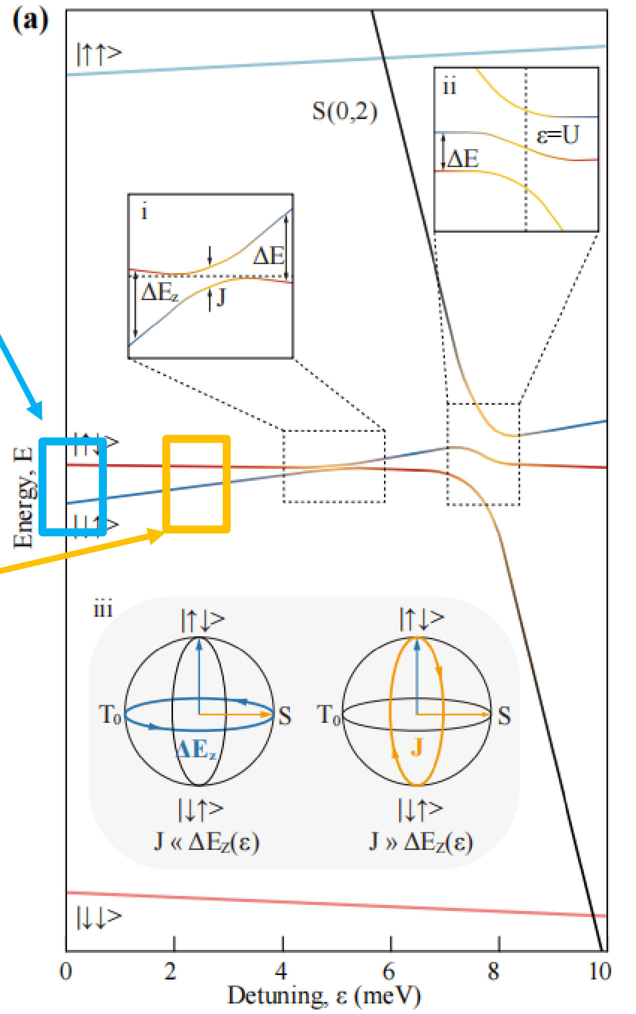


When we turn on the exchange coupling between qubit:

$$\Delta E = E_{|\widetilde{\downarrow\uparrow}\rangle} - E_{|\widetilde{\uparrow\downarrow}\rangle} = \sqrt{\Delta E_Z^2 + J^2}$$

Exchange coupling measurement





- $P(t_{evol}) = V\sin(\pi J_{off} t_{evol})$

J_{off}: the lowest J when we turn off the exchange at symmetric operation point

- $P(t_{evol}) = (V\sin(2\pi J_{on} t_{evol}) + B) \times exp\left(-\frac{2t_{vol}}{T_2}\right)^2$

