

# Cascaded superconducting junction refrigerators: Optimization and performance limits

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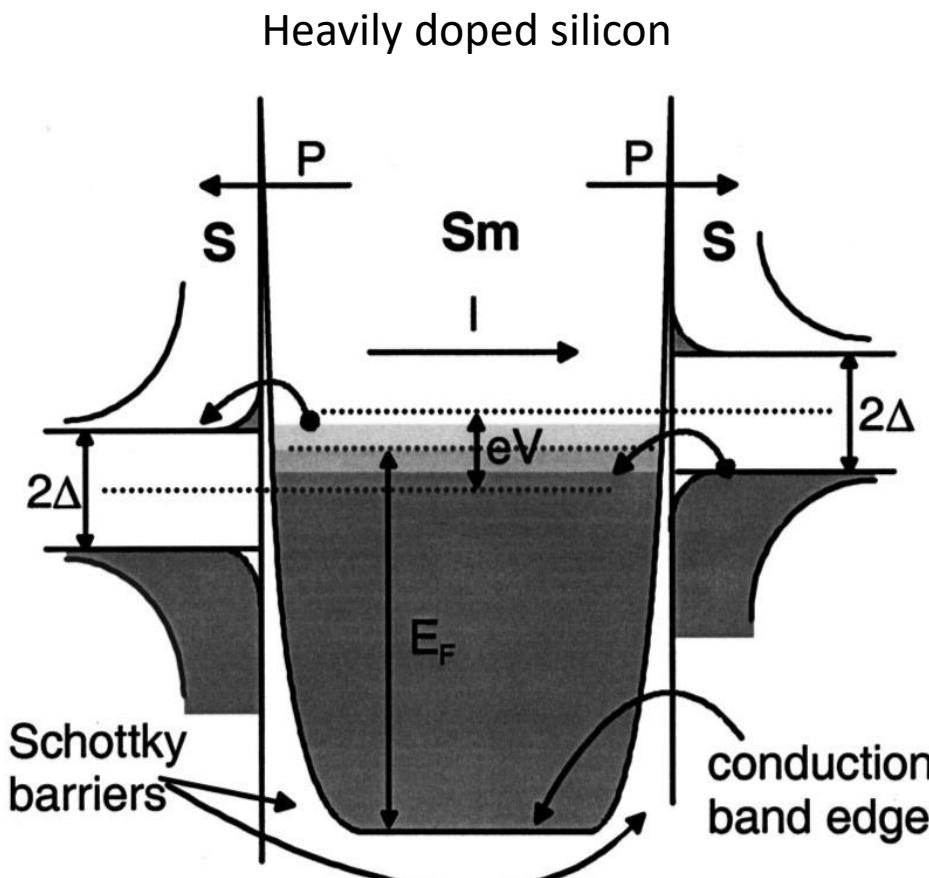
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## AFFILIATIONS

