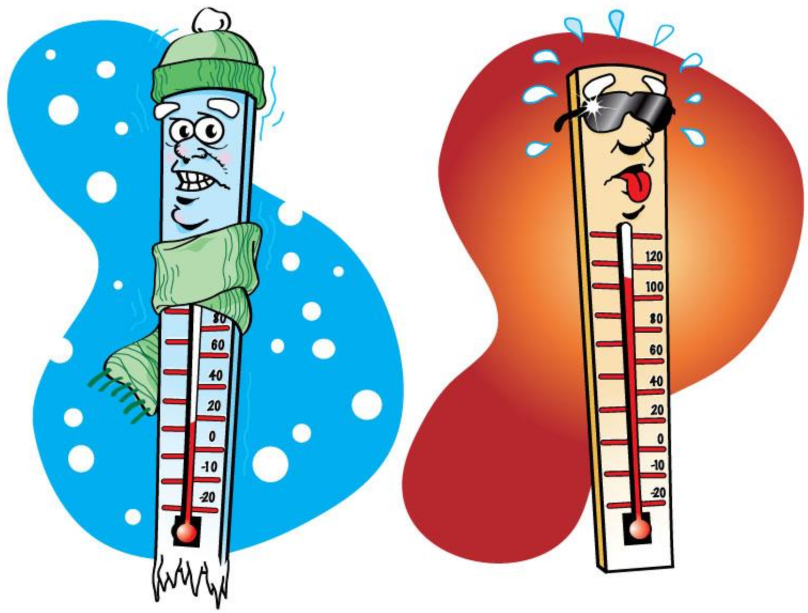
University of Basel
Department of Physics

QUANTUM COHERENCE LAB
ZUMBÜHL GROUP

Thermometry: From RT to μK

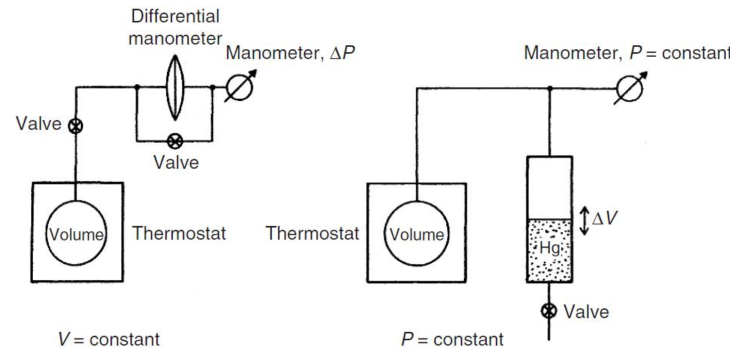


Overview

- Ideal gas thermometer / vapour pressure thermometer
- Platinum thermometer / Doped Ge thermometer
- Oxide compounds: Cernox / RuO_2
- Fixed point device
- CMN thermometer
- Noise thermometer
- GaAs quantum dot (fermi distribution)
- CBT
- NIS junction as a thermometer

Gas therm. / Helium vapor pressure

- Gas thermometry
  - Ideal gas law: pV=nRT (primary thermometer)
  - Operation at constant p / V



- Problems
  - Corrections non-ideal gas
  - Corrections dead volumes (valves, tubes, manometers), need to be small, constant
  - Changes in volumes (thermal, elastic)
  - Absorption, desorption surfaces

⇒ Measurements tedious, mostly used national laboratories for calibration purposes (other versions exist, e.g. sound velocity or dielectric constant)

- Vapour pressure thermometer (H_2, Ne, N_2, O_2, and He from 0.5K – 5K)
  - Used as secondary thermometer
  - Problem: Low therm. cond. of liquids (^{3}He) leads to temperature gradients
  - ^{4}He: superfluid => no temperature gradients, but creeping up walls can cause problems

^{4}He vapour pressure values

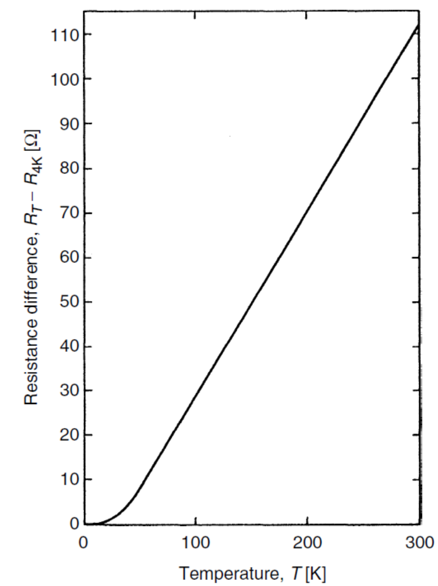
Table 11.4. Helium-4 vapour pressure (kPa) according to ITS-90 [11.4, 11.5]