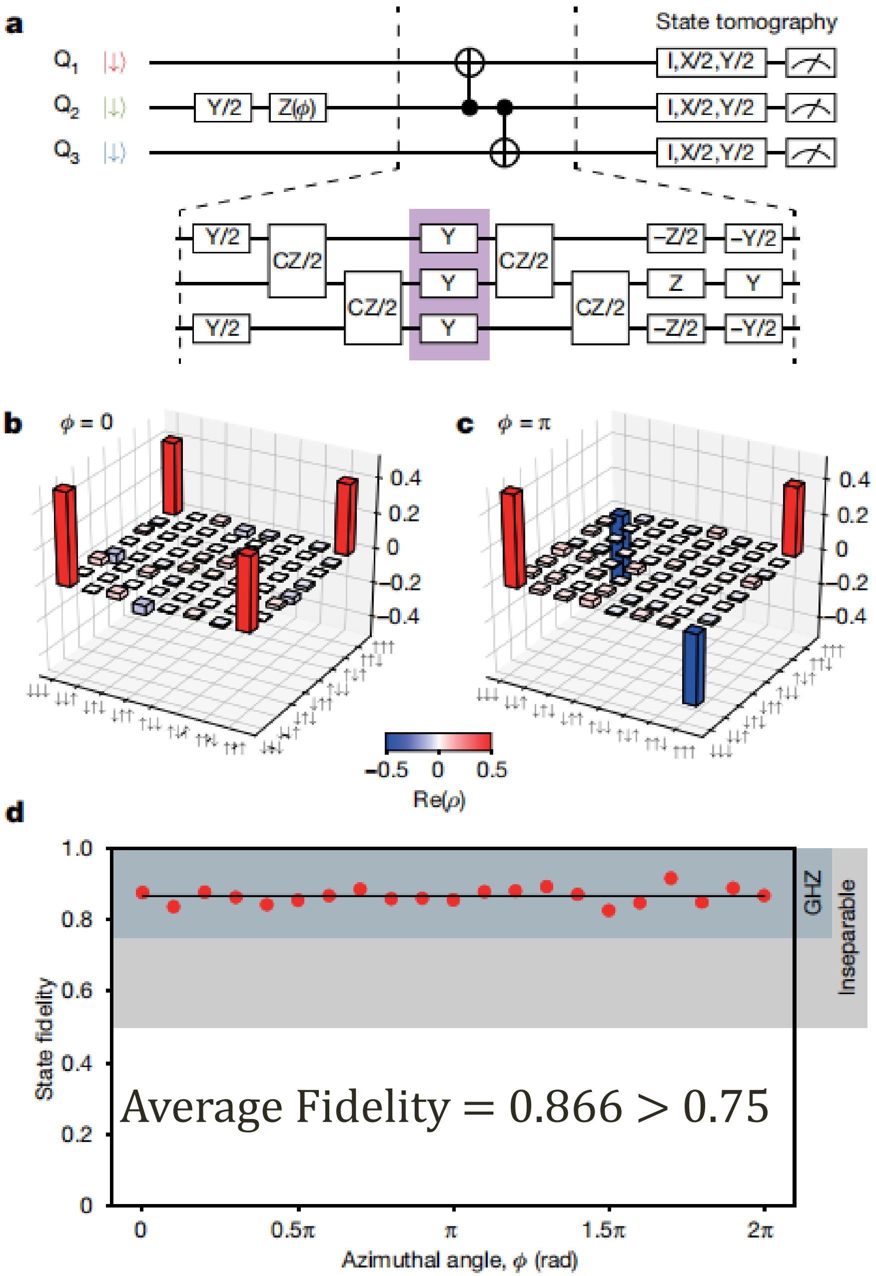
T_2: Gaussian decay time $\sim\mu s$

J_{on} : exchange interaction when we turn on exchange away from symmetric operation point

T. F. Watson *et al*, Nature 555, 633–637 (2018)

Ming Ni *et al*, arXiv 2310.06700 (2023)

GHZ entangle states



Average Fidelity = 0.866 > 0.75

$$Y\left(\frac{\pi}{2}\right) = \frac{\exp\left(i\frac{\pi}{4}\right)}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}$$

here we omit the phase $\exp\left(i\frac{\pi}{4}\right)$

$$\text{CNOT gate} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$$

$$Q_1Q_2Q_3 = |\downarrow\rangle \otimes |\downarrow\rangle \otimes |\downarrow\rangle \xrightarrow{\frac{Y}{2} + Z(\phi)\, on\ Q_2} |\downarrow\rangle \otimes \frac{1}{\sqrt{2}}(|\downarrow\rangle + e^{i\phi}|\uparrow\rangle) \otimes |\downarrow\rangle$$

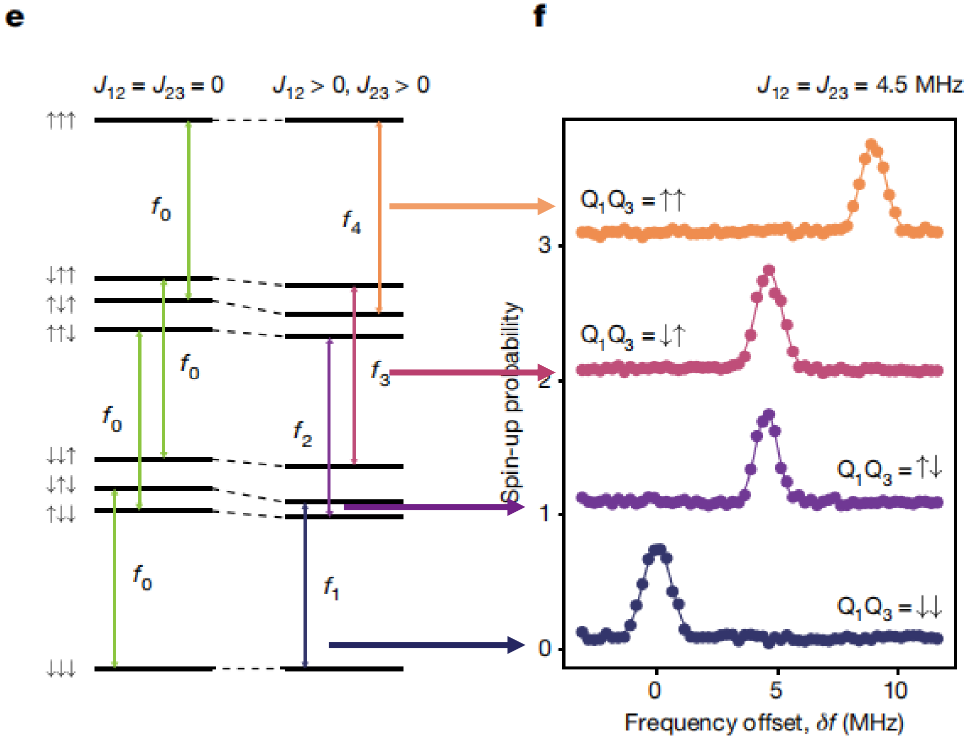
$$\xrightarrow{CNOT on\ Q_2Q_1} \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle + e^{i\phi}|\uparrow\uparrow\rangle) \otimes |\downarrow\rangle \xrightarrow{CNOT on\ Q_2Q_3} \frac{1}{\sqrt{2}}(|\downarrow\downarrow\downarrow\rangle + e^{i\phi}|\uparrow\uparrow\uparrow\rangle)$$

