"Everything I know about coherence & noise in a nutshell"

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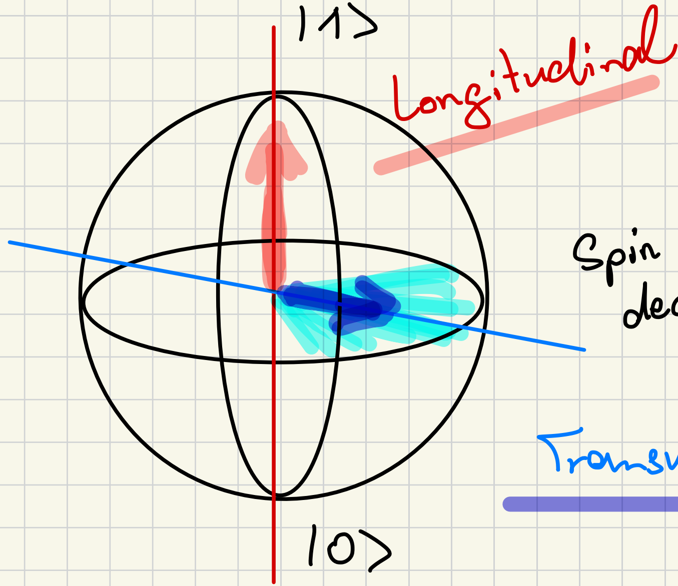


Spin decay

Transversel

generally: $\left(\Gamma := \frac{1}{T}\right)$

$$\Gamma_2 = \frac{1}{2}\Gamma_1 + \Gamma_\varphi$$

Ithier *et al.*, PRB **72**, 2005

particularly Refs. 32,33 on ***Bloch-Redfield Theory.***

$$|\psi\rangle = \alpha|1\rangle + \beta|0\rangle \; ; \quad \alpha = \alpha_0 e^{-i\cancelto{0}{\varphi_1}} , \quad \beta = \beta_0 e^{-i\cancelto{\varphi}{\varphi_2}}$$

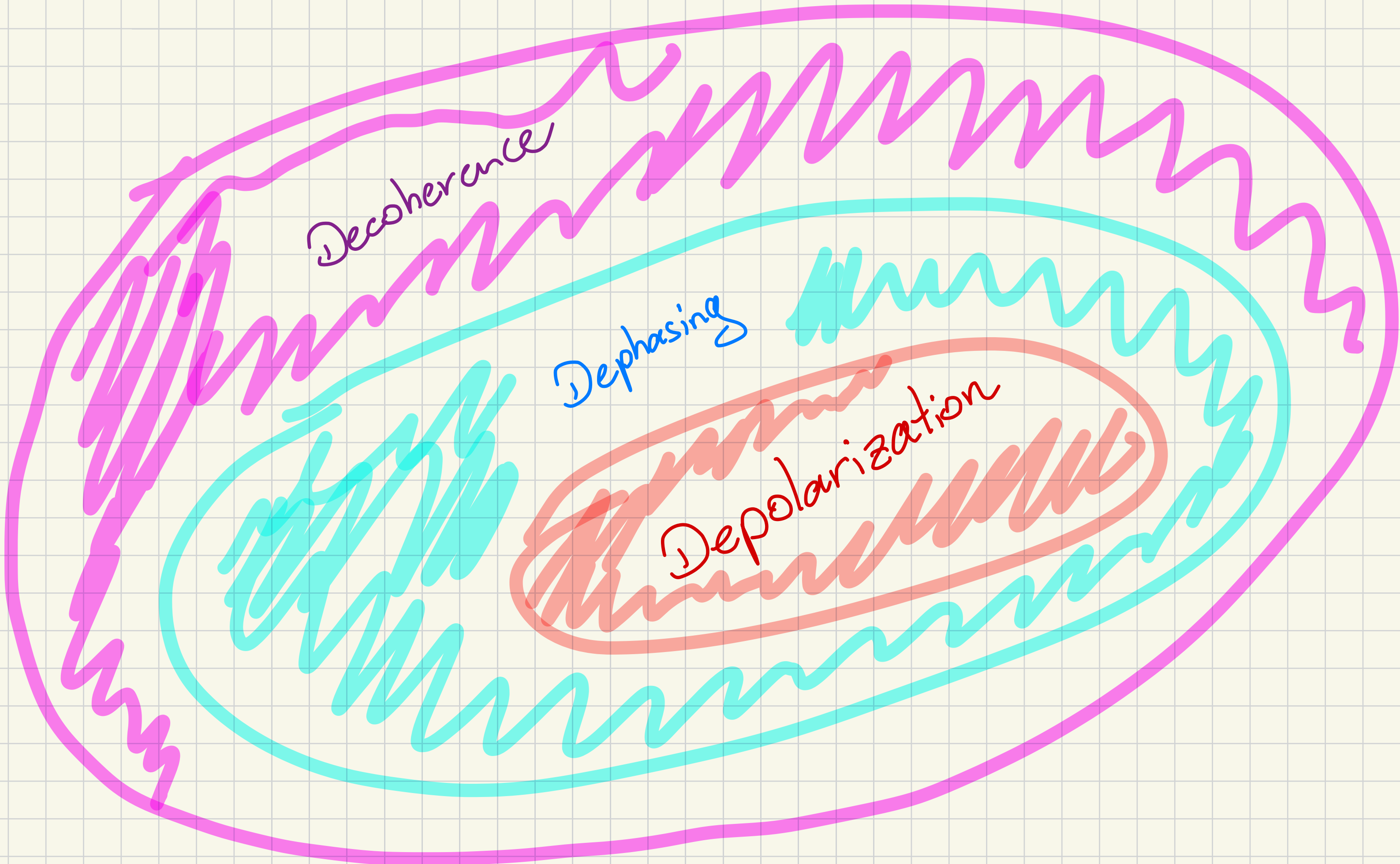
$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} = \begin{pmatrix} |\alpha_0|^2 & \alpha_0\beta_0 e^{-i\varphi} \\ \alpha_0\beta_0 e^{+i\varphi} & |\beta_0|^2 \end{pmatrix}$$