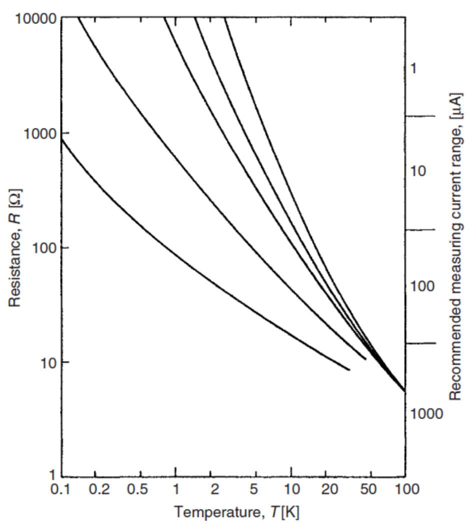
T (K)	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.2	0.082	0.087	0.093	0.100	0.107	0.115	0.123	0.131	0.139	0.148
1.3	0.158	0.168	0.178	0.189	0.201	0.213	0.226	0.239	0.252	0.267
1.4	0.282	0.298	0.314	0.331	0.348	0.367	0.387	0.407	0.428	0.449
1.5	0.472	0.495	0.519	0.544	0.570	0.597	0.625	0.654	0.684	0.715
1.6	0.747	0.780	0.814	0.849	0.885	0.922	0.961	1.001	1.042	1.084
1.7	1.128	1.173	1.219	1.266	1.315	1.365	1.417	1.470	1.525	1.581
1.8	1.638	1.697	1.758	1.820	1.883	1.948	2.015	2.084	2.154	2.226
1.9	2.299	2.374	2.451	2.530	2.610	2.692	2.776	2.862	2.949	3.039
2.0	3.130	3.223	3.317	3.414	3.512	3.613	3.715	3.818	3.925	4.032
2.1	4.141	4.253	4.366	4.481	4.597	4.716	4.836	4.958	5.082	5.207
2.2	5.335	5.465	5.597	5.731	5.867	6.005	6.146	6.288	6.433	6.580
2.3	6.730	6.882	7.036	7.192	7.351	7.512	7.675	7.841	8.009	8.180
2.4	8.354	8.529	8.708	8.889	9.072	9.258	9.447	9.638	9.832	10.03
2.5	10.23	10.43	10.64	10.84	11.05	11.27	11.48	11.70	11.92	12.15
2.6	12.37	12.60	12.84	13.07	13.31	13.55	13.80	14.05	14.30	14.55
2.7	14.81	15.07	15.33	15.60	15.87	16.14	16.42	16.70	16.98	17.26
2.8	17.55	17.84	18.14	18.44	18.74	19.05	19.36	19.67	19.98	20.30
2.9	20.63	20.95	21.28	21.61	21.95	22.29	22.64	22.98	23.33	23.69
3.0	24.05	24.41	24.77	25.14	25.52	25.89	26.27	26.66	27.05	27.44
3.1	27.84	28.24	28.64	29.05	29.46	29.87	30.29	30.72	31.14	31.58
3.2	32.01	32.45	32.89	33.34	33.79	34.25	34.71	35.17	35.64	36.11
3.3	36.59	37.07	37.56	38.05	38.54	39.04	39.54	40.05	40.56	41.08
3.4	41.60	42.12	42.65	43.18	43.72	44.26	44.81	45.36	45.92	46.48
3.5	47.05	47.62	48.19	48.77	49.35	49.94	50.54	51.13	51.74	52.35
3.6	52.96	53.57	54.20	54.82	55.46	56.09	56.73	57.38	58.03	58.69
3.7	59.35	60.02	60.69	61.37	62.05	62.73	63.43	64.12	64.83	65.53
3.8	66.25	66.96	67.69	68.41	69.15	69.89	70.63	71.38	72.14	72.90
3.9	73.66	74.43	75.21	75.99	76.78	77.57	78.37	79.17	79.98	80.80
4.0	81.62	82.44	83.27	84.11	84.95	85.80	86.66	87.52	88.38	89.26
4.1	90.13	91.02	91.91	92.80	93.70	94.61	95.52	96.44	97.37	98.30
4.2	99.23	100.18	101.13	102.08	103.04	104.01	104.98	105.96	106.95	107.94
4.3	108.94	109.94	110.95	111.97	113.00	114.03	115.06	116.11	117.15	118.21
4.4	119.27	120.34	121.42	122.50	123.51	124.68	125.79	126.89	128.01	129.13
4.5	130.26	131.40	132.54	133.69	134.84	136.01	137.18	138.36	139.54	140.73
4.6	141.93	143.13	144.35	145.57	146.79	148.03	149.27	150.52	151.77	153.04
4.7	154.31	155.58	156.87	158.16	159.46	160.77	162.09	163.41	164.74	166.08
4.8	167.42	168.78	170.14	171.51	172.89	174.27	175.66	177.07	178.47	179.89
4.9	181.32	182.75	184.19	185.64	187.10	188.56	190.04	191.52	193.01	194.51
5.0	196.08	197.53	199.06	200.59	202.13	203.68	205.24	206.81	208.39	209.97

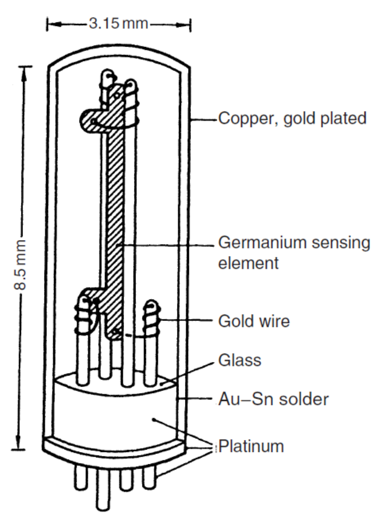
Platinum thermometer & Doped Ge

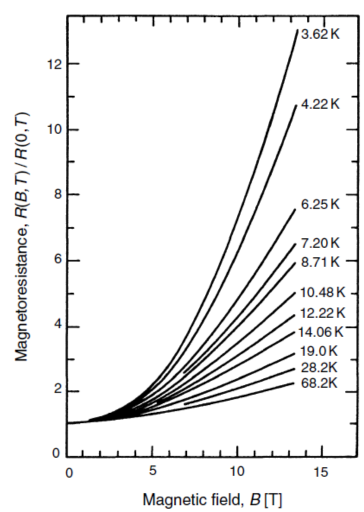
- Platinum
  - Chemically resistant
  - Can be obtained in high purity
  - Ductile => can make thin wires
  - Reasonably large temperature coeff.
  - R linear in T down to 50K
- Doped Germanium (semiconductor thermometer)
  - $R(T) = \alpha * \exp\left(\frac{\Delta E}{2k_B T}\right)$
  - Does not agree well in reality, need empirical formulas
  - Resistance very high at low-T, can be used from 50mK-40K
  - Good stability, ca 1mK at 4K
  - Strong, orientation dependent magnetoresistance