$$\phi = 0, GHZ = \frac{1}{\sqrt{2}}(|\downarrow\downarrow\downarrow\rangle + |\uparrow\uparrow\uparrow\rangle)$$

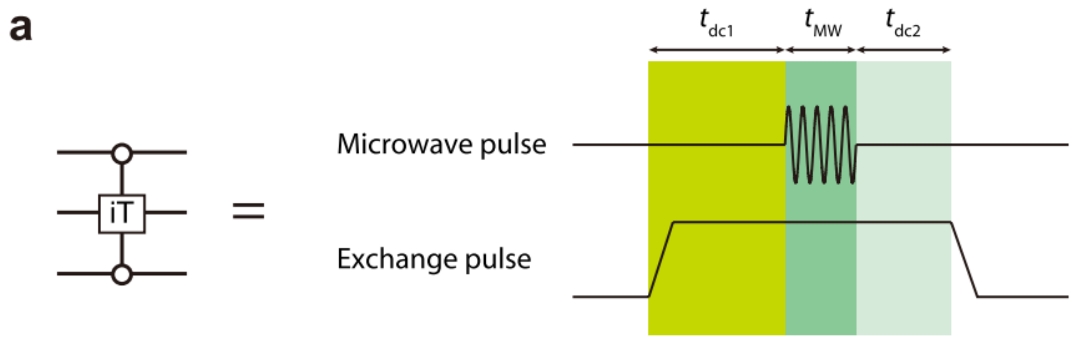
$$\phi = \pi, GHZ = \frac{1}{\sqrt{2}}(|\downarrow\downarrow\downarrow\rangle - |\uparrow\uparrow\uparrow\rangle)$$

$$F = \langle GHZ_\phi|\rho|GHZ_\phi\rangle$$

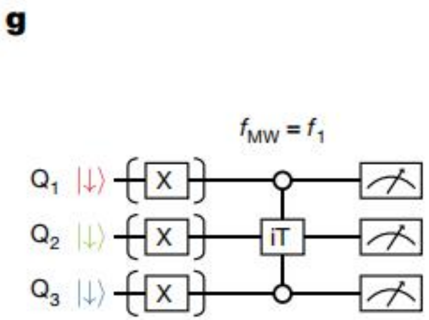
Resonantly driven iToffoli gate

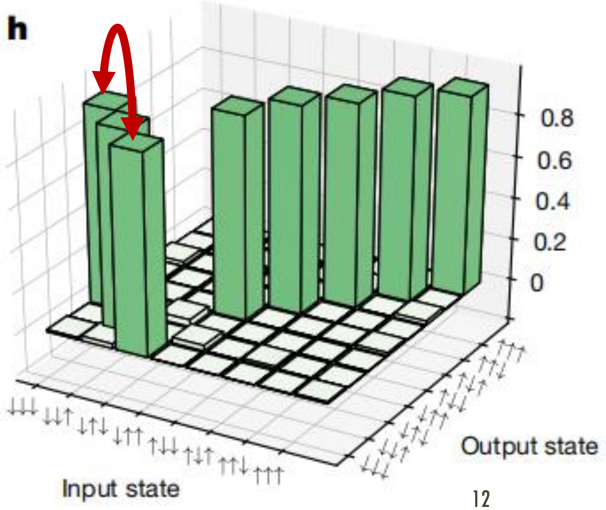


- iToffoli: an extra phase factor of I on the ancilla qubits
- $Q_1 Q_3 = |{\downarrow\downarrow}\rangle$, flip Q_2. The population of $|{\downarrow\downarrow\downarrow}\rangle$ and $|{\downarrow\uparrow\downarrow}\rangle$ are swapped
- $f_{MW} = f_1$ for highest fidelity
- $\mathrm{F}_{iToffoli} = \frac{Tr(U_{expt} U_{ideal})}{8} = 0.96$