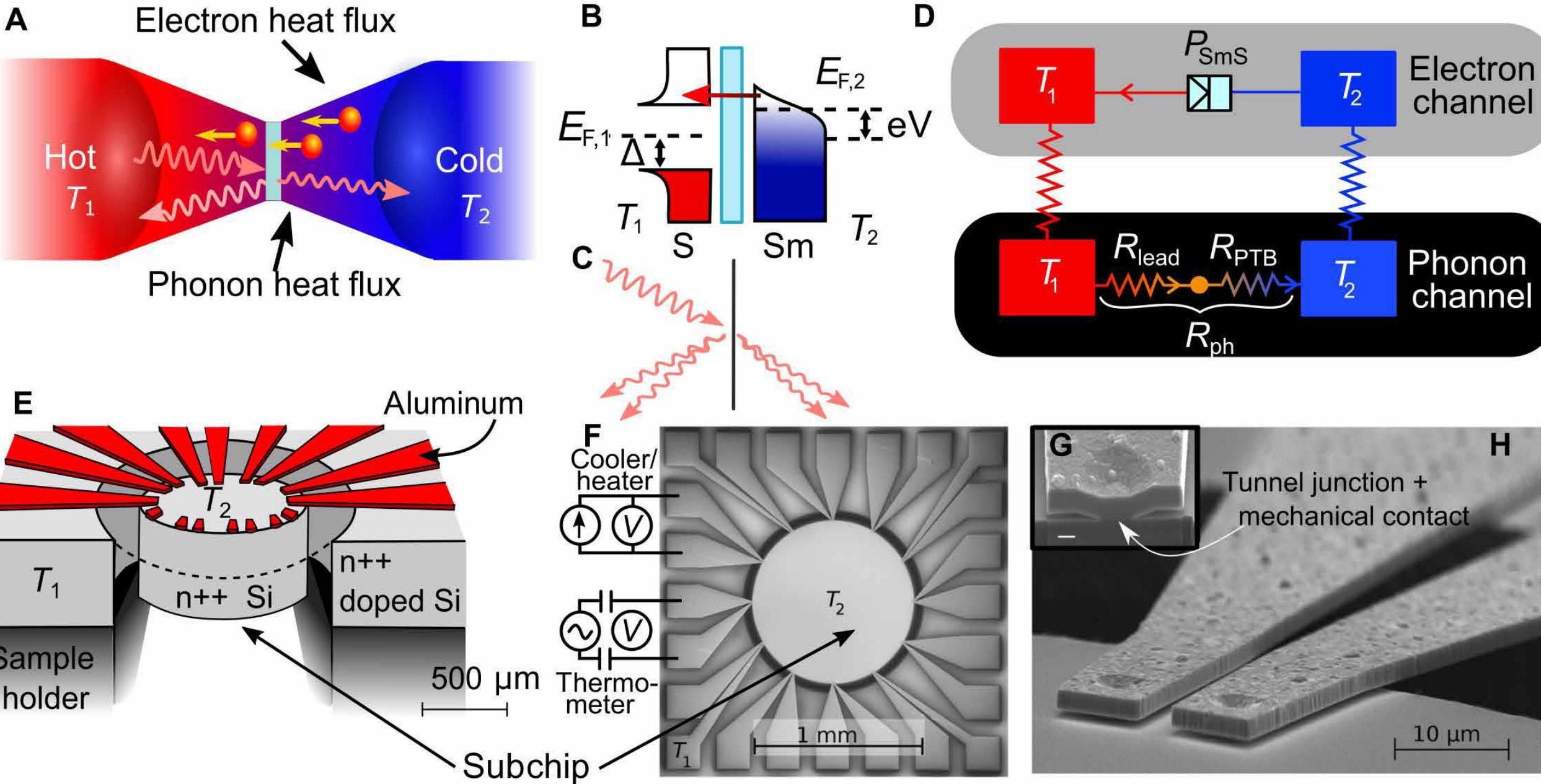
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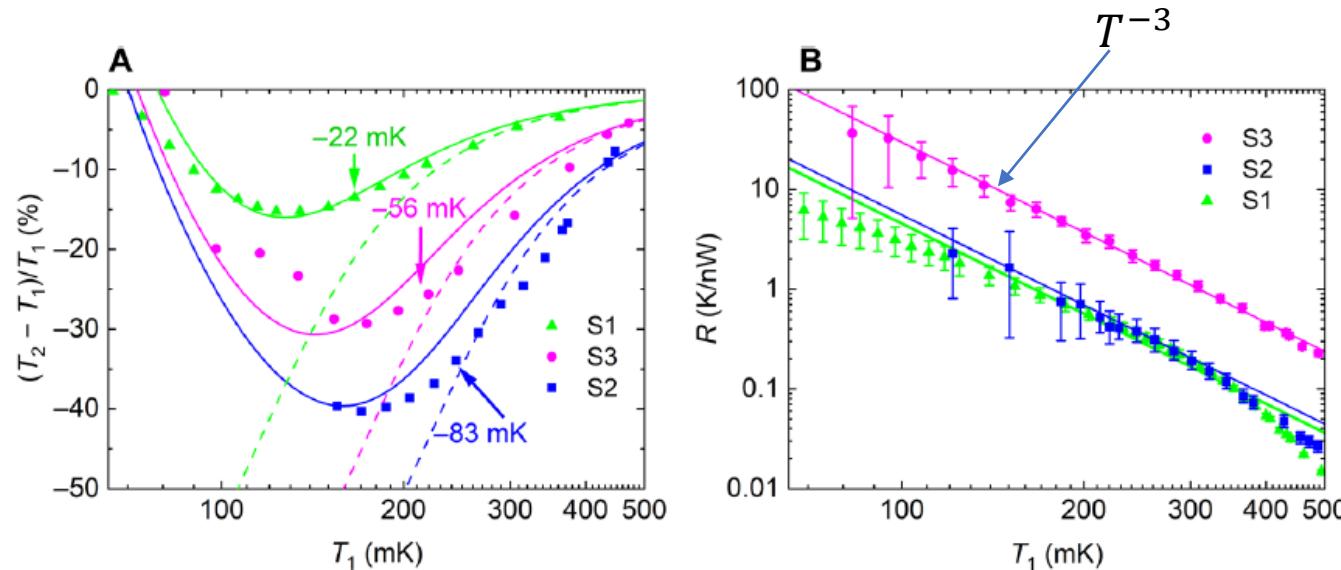
Metallurgical Schottky contact at  
S-Sm forms a tunnel barrier