Platinum PT100 resistor



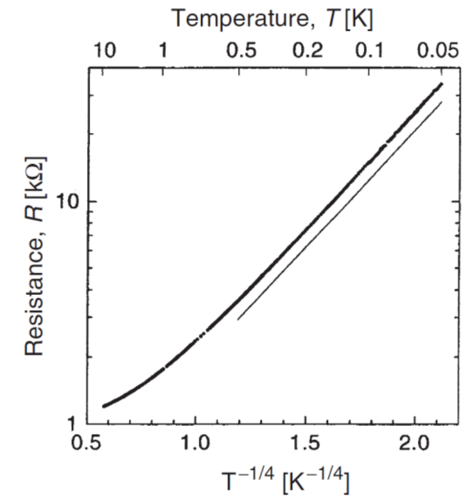




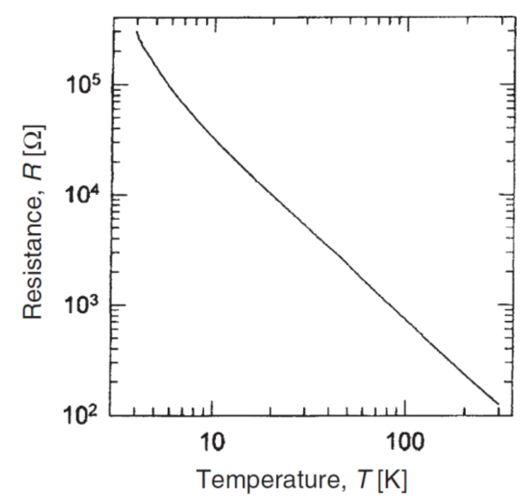


Oxide compounds: Cernox & RuO_2

- Thick film RuO_2 resistor
  - Metal ceramic composites
  - Conductive RuO_2 and Bi_2RuO_2 embedded in lead silicate glass
  - Deposit on Al, heat above its glass point
  - Reproducible characteristics
  - Weak magnetoresistance
  - Small size / mass
  - Resistance fitted with empirical equation
  - Conduction through variable range hopping in 3D
  - In limited T-range the resistance can be described by $R = R_0 \exp\left(\frac{T_0}{T}\right)^{0.25}$

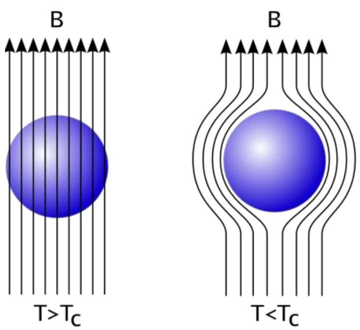


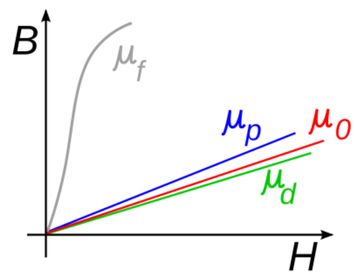
- Thin film ceramic zirconium oxynitrate (CERNOX) resistors
  - Sputtering of zirconium onto 0.3mm thick sapphire substrate => 0.3μm film
  - Sputtering in atmosphere containing Ar, N_2, O_2
  - Resistance from metallic to insulating with increasing O_2 content: incorporation into ZrN lattice enlarges spacing
  - Robust, small, fast, insensitive to B-field, 0.1K to above RT



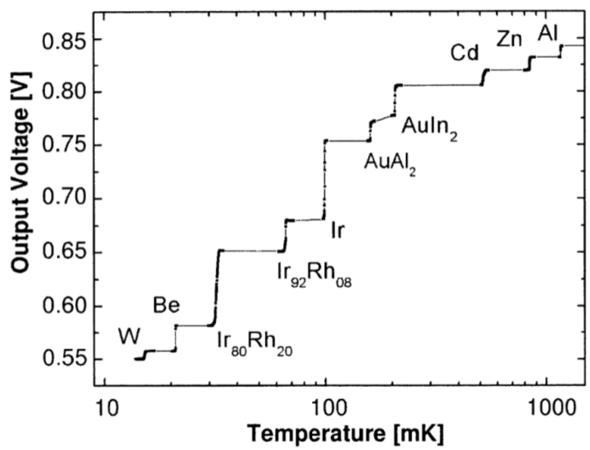
Fixed point device

- Use known superconducting transition temperatures (sensitive to ext. fields, needs shielding)
- How to measure?
  - Resistance drop (invasive, heating when coming from high temperature)
  - Measure inductance (supercond. ⇔ diamagnet, Meissner effect)





- Inductance of a coil: $L = \frac{\mu_0 \mu_r N^2 A}{l}$

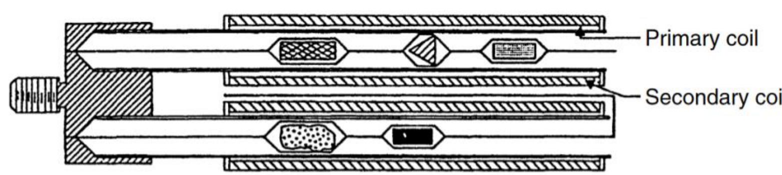


SRD 1000 (see Pobell)

Element	T_c (K)
Nb	9.3
Pb	7.2
In	3.415
Al	1.18
Cd	0.52
$AuIn_2$	0.204
$AuAl_2$	0.16
$Ir_{0.8}Ru_{0.2}$	0.1
Be	0.0227
W	0.0155

Data from Pobell

Commercial fixed point device (mutual inductance measurement)



SRM 768 (see Pobell)

CMN (cerium magnesium nitrate)

- CMN: paramagnetic salt
- ferromagn. ordering $T_C = 1 - 2$ mK
- para phase $\rightarrow$ Curie-Weiss law:

  Susceptibility $\chi_m(T) = \frac{C}{T-\Delta}$

- Δ: Weiss constant (depends on exp. details: sample shape, crystal symmetry of the salt)
- C: Curie constant

  amount of salt used

  $C = N_0 J(J+1)\mu_0 \mu_B^2 g^2 / 3k_B$

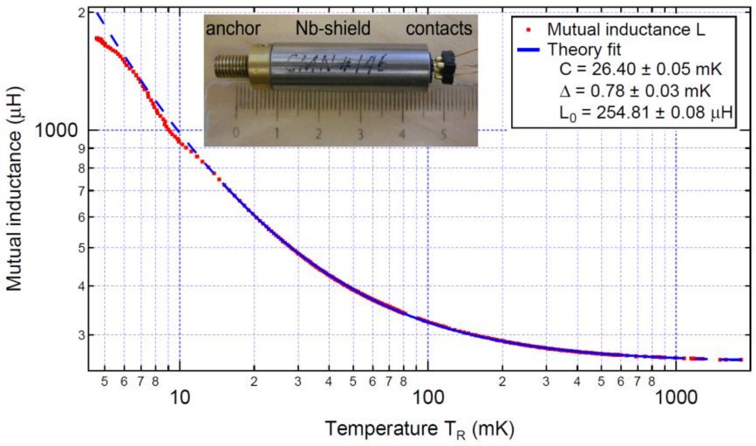
  N_0: number of moles

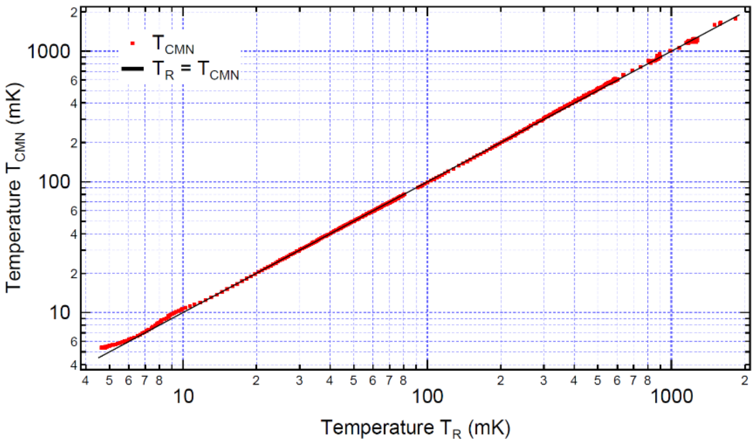
  μ_B : Bohr magneton 59 μeV/T

  J: Total angular momentum quantum number

  k_B: Boltzmann constant

  μ_0: Vacuum permeability





- Susceptibility via mutual inductance (pair of concentric coils)

  alternating current $I(\omega)$ on prim. coil

  oscillating field $B(t) = (1 + \chi_m) B_0(t)$

  $B_0(t)$: field without salt or high-T limit

- Lentz rule $\rightarrow$ voltage in second. Coil

  $$U_{ind} = -L\frac{dI}{dt} = -\left(1 + \frac{C}{T-\Delta}\right) * \frac{\mu_0 N_P N_S A}{l} * \frac{dI}{dt}$$

$I(\omega) = I_0 * \sin(\omega t)$

N_P, N_S: windings prim./second. Coil

A: cross section

l: length of coil

- Nb shield ($B_C \sim 80$mT) to suppress external B-fields

Noise thermometry (MFFT)

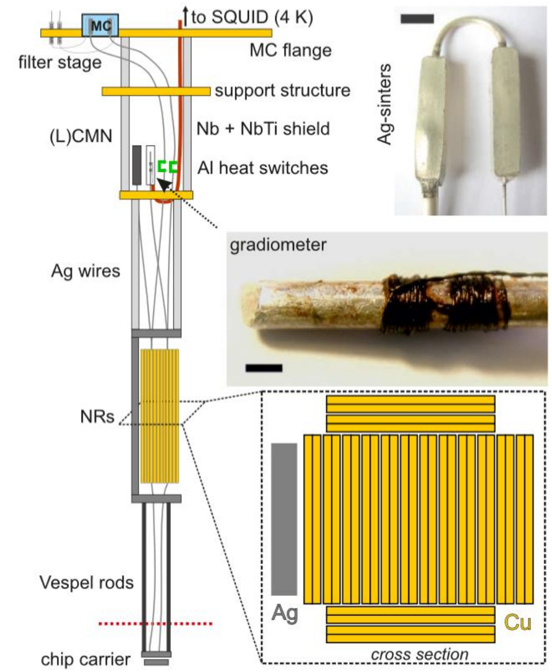
- MFFT: magnetic field fluctuation thermometer
- Many thermometer suffer from self heating, susceptible to heat leaks → saturation at low-T

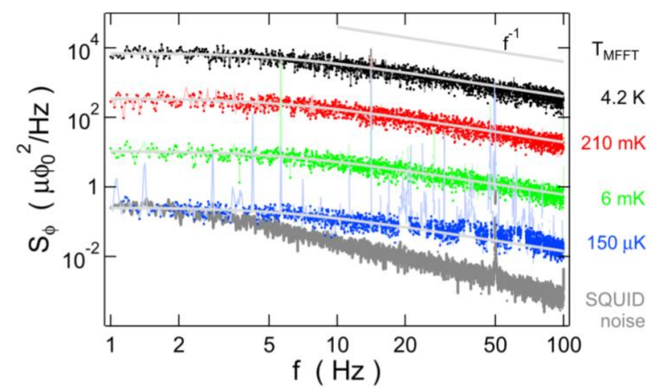
- Brownian motion of electrons in high purity 5N Ag wire
  → current noise (Johnson Nyquist noise)
  $S_I = 4k_B T/R$

- Varying currents → varying magnetic fields
  → readout via supercond. pickup coil (gradiometer)
  → non-inductive winding (not sensitive to global ext. magn. fields, but to local current noise)
  → Nb, NbTi shields to suppress ext. fields

- Small signal → sensitive readout: SQUID amplifier (no bias applied)