Density Matrix

Most systems, specially e^- & h^+

$T_2 \ll T_1$

$$\boxed{T_2 \leq 2\,T_1}$$

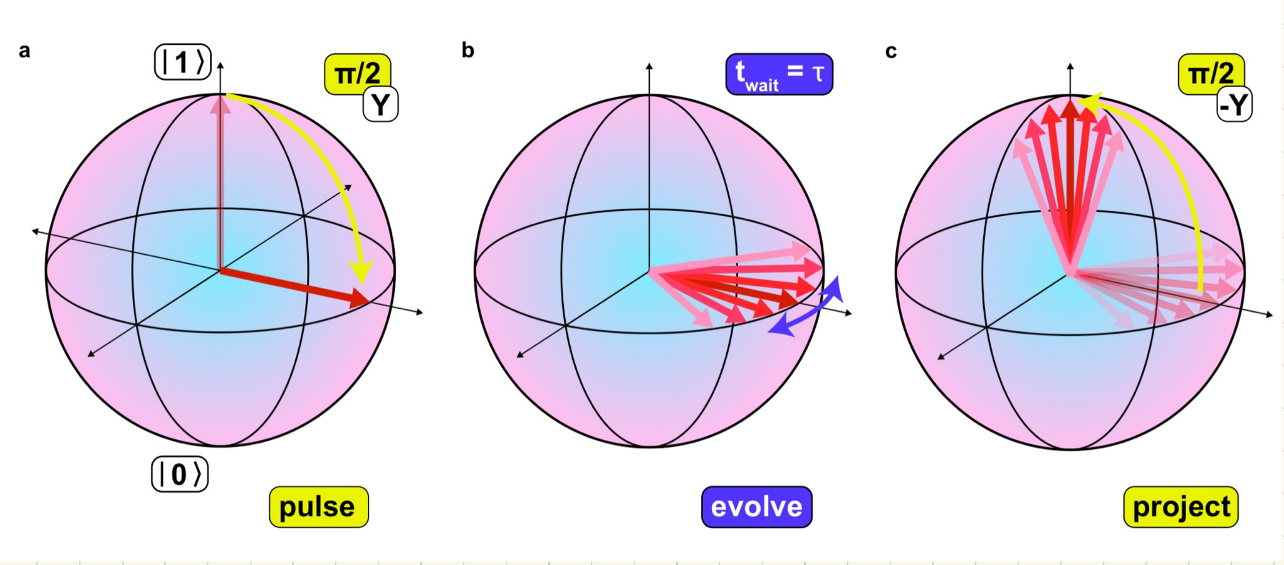

Decoherence
Dephasing
Depolarization

Since for us: $\boxed{T_2 \ll T_1}$ we focus on T_2

Check by performing a Ramsey experiment:

We check probability to get/stay in e.g. $|+\rangle$ state.