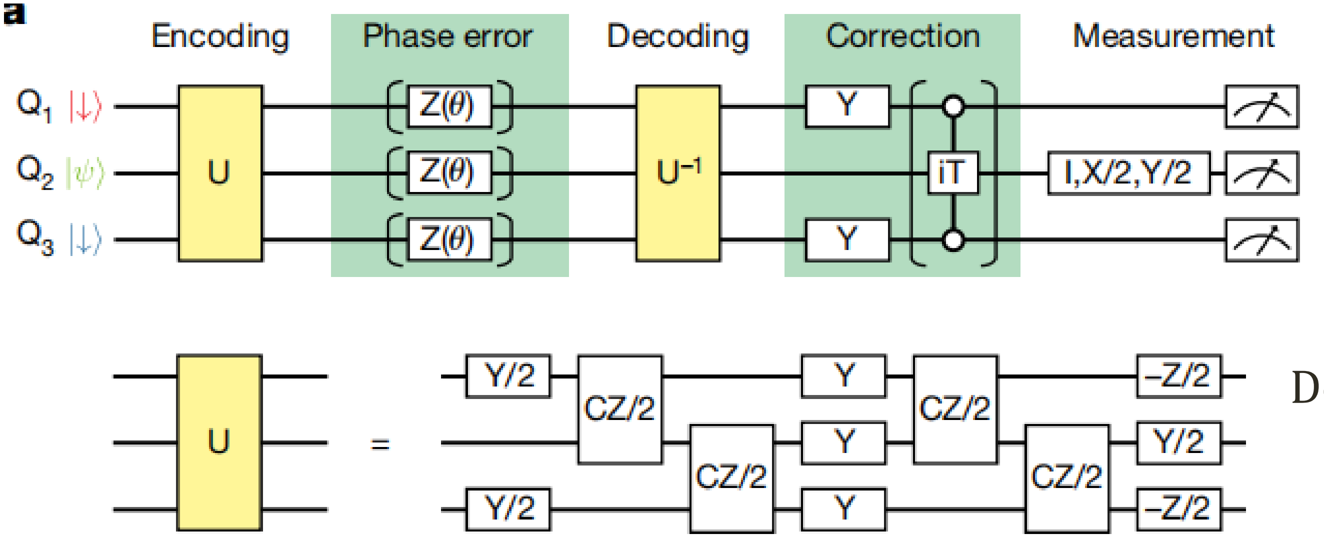
- $J_{12} = J_{23} = 0$, four transition associated with the Q_2 rotation are degenerated with a resonance frequency of f_0
- Resonance peaks of Q_2 for four different control qubit states at the exchange couplings $J_{12} = J_{23} = 4.5\ MHz$





One qubit error correction

a



U gate

$|{\downarrow\downarrow\downarrow}\rangle \rightarrow |{+}{+}{+}\rangle$

$|{\uparrow\uparrow\uparrow}\rangle \rightarrow |{-}{-}{-}\rangle$

$$Q_1 = |{\downarrow}\rangle, Q_2 = \alpha|{\downarrow}\rangle + \beta|{\uparrow}\rangle, Q_3 = |{\downarrow}\rangle$$

↓

Encode: $\alpha|{+}{+}{+}\rangle + \beta|{-}{-}{-}\rangle$

↓

Phase error with a flip rate of p on Q_2

$$p = \sin^2(\frac{\theta}{2})$$

↓

Decode: $\sqrt{1-p}|{\downarrow}\rangle(\alpha|{\downarrow}\rangle + \beta|{\uparrow}\rangle)|{\downarrow}\rangle + \sqrt{p}|{\uparrow}\rangle(\beta|{\downarrow}\rangle + \alpha|{\uparrow}\rangle)|{\uparrow}\rangle$