- Power spectral density of magnetic flux noise:
  Low frequency: $S_\phi(0,T) = 4k_B T\sigma G\mu_0^2 r^3/2\pi$
  Higher frequency (skin effect): $S_\phi(f,T) = \frac{S_\phi(0,T)}{\sqrt{1+\left(\frac{f}{f_c}\right)^2}}$





M. Palma et al., Rev. Sci. Instr. **88**, 043902 (2017)

Quantum dot thermometry

Lifetime broadended → Lorentzian

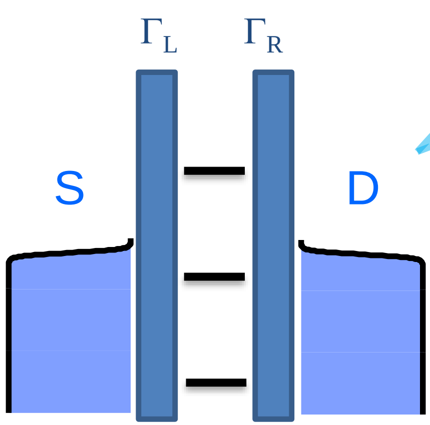
- QD at large Γ >> kT/h
- Resonances („particles“) in particle physics
- Lorentzian = FFT of exp. decay

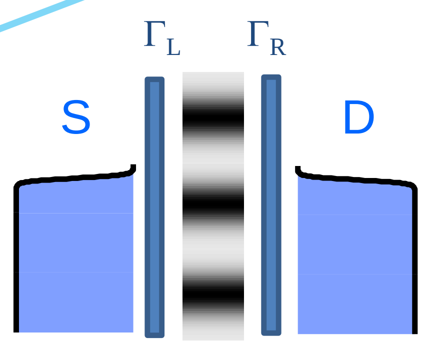
$$G = \frac{2e^2}{h}\frac{\Gamma_s\Gamma_d}{\Gamma_s+\Gamma_d}\left(\frac{\Gamma_t}{(\frac{\Gamma_t}{2})^2+(\frac{e\alpha_g}{\hbar}V_g)^2}\right)$$

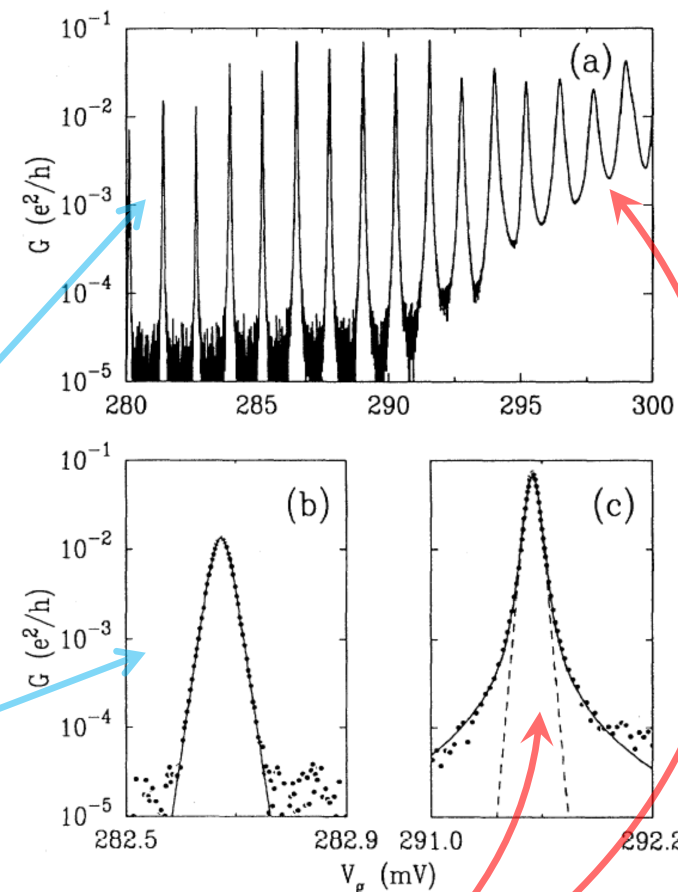
T broadended → $\cosh_{-2}$

- Derivative of FD

$$G = \frac{2e^2}{h}\frac{\Gamma_s\Gamma_d}{\Gamma_s+\Gamma_d}\frac{h}{4k_BT}\cosh^{-2}\left(\frac{e\alpha_g V_g}{2k_BT}\right)$$







T-broadened Lifetime broadened

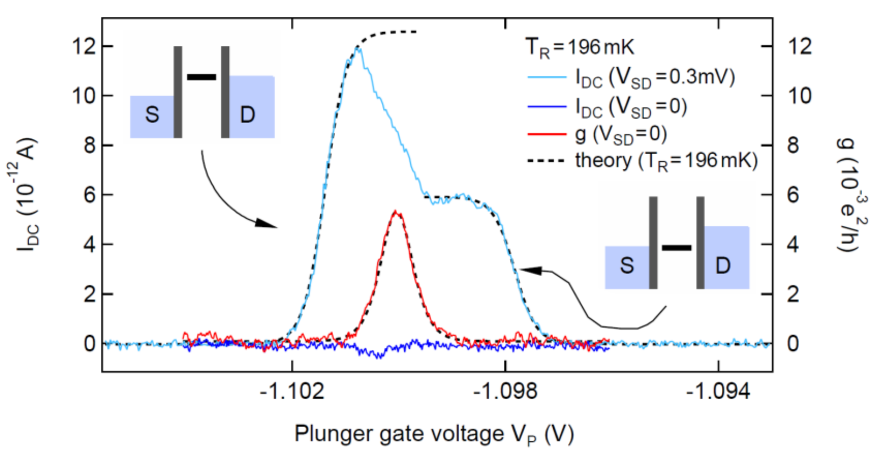
Foxman et al., PRB **47**, 10020 (1993)

Quantum dot thermometry

Energy profile in the leads?