Def: $T_2^{\text{RAMSEY}} := T_2^*$



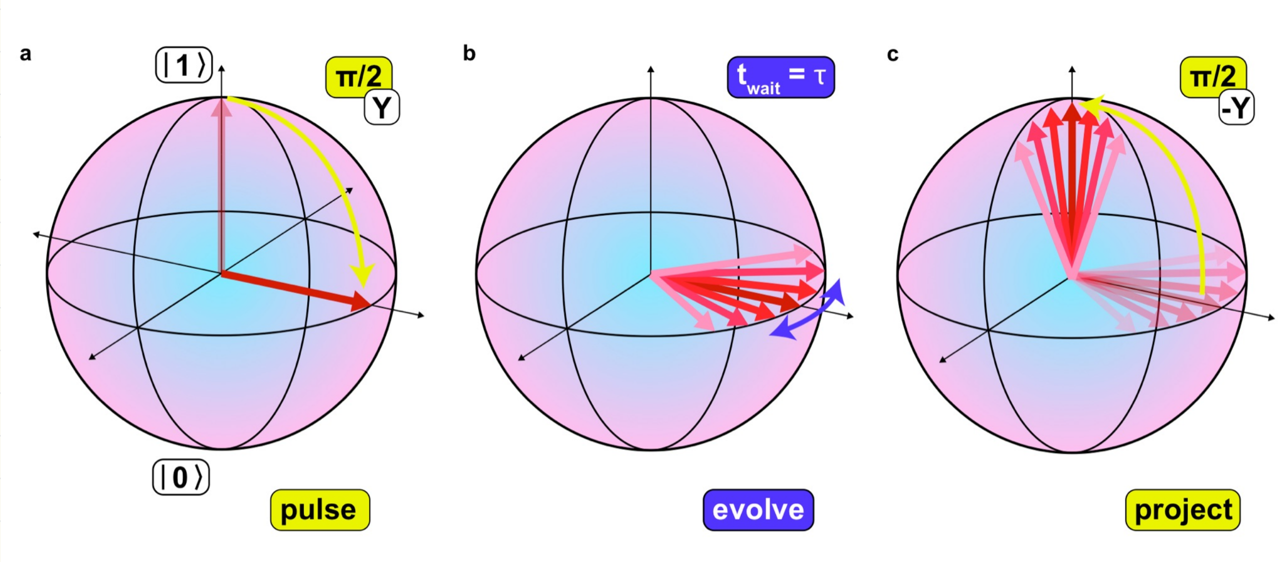
It's an EXPERIMENT! The „T_2“ is an aggregate effect of the „problems" we can DO something about (quasi-static) & the „problems" we CAN NOT tackle (they happen too fast).

(Think of the rate of decay, $\Gamma_2 = \frac{1}{T_2}$)

$$\frac{1}{T_2^*} = \boxed{\frac{1}{T_2}} + \boxed{\frac{1}{T_2'}}$$

effective or pure transverse spin dephasing

From e.g. static inhomogenieties such as static B-field inhomog. in solids.



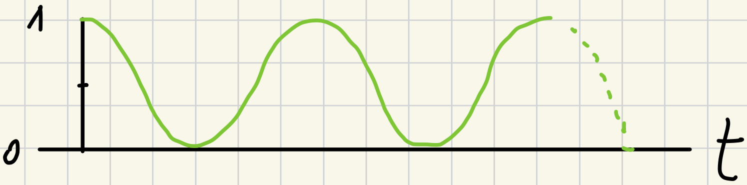
No noise: $H = \frac{\Delta}{2}\sigma_z$, $H|0\rangle = -\frac{\Delta}{2}|0\rangle$, $H|1\rangle = +\frac{\Delta}{2}|1\rangle$

use $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$; $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$

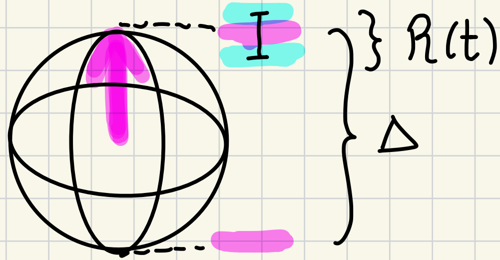
Time evolution: $|\psi(t)\rangle = e^{-iH/\hbar}|+\rangle = \alpha(t)|+\rangle + \beta(t)|-\rangle$

& calculate $|\langle + | \psi(t)\rangle|^2 = |\alpha(t)|^2$

$$|\alpha(t)|^2 =: P_+ = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\Delta\cdot t}{\hbar}\right)$$



Pure dephasing Hamiltonian:



$H = \frac{\Delta + R(t)}{2}$, skipping lots of math:

$$P_+ = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\Delta\cdot t}{\hbar} + \delta\phi(t)\right)$$

$$\& \quad \delta\phi(t) = \frac{1}{\hbar}\int_0^t dt'\, R(t')$$

here $R(t)$ is single realization of noise

→ need ensemble avg. of $\delta\phi(t)$

$$P_+ = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\Delta\cdot t}{\hbar} + \delta\phi(t)\right)$$

→ one acceptable assumption: $\delta\phi(t)$ is normal distributed.