↓

Correction: $Q_1Q_3 = |{\downarrow\downarrow}\rangle$, flip Q_2

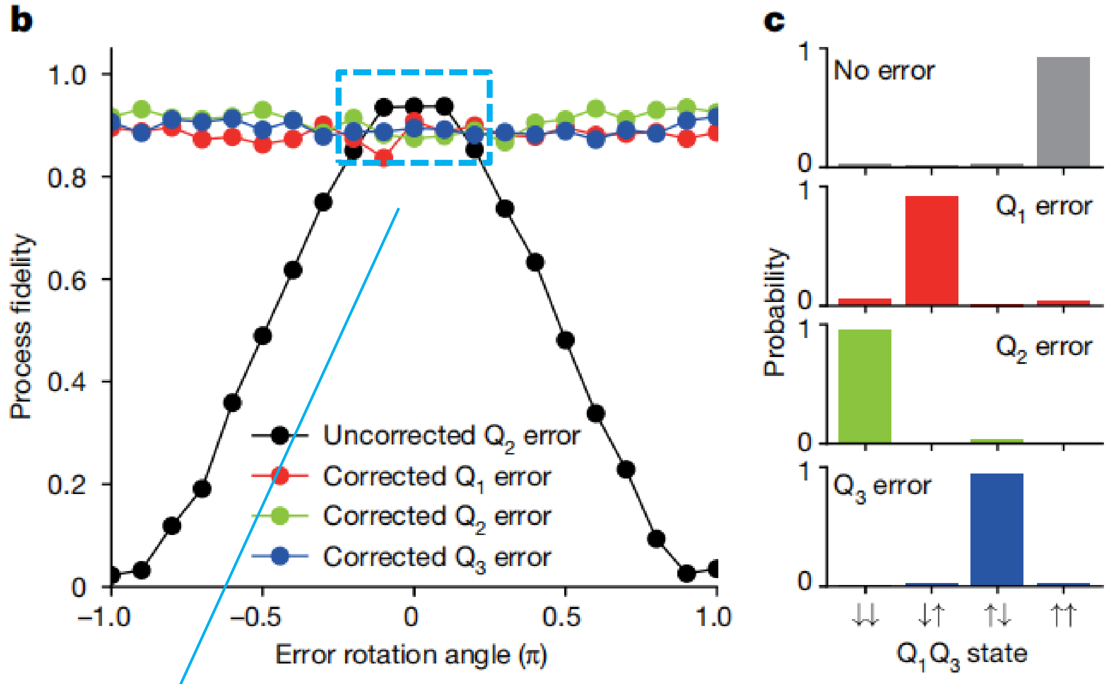
↓

Product state:

$Q_2 = \alpha|{\downarrow}\rangle + \beta|{\uparrow}\rangle$ and $Q_1Q_3 = \sqrt{1-p}|{\uparrow\uparrow}\rangle + i\sqrt{p}|{\downarrow\downarrow}\rangle$