- T=0 electrons up to EF
- Current I(E) ~ DOS(E)*f(E)
- DOS2D = const

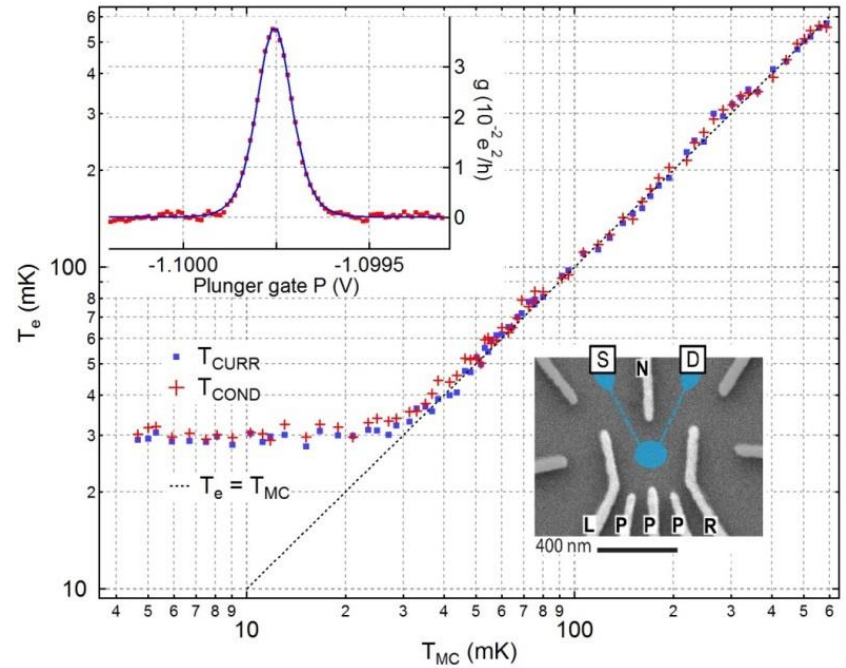
  => I(E) ~ f(E); f(E) = 1/(1+exp(E/kT))
- Requirement: dE << kT i.e. $\Gamma_L, \Gamma_R << kT/\hbar$



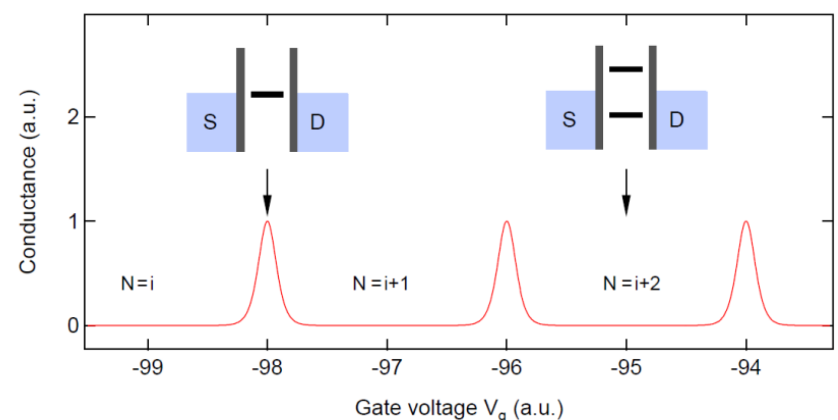
QD as a thermometer

- Width of FD, convert using known leverarm (DC-bias)
- Individual FD-fits for S/D => S/D temperatures

- Similar for diff. cond. => folding of S/D temp.

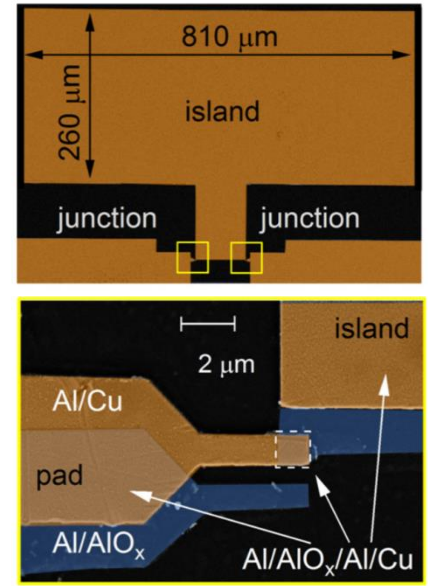


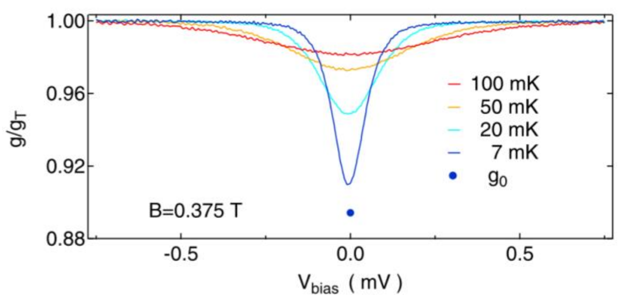
C.P. Scheller et al., APL **104**, 211106 (2014)

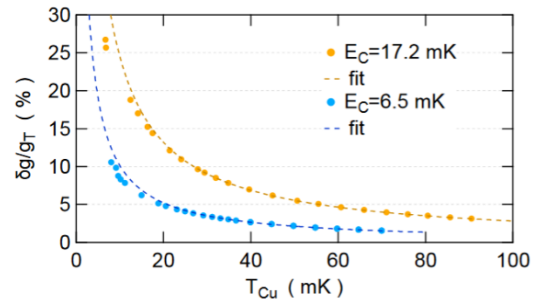


Coulomb blockade thermometry (CBT)