Gaussian Noise Approximation:

$$p_{\delta\phi} = \frac{1}{\sqrt{2\pi\sigma_{\delta\phi}}}\exp\left(-\frac{\delta\phi^2}{2\sigma_{\delta\phi}}\right)$$

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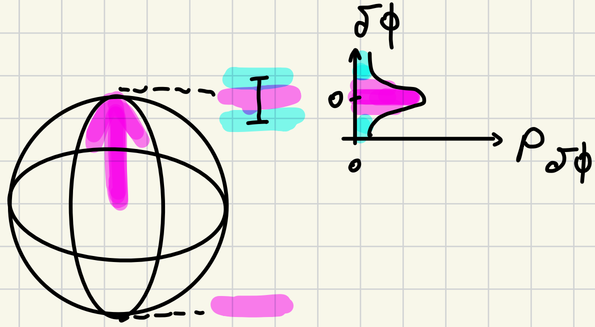


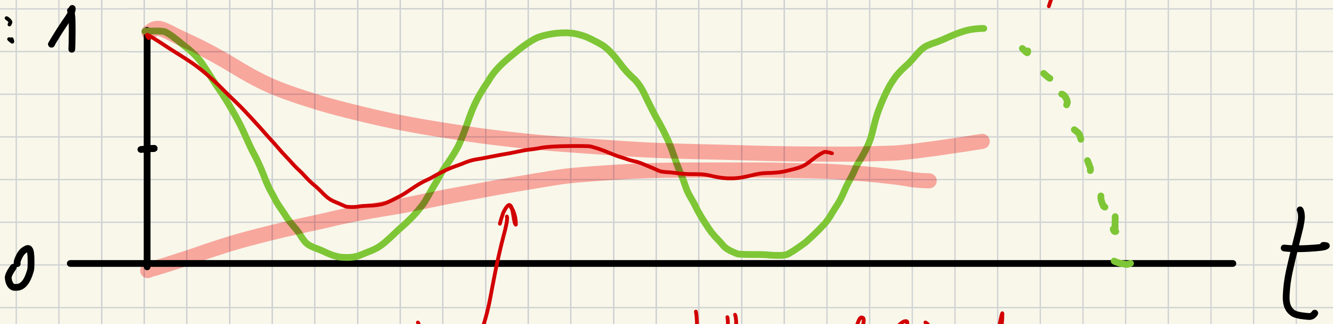
Note:

$$p_{\delta\phi} = p\left(\delta\phi(t) = \delta\phi,\ \sigma_{\delta\phi}(t) = \sigma_{\delta\phi},\ t\right)$$

$$\langle P_+\rangle = \int_{-\infty}^{\infty} d\delta\phi\; p_{\delta\phi}\, P_+ = \frac{1}{2} + \frac{1}{2}\, e^{-\sigma_{\delta\phi}/2}\cos\left(\frac{\Delta\cdot t}{\hbar}\right)$$

Before: Now with $\exp(-\sigma_{\delta\phi}/2)$ decay

Example:



But! Decay still undefined.

USE DEF of VARIANCE $\sigma_{\delta\phi} = \langle \delta\phi^2 \rangle$,

RECALL

$$\delta\phi(t) = \frac{1}{\hbar}\int_0^t dt'\, R(t')$$

$$\sigma_{\delta\phi} = \frac{1}{\hbar^2}\left\langle \int_0^t dt'\, R(t') \int_0^t dt''\, R(t'') \right\rangle = \frac{1}{\hbar^2}\int_0^t dt' \int_0^t dt'' \left\langle R(t')R(t'') \right\rangle$$

Auto korrelator of the noise. I.e.

Markovian : NO MEMORY

Non-Markovian: DEPENDS ON PAST

$$\left\langle R(t')R(t'') \right\rangle =: G_{RR}(t'-t'') = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega(t'-t'')} S_{RR}(\omega)$$

Fourier Theorem

$$G_{RR}(t) = \left\langle R(t)\,R(0) \right\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\, S_{RR}(\omega)$$

$$\sigma_{\delta\phi} = \frac{1}{2\pi\hbar^2}\int_{-\infty}^{\infty} d\omega\, S_{RR}(\omega)\, \frac{4\sin^2(\omega t/2)}{\omega^2} \qquad F(\omega t)$$