Q_2 is the same with input regardless of p

One qubit error correction

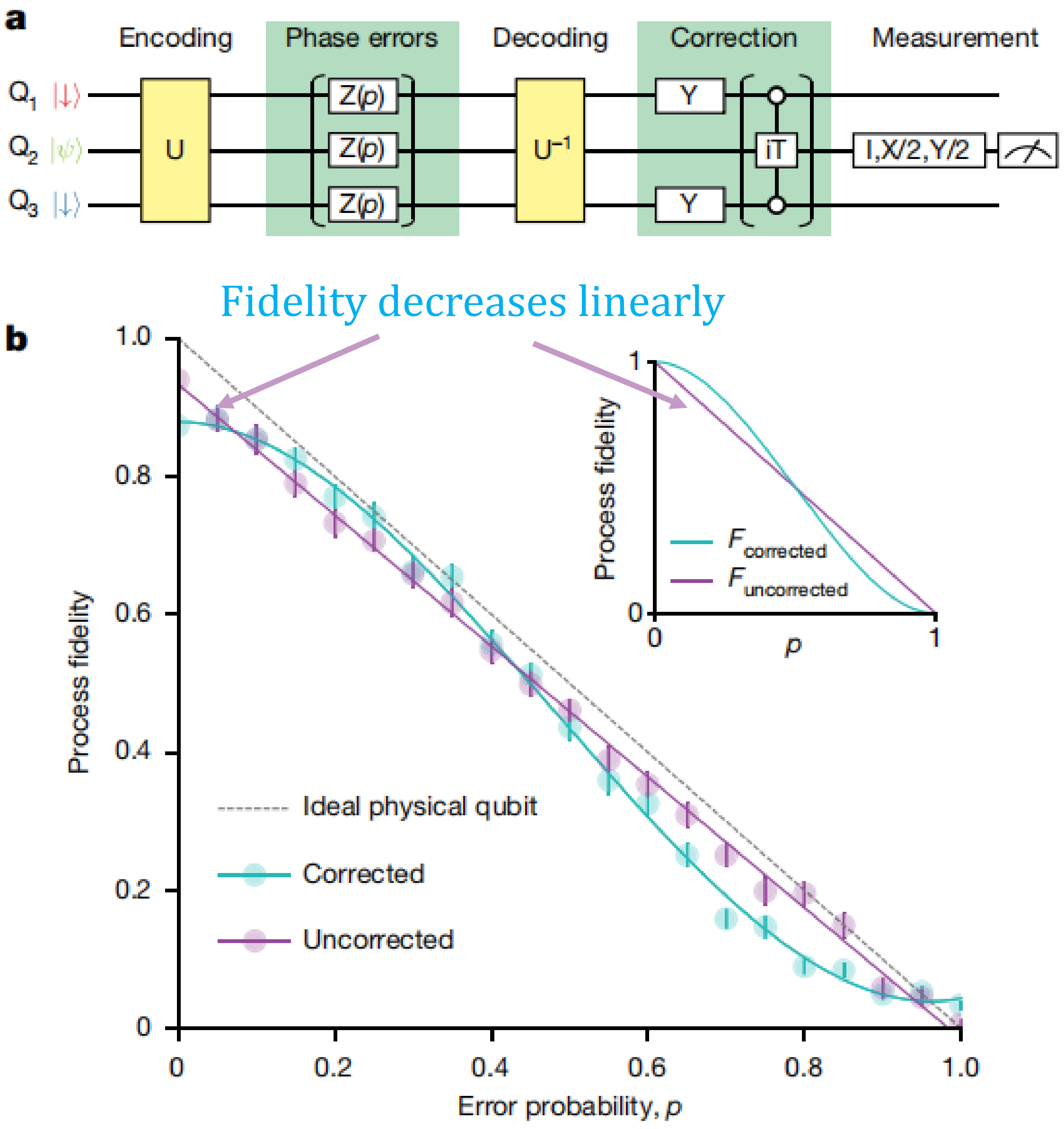


Infidelity of iToffoli gate projected to the data qubit subspace

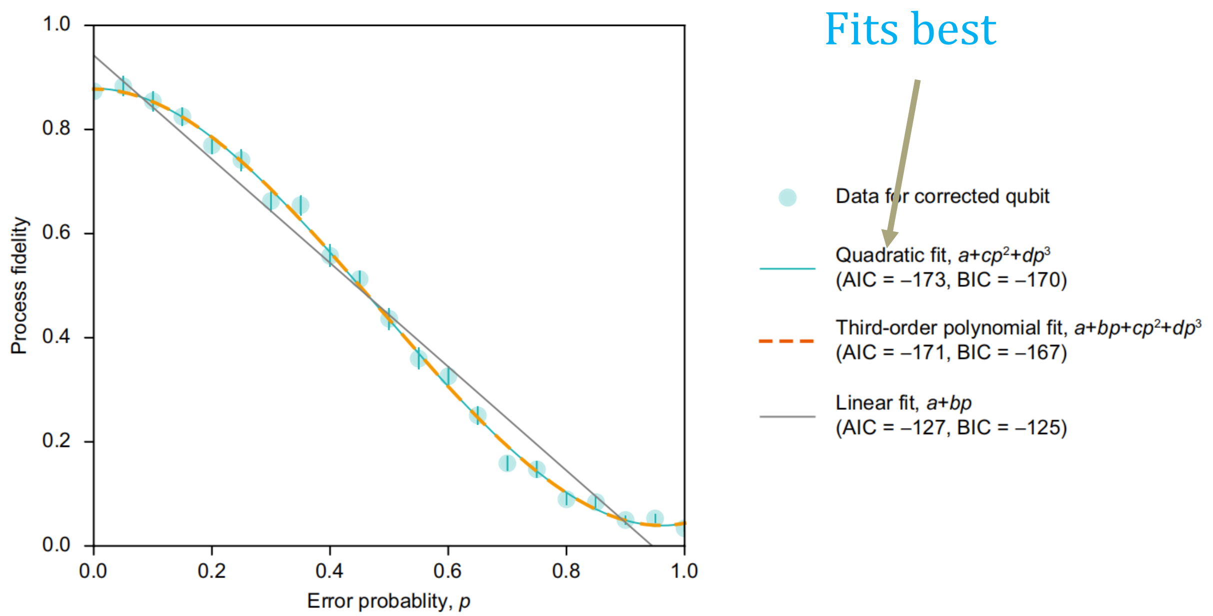
- One-qubit error: use a phase rotation with a known rotation angle θ to simulate the phase error
- Phase-flip error with $p = \sin^2(\frac{\theta}{2})$
- The ancilla states shows where the error happens