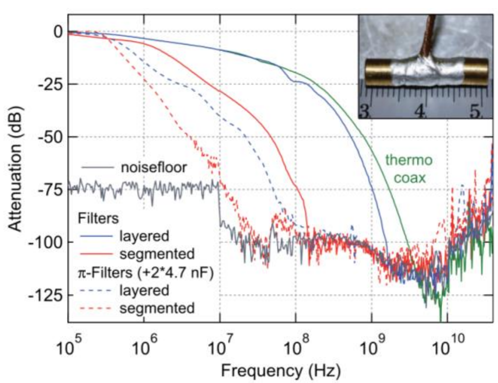
- CBT: Array of (small) tunnel junctions (shadow mask evaporation)
- QD: $E_C \gg k_B T$ CBT: $E_C \ll k_B T$
- Zero bias dip of FWHM: $V_{FWHM} = 5.439 \frac{N k_B T}{e}$
- Overheating at finite bias => use zero bias dip (secondary mode → precalibration: $\frac{\Delta G}{G_T} = \frac{1}{6} u_N - \frac{1}{60} u_N^2 + \frac{1}{630} u_N^3,\ u_N = E_C / k_B T$)

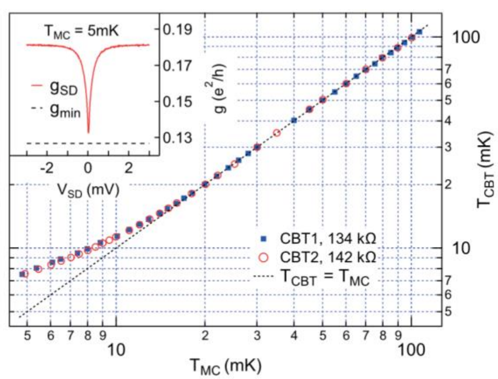






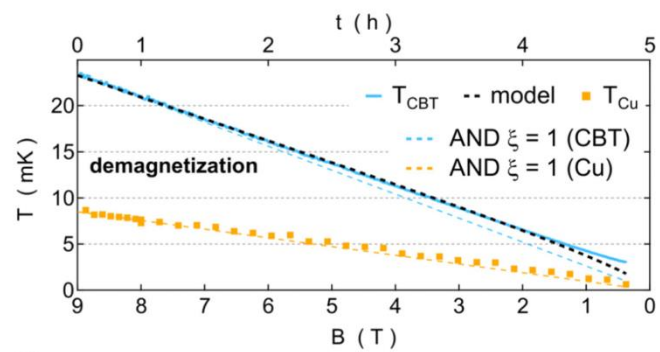
- Filtering & thermalization in order to reach low T
- Thermocoax: 100 dB @ 3 GHz (⇔ 0.14 K)
- Microwave filter: 100 dB @ 150 MHz (⇔ 7 mK)
- 7.5mK CBT temperature in standard fridge





C.P. Scheller et al., APL **104**, 211106 (2014)

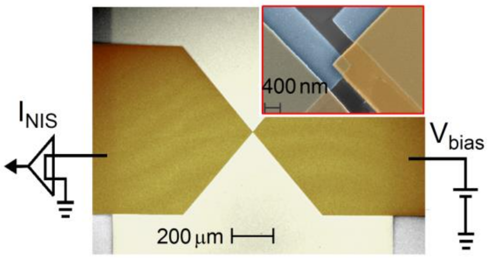
2.8mK (1.8mK) CBT temperature after AND

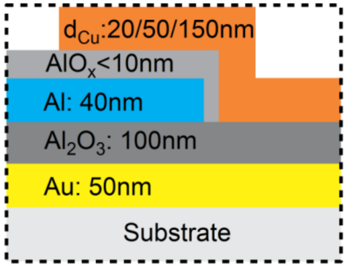


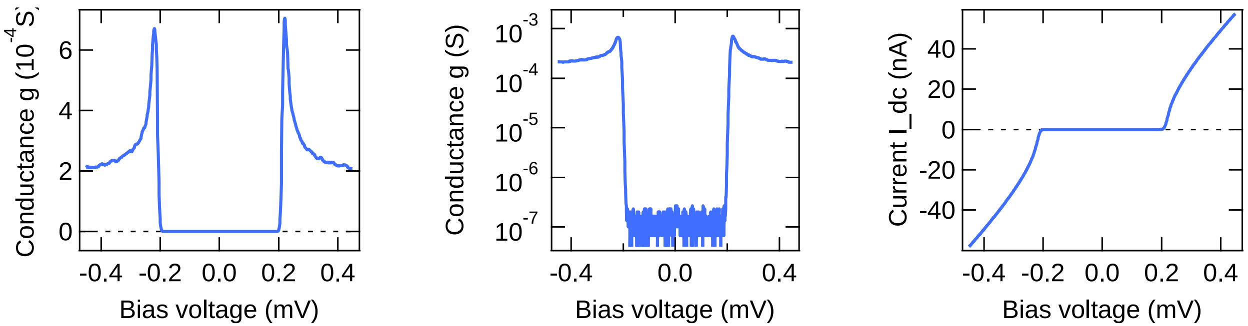
C.P. Scheller, M. Palma et al., APL **111**, 253105 (2017)

NIS thermometry

- Resistive junction (4 – 10 kΩ) => negligible higher order processes (Andreev reflection)
- Excellent filtering/shielding/thermalization
  => photon assisted tunnelling **(PAT) negligible**
- **Hard gap**: R_gap/R_normal >1000, small dynes parameter γ=(2−8)·10^{-5}