$F(\omega t)$ is measurable & specific to the sequence.
Here we considered a free evolution of the spin in the equator, looking at $|\langle + | \psi(t) \rangle|^2$

$$\langle P_+ \rangle = \int_{-\infty}^{\infty} d\delta\phi\; p_{\delta\phi}\, P_+ = \frac{1}{2} + \frac{1}{2} e^{-\sigma_{\delta\phi}/2} \cos\left(\frac{\Delta \cdot t}{\hbar}\right)$$

Now specified decay as integral over spectrum S_{RR} & filter/experiment $F(\omega t)$

Same filter of before, i.e. Ramsey, but different spectra:

<u>Quasi static:</u> $S_{RR}(\omega) = \frac{2\pi\hbar^2}{(T_2^*)^2}\delta(\omega)$ const in t-domaine

$$\langle P_+ \rangle = \frac{1}{2} + \frac{1}{2} e^{-\frac{t^2}{2(T_2^*)^2}} \cos\left(\frac{\Delta t}{\hbar}\right)$$

Pink noise : $S_{RR}(\omega) = \hbar^2 / (T_2^*)^2 |\omega|$

Different TLFs contribute to $R(t)$, where probability of finding TLF with switch rate γ is $P(\gamma) \propto 1/\gamma$

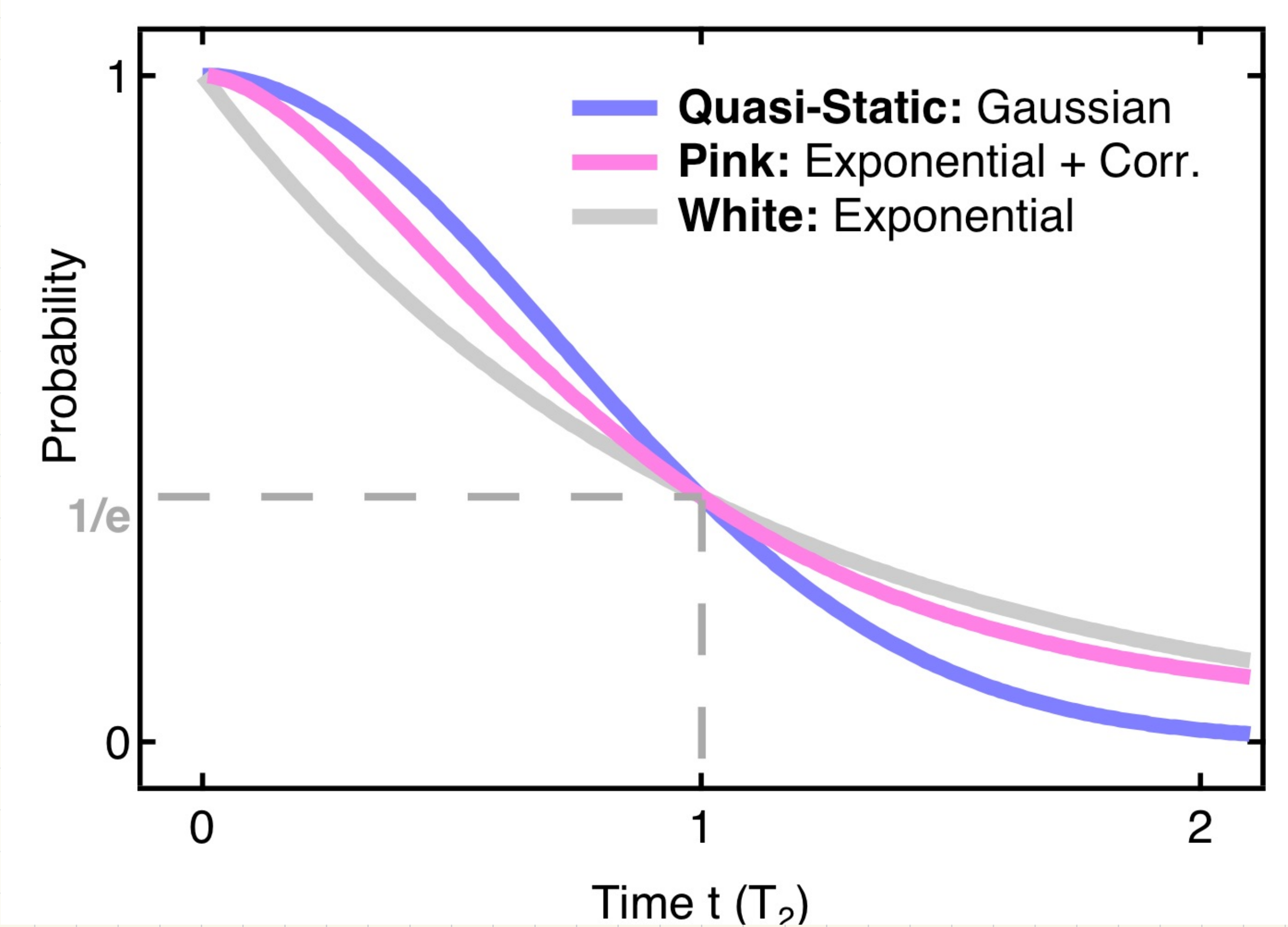
$$\langle P_+ \rangle = \frac{1}{2} + \frac{1}{2} e^{-\frac{t^2}{2\pi (T_2^*)^2} \ln(\frac{1}{\omega_{co} t})} \cos(\frac{\Delta t}{\hbar})$$

cut-off, can't go to arbitrarily low ω.