	Q_1 error	Q_2 error	Q_3 error
Encoded		$\alpha\lvert +++\rangle + \beta\lvert ---\rangle$	
Error	$\alpha(\sqrt{1-p}\lvert +++\rangle + \sqrt{p}\lvert -++\rangle)$ $+\beta(\sqrt{1-p}\lvert ---\rangle + \sqrt{p}\lvert +--\rangle)$	$\alpha(\sqrt{1-p}\lvert +++\rangle + \sqrt{p}\lvert +-+\rangle)$ $+\beta(\sqrt{1-p}\lvert ---\rangle + \sqrt{p}\lvert -+-\rangle)$	$\alpha(\sqrt{1-p}\lvert +++\rangle + \sqrt{p}\lvert ++-\rangle)$ $+\beta(\sqrt{1-p}\lvert ---\rangle + \sqrt{p}\lvert --+\rangle)$
Decoded ($Q_2Q_1Q_3$)	$(\alpha\lvert\downarrow\rangle + \beta\lvert\uparrow\rangle)(\sqrt{1-p}\lvert\downarrow\downarrow\rangle + \sqrt{p}\lvert\uparrow\downarrow\rangle)$	$\sqrt{1-p}(\alpha\lvert\downarrow\rangle + \beta\lvert\uparrow\rangle)\lvert\downarrow\downarrow\rangle$ $+\sqrt{p}(\beta\lvert\downarrow\rangle + \alpha\lvert\uparrow\rangle)\lvert\uparrow\uparrow\rangle$	$(\alpha\lvert\downarrow\rangle + \beta\lvert\uparrow\rangle)(\sqrt{1-p}\lvert\downarrow\downarrow\rangle + \sqrt{p}\lvert\downarrow\uparrow\rangle)$
Corrected ($Q_2Q_1Q_3$)	$(\alpha\lvert\downarrow\rangle + \beta\lvert\uparrow\rangle)(\sqrt{1-p}\lvert\uparrow\uparrow\rangle + \sqrt{p}\lvert\downarrow\uparrow\rangle)$	$(\alpha\lvert\downarrow\rangle + \beta\lvert\uparrow\rangle)(\sqrt{1-p}\lvert\uparrow\uparrow\rangle + i\sqrt{p}\lvert\downarrow\downarrow\rangle)$	$(\alpha\lvert\downarrow\rangle + \beta\lvert\uparrow\rangle)(\sqrt{1-p}\lvert\uparrow\uparrow\rangle + \sqrt{p}\lvert\uparrow\downarrow\rangle)$
Error syndrome	$\lvert\downarrow\uparrow\rangle$	$\lvert\downarrow\downarrow\rangle$	$\lvert\uparrow\downarrow\rangle$[14]