**Table 1. Sample parameters and literature values for thermal boundary resistance.** Here,  $A$  is the area of a tunnel junction,  $d$  is the diameter of the subchip,  $R_A = R_T A$  is the characteristic junction resistance, and  $R_T$  is the junction resistance. Thermal resistance prefactors ( $\alpha$ ) are the values obtained from the fits of Fig. 2, whereas  $\alpha_{\text{AMM}}$  and  $\alpha_{\text{DMM}}$  correspond to theoretical  $R_{\text{PTB}}$  (35).

Sample	$A$ $\mu\text{m}^2$	$d$ $\mu\text{m}$	$R_A = R_T A$ $\Omega \mu\text{m}^2$	$\alpha$ $\text{K}^4/\mu\text{W}$	$\alpha_{\text{AMM}}$ $\text{K}^4/\mu\text{W}$	$\alpha_{\text{DMM}}$ $\text{K}^4/\mu\text{W}$	Max cooling %	Max cooling mK
S1	2.0/137*	300	538	4.5	—	—	15 (@ 140 mK)	22 (@ 166 mK)
S2	7.5	1000	476	5.6	6.5	8.8	40 (@ 170 mK)	83 (@ 244 mK)
S3	3.2	300	1945	29	16	21	29 (@ 173 mK)	56 (@ 220 mK)

\*Sample S1 had silicon oxide between leads and subchip, and therefore, the effective area of the phononic heat contact,  $137 \mu\text{m}^2$ , is larger than the junction area  $2.0 \mu\text{m}^2$ .



# Conclusions

1. Since cooling power and phonon leaks are proportional to  $A$ , one should decrease  $R_A = R_T A$
2. Suppression of phonon thermal conductivity above 500 mK requires “phonon engineering”
3. Above 500 mK, a superconductor with larger  $\Delta$  is required.  
Vandilium.