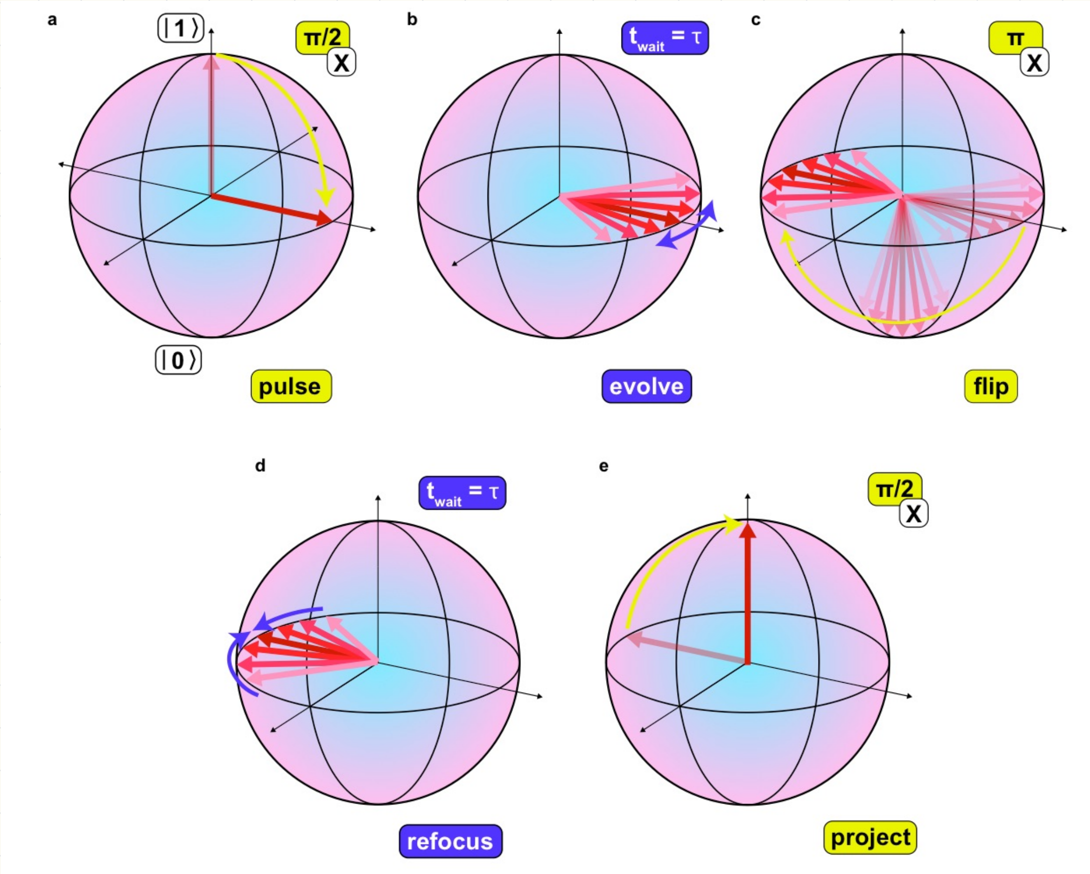
White noise : $|t' - t''| >>$ correlation time, the correlation between $R(t')$ & $R(t'')$ becomes negligible.

$\rightarrow$ MARKOVIAN $\langle R(t') R(t'') \rangle \propto \delta(t) \Rightarrow S_{RR}(\omega) = \frac{\hbar^2}{T_2^*}$

$$\langle P_+ \rangle = \frac{1}{2} + \frac{1}{2} e^{-\frac{t}{2T_2^*}} \cos\left(\frac{\Delta t}{\hbar}\right)$$


Quasi-Static: Gaussian
Pink: Exponential + Corr.
White: Exponential
Probability
1
1/e
0
0
1
2
Time t (T_2)

As in politics, it's sometimes easier to turn a blind eye to your „noise" & remain in your „echo"-chamber.