Three-qubit phase error correction

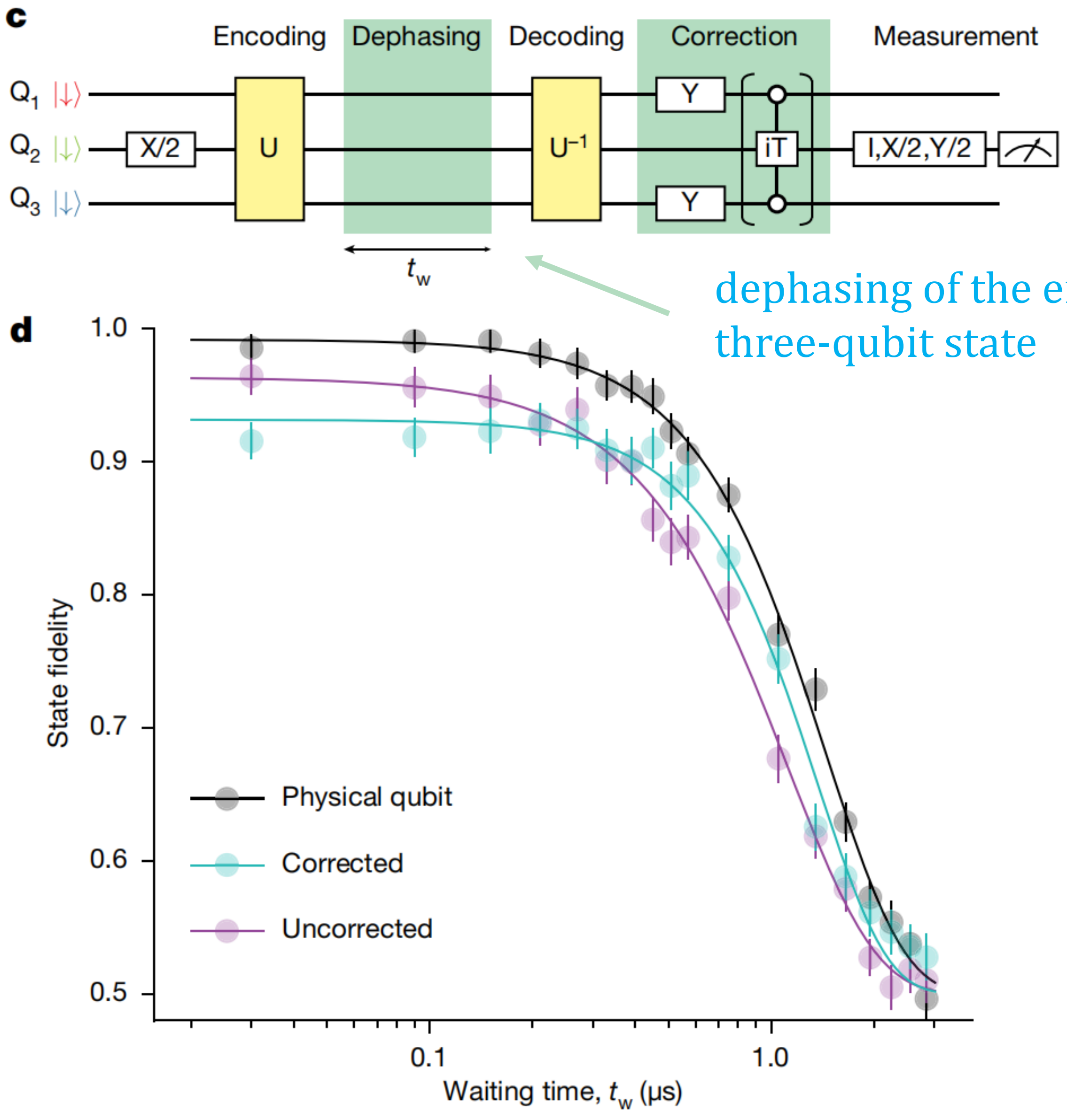


- ❖ Three-qubit error: use a phase rotation with a known rotation angle θ to simulate the phase error in all three qubits
- ❖ Same effective error rate on all qubits: $p = \sin^2(\frac{\theta}{2})$
- ❖ Without the correction: fidelity linearly decreases as p increases.
- ❖ Quadratic fitting: $F(p) = 0.897 - 2.72p^2 + 1.89p^3$



The correction ideally results in an improvement of the fidelity for $p < 0.429 \pm 0.003$.

Three-qubit phase error correction: Ramsey



dephasing of the encoded three-qubit state

Nature Si: nuclei spin leads to phase shift in spin.

$$Q_1 = |\downarrow\rangle, Q_2 = \frac{1}{\sqrt{2}}(|\downarrow\rangle + |\uparrow\rangle), Q_3 = |\downarrow\rangle$$

↓

Encode: $\frac{1}{\sqrt{2}}(|+++\rangle + |---\rangle)$

↓

Dephasing on equatorial plane t_w

↓

Decode

↓

Correction: $Q_1 Q_3 = |\downarrow\downarrow\rangle$, flip Q_2

↓

Decay function $F(t) = \frac{1}{2}\left(1 + \alpha \exp\left(-\frac{t}{T_2}\right)^n\right)$

For physical qubit, corrected qubit and uncorrected qubit:

$\alpha = 0.492 \pm 0.005, 0.432 \pm 0.008, 0.464 \pm 0.007$

$T_2 = 1.44 \pm 0.02, 1.36 \pm 0.04, 1.12 \pm 0.03\ \mu s$

$n = 1.90 \pm 0.08, 2.1 \pm 0.2, 1.68 \pm 0.08$

Conclusion

Conclusion

- Generation of the various three-qubit entangled states
- Demonstration of the effective single-step resonantly driven iToffoli gate
- Demonstration of the fundamental properties of three-qubit QEC in silicon

Outlook

- Slow spin measurement and initialization by energy-selective tunnelling can be improved by switching to the singlet-triplet readout ($\sim \mu s$)