$$P_{\Sigma 2} = P_{H3} - P_{C2} + P_{L2} - P_{L3}$$

Progressively smaller area is good

$$P_{H3} = P_{C3} \propto A_3$$

Less efficient at high T

$$P_L \propto T^4$$

$$P_C \propto T^2$$

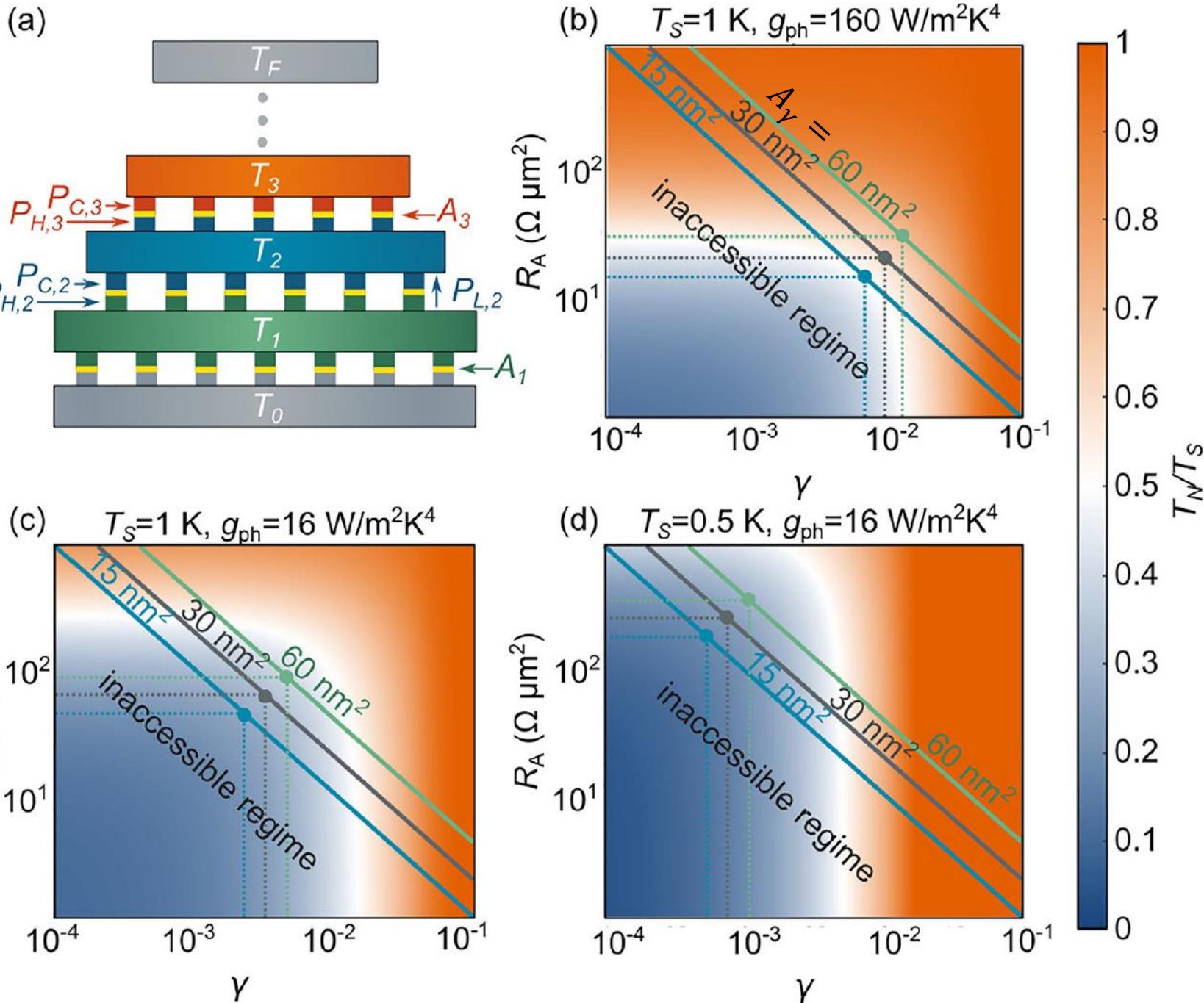
$R_0$  is sub-gap leakage resistance

$A_{ch}$  is effective tunneling area/ $G_0$

$$\gamma = \frac{R_0}{R_T} = \frac{A_{ch} R_K}{4 R_A}$$

$$P_L = g_{ph} A (T_S - T_N)^4$$

$$A_\gamma = \frac{4\gamma R_A}{R_K} \text{ phenomen. leakage area}$$



Lower  $R_A$  at the cost of high  $\gamma$  is preferred.

A good refrigerator is energy efficient too.

Sufficient cooling power:

$$\frac{T_N^2}{T_S^2} \approx \frac{P_C}{P_H}$$

Maximum heat load:  $P_{in} = P_C - P_L$

$$a = \frac{A_{n-1}}{A_n} = \frac{P_H T_N^2}{P_{in} T_S^2}$$

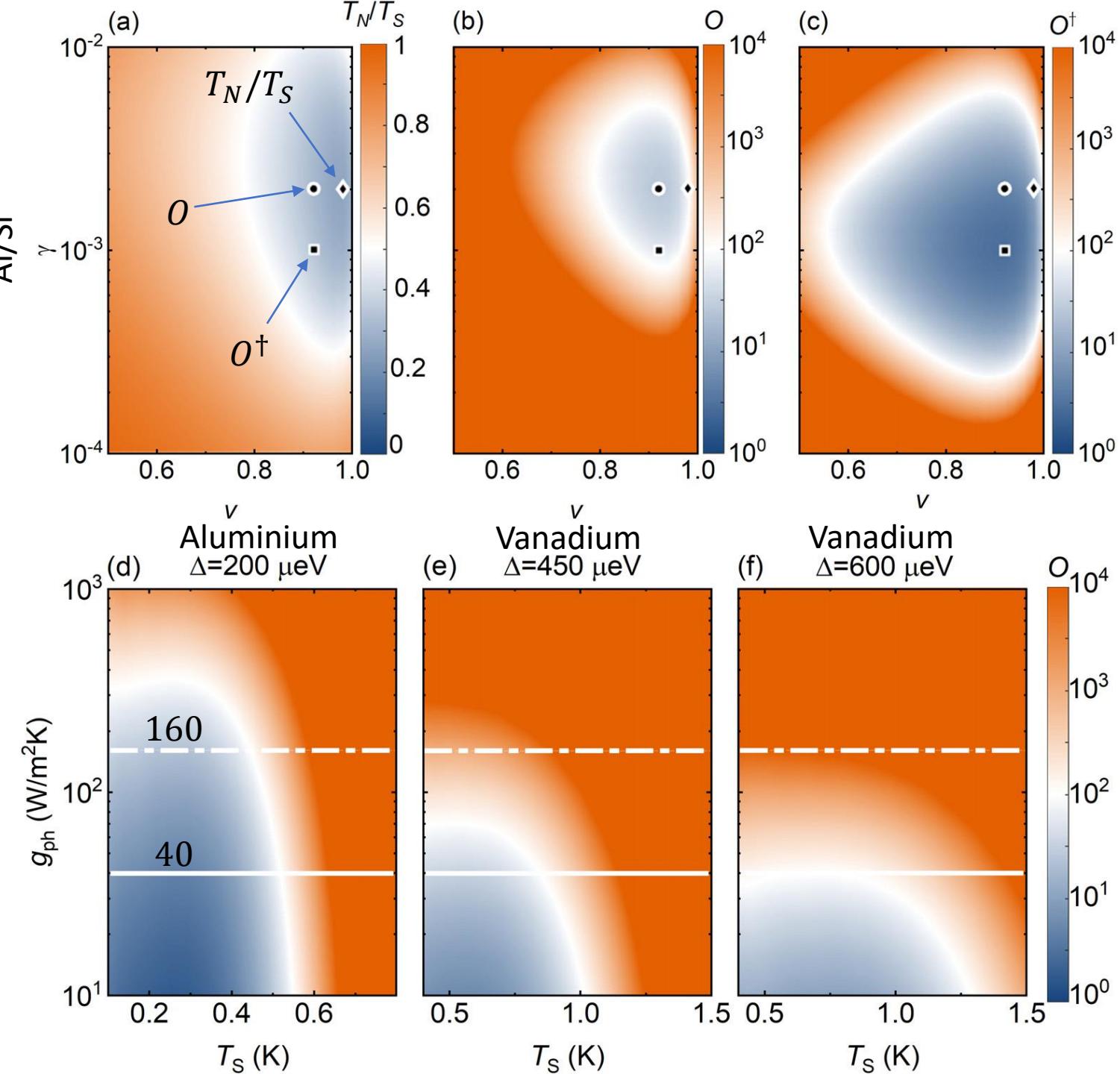
$$O = a^{1/\log(\frac{T_S}{T_N})}$$

For small phonon leak (low T or good ph-filtering):