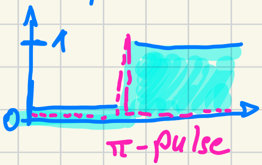


$$\delta\phi(t) = \frac{1}{\hbar}\int_0^t dt' R(t') \underbrace{f_n(t;t')}_{\text{before} = 1}$$

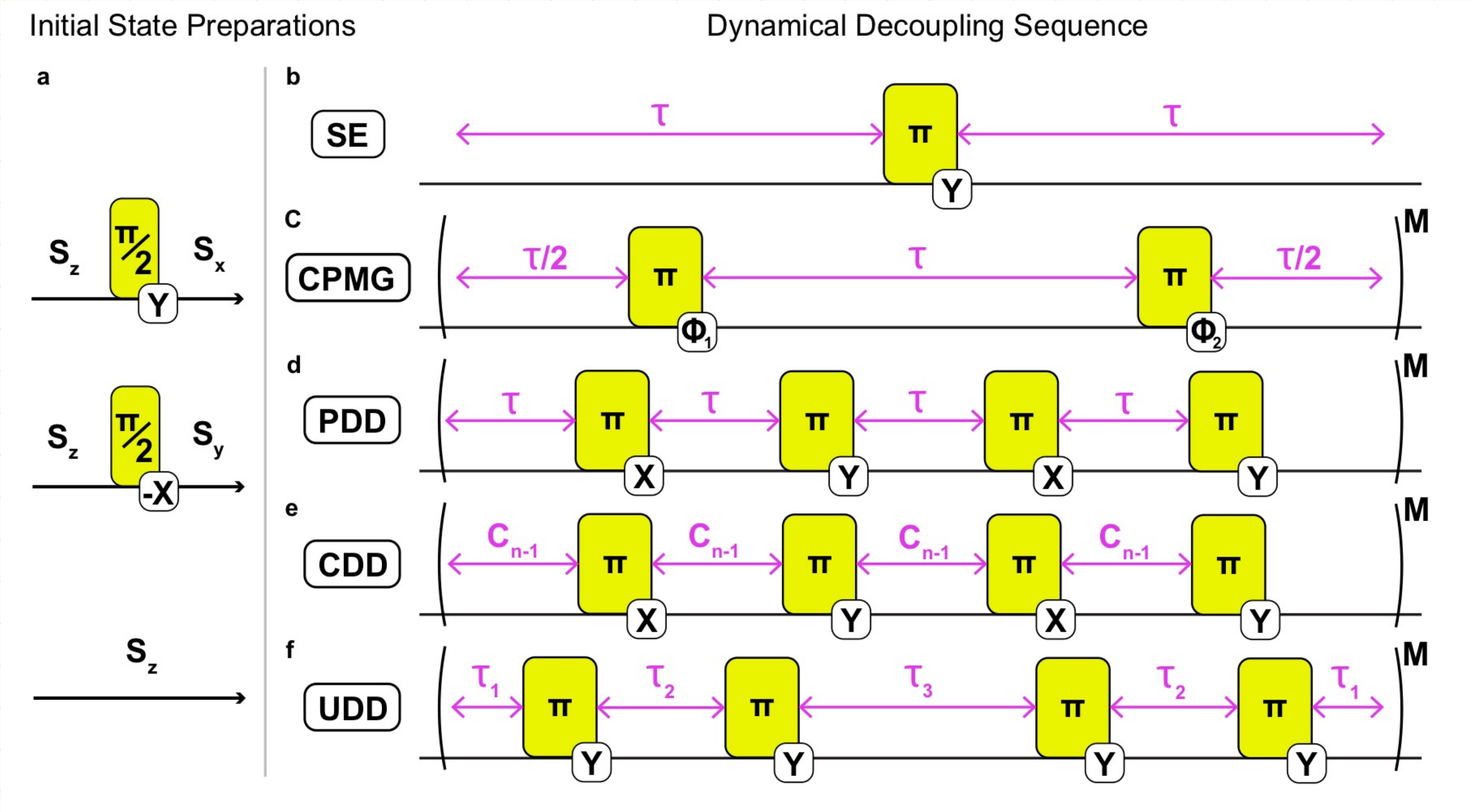
$$f_n(t;t') = \sum_{k=0}^{n} (-1)^k \Theta(t_{k+1}-t')\underbrace{\Theta(t'-t_k)}_{\begin{cases}0, & t'<t_k\\ 1, & t'\geq t_k\end{cases}}$$