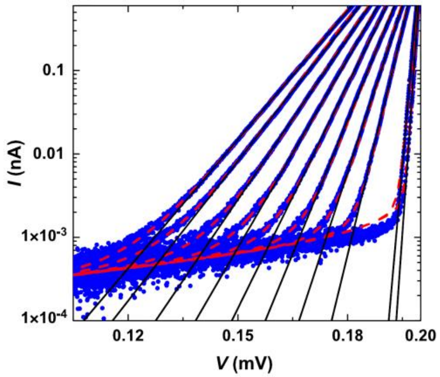


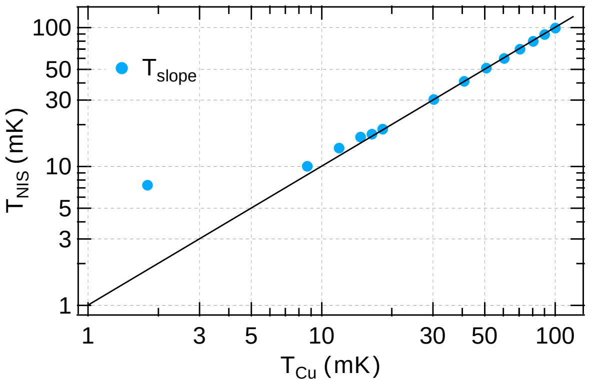


- Exponentially suppressed quasiparticle current => primary NIS junction thermometry

$$I = I_0 e^{\frac{V - V_\Delta}{k_B T}}$$

$$T = \frac{e}{k_B} \frac{dV}{d\,ln(I)}$$

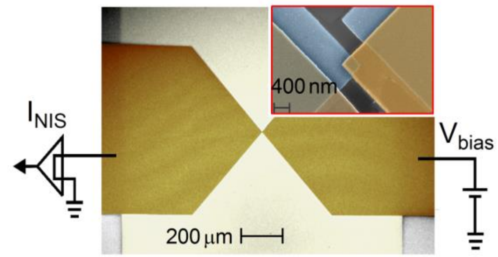


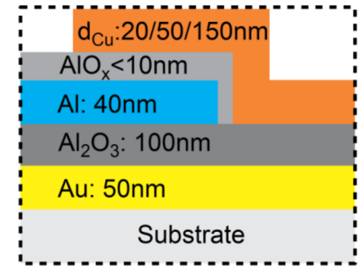


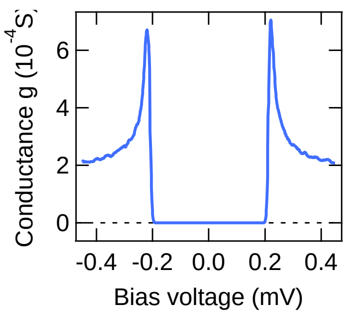
A.V. Feshchenko et al., PRAppl **4**, 034001 (2015)

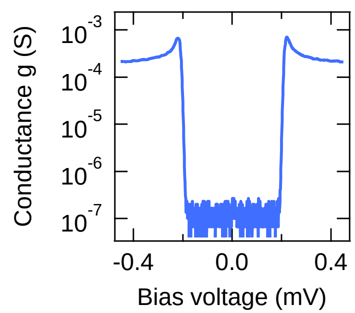
NIS subgap-state thermometry

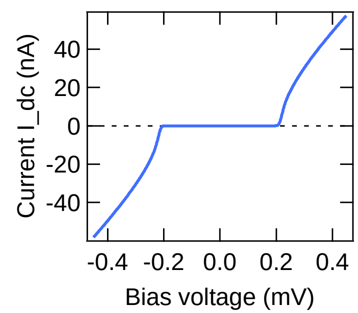
- Resistive junction (4 – 10 kW) => negligible higher order processes (Andreev reflection)
- Excellent filtering/shielding/thermalization
  => photon assisted tunnelling **(PAT) negligible**
- **Hard gap**: R_{gap}/R_{normal} >1000, small dynes parameter γ=(2–8)·10^{-5}



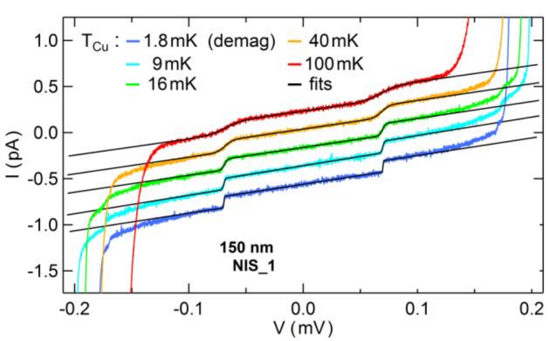


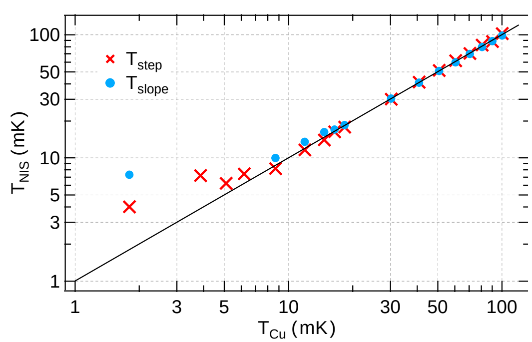


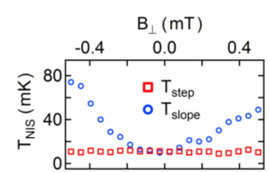




- Thermally broadened subgap states => also allows for primary thermometry
  - Symmetric in bias (electron – hole symmetry in superconductor)
  - Insensitive to ext. magnetic field (unlike quasiparticle current)
  - Reads down to 4mK, the coldest NIS junction !







Summary

- Thermometers are a dime a dozen (gibt es wie Sand am Meer)
  $\rightarrow$ Pick the right one for your specific application
- Gas thermometer / He vapour pressure: Calibration, national institutes (cumbersome, limited T-range, not useful for us)
- Platinum (doped Ge) limited T-range
- Oxide compounds: RuO_2 (Cernox) our standard MC chamber thermometer. Easy use, down to mK, not sensitive to ext. B-field
- Fixed point device: useful to check calibration of other thermometer
- CMN thermometer: Easy use, down to mK
- Noise thermometer: Need squid, shielding $\rightarrow$ difficult, but down to μK
- On chip (samples): GaAs quantum dot, CBT, NIS (fermi distribution). CBT easy to use