$$a^\dagger = \frac{P_H T_N^2}{P_C T_S^2}$$

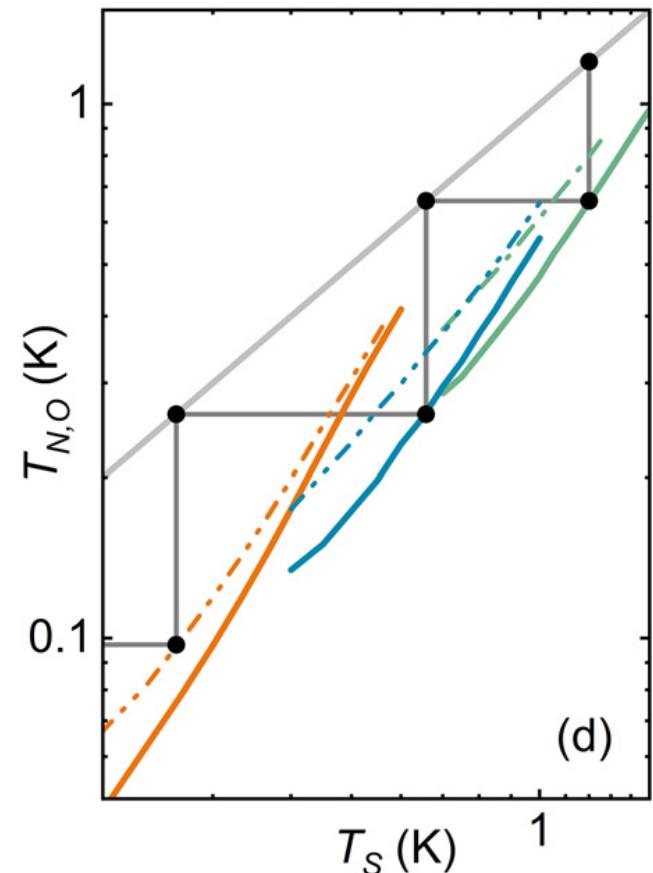
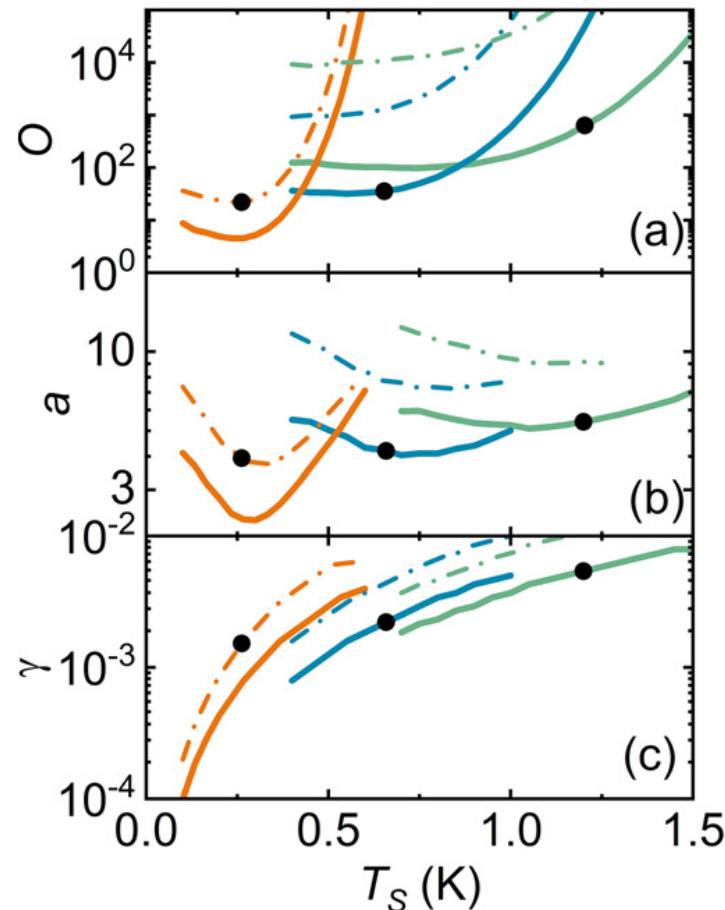
$$O^\dagger = a^{\dagger 1/\log(\frac{T_S}{T_N})}$$

Minimization parameters:  $\nu, \gamma, g_{ph}, T_S, T_N$



# The Plan

$\Delta = 200 \mu eV$   
 $\Delta = 450 \mu eV$   
 $\Delta = 600 \mu eV$



$n$	$g_{ph}$ (W/m <sup>2</sup> K <sup>4</sup> )	$A_n/A_3$		$\gamma (10^{-3})$		$\nu$		$T$ (m K)	
		A	B	A	B	A	B	A	B
1	40	32	90	5	5	0.88	0.94	598	571
2	40	8	17	2	1.6	0.94	0.98	196	161
3	160	1	1	0.8	0.6	0.96	0.98	89	91