Ramsey: $n=0$, $f_0=1$

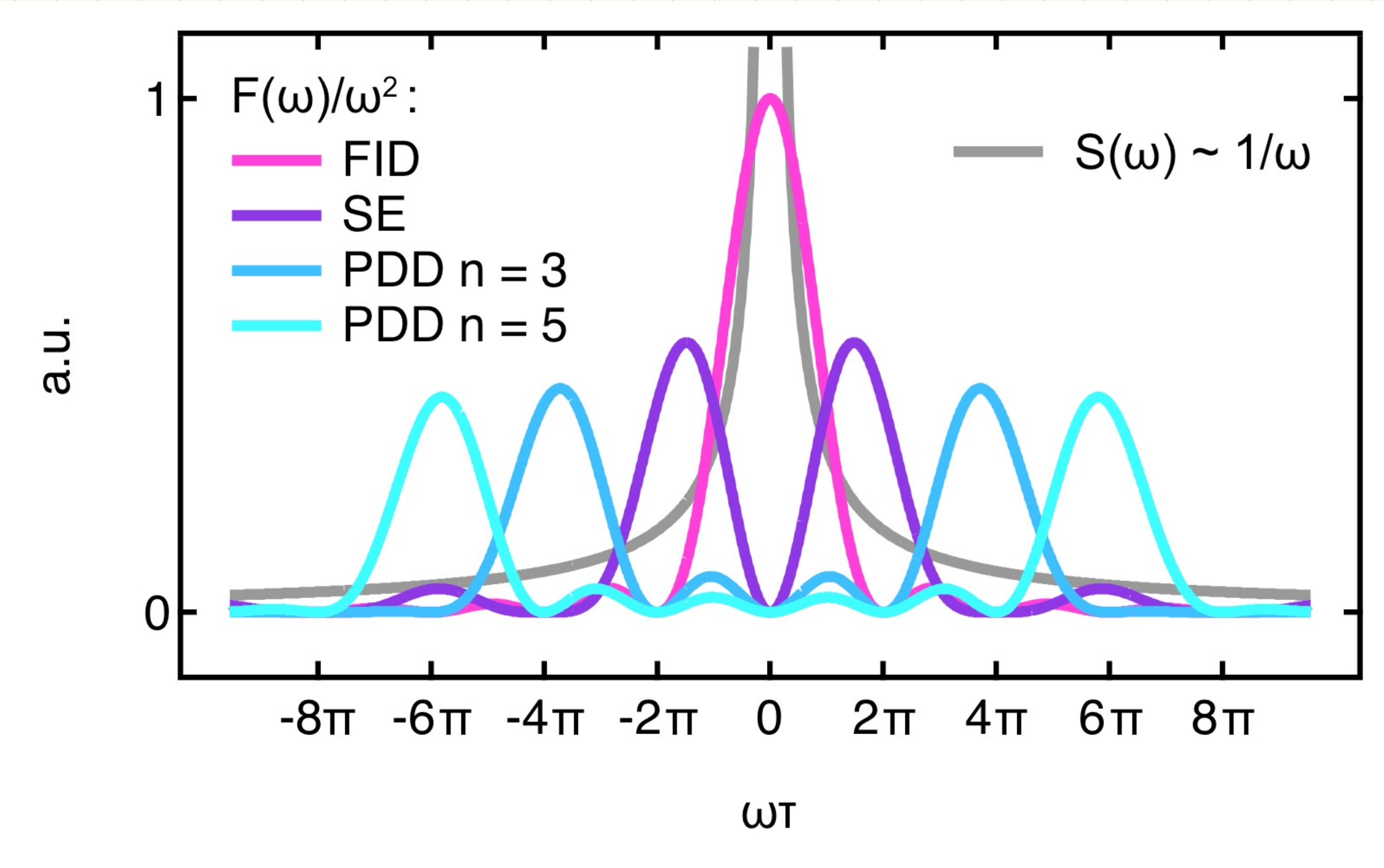
for Hahn: $n=1$, f_1:



$$\delta\phi(2\tau) = \frac{1}{\hbar}\int_0^\tau dt' R(t') - \frac{1}{\hbar}\int_\tau^{2\tau} dt' R(t')$$



Carr-Purcell-Meiboom-Gil, Periodic DD,
Concatenated DD (recursive), Uhrig DD


F(ω)/ω²:
FID
SE
PDD n = 3
PDD n = 5
S(ω) ~ 1/ω
1
0
a.u.
-8π
-6π
-4π
-2π
0
2π
4π
6π
8π
ωτ

- T_1-processes induce T_2 processes. Typically $T_2 << T_1$
- T_2 is an aggregate effect of noise you can „run away" from & parts you can't.
- The above depends on your specific pulse-sequence or experiment you perform defining $F(\omega t)$, the filter function.
- If you were able to probe S_{RR} with arbitrary accuracy, you could identify the soure of evil, like the FBI using fingerprint scans.