Quantized conductance of magnetoelectric subbands in ballistic point contacts

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The two-terminal conductance of ballistic point contacts in the two-dimensional electron gas of a high-mobility GaAs/Al$_x$Ga$_{1-x}$As heterostructure has been studied in quantizing electric and magnetic fields. The conductance is found to be quantized at multiples of $e^2/h$, exclusively determined by the number of occupied magnetoelectric subbands in the constriction. The experiment provides the first direct demonstration of magnetic and electric depopulation of one-dimensional subbands in a single wire.

The study of ballistic electron transport in small semiconductor devices is a rapidly developing field of research. Combining present-day microfabrication technology and advanced material growth techniques, such as molecular-beam epitaxy, devices can be made through which electrons can travel with a minimal amount of impurity scattering. These systems are ideal for the study of quantum transport, which occurs in devices with dimensions comparable to the wavelength of the carriers. A clear manifestation of quantum transport has been reported in Ref. 6. The two-terminal conductance of narrow and short ballistic constrictions (point contacts) in a two-dimensional electron gas (2DEG) changes in steps of $2e^2/h$, when the width is varied by means of a gate.

Because of the ballistic transport in the constriction, a direct correspondence between the conductance and the number of occupied subbands in the constriction is observed, each subband contributing an amount $2e^2/h$ to the conductance. This makes these devices very suitable for the study of the (quasi)-one-dimensional subband structure in narrow wires. This can be done either by electric depopulation, reducing the number of occupied subbands by decreasing the width (or the electron density) of the wire, or by magnetic depopulation. A perpendicular magnetic field forms hybrid magnetoelectric subbands, which can be depopulated by increasing the field.

Both also require a multiwire system to resolve the signal originating from the depopulation of subbands.

Magnetic depopulation has been studied in single wires by measuring the deviations from the $1/B$ periodicity of the Shubnikov–de Haas resistance oscillations at low fields, the observation of which is also made difficult by the irregular structure in the magnetoresistance. Moreover, it is not possible to determine the number of occupied subbands at zero field from the measurements.

In the point contacts we study in this paper, the number of occupied subbands can be determined accurately as a function of gate voltage and magnetic field by simply counting the number of conductance steps occurring until the point contact is pinched off. We thus study the transition between the zero-field conductance quantization of the quantum Hall effect in narrow ballistic devices. As will be shown, these effects are the limiting cases of a more general quantization phenomenon. We observe a continuous transition from electric quantization at zero magnetic field to fully magnetic quantization at high fields. The conductance is determined exclusively by the number of quantum channels, which can be identified with the hybrid magnetoelectric subbands in the constriction.

The devices are made on high-mobility GaAs/Al$_x$Ga$_{1-x}$As heterostructures. The electron density of the material is $3.56 \times 10^{15}/m^2$ and the mean free path $l_e$ is 8.5 $\mu$m (at 0.6 K). A standard Hall bar geometry is defined by wet etching. Using e-beam lithography, a metal gate is made on top of the heterostructure, with an opening about 250 nm wide (inset Fig. 1.) The point contacts are defined by applying a negative voltage to the gate. At $V_g = -0.6$ V the electron gas underneath the gate is fully depleted, the conduction taking place through the point contact only. At this voltage the point contact has its maximum width, about equal to the defined width.
performed in magnetic fields for which $\rho_{xx}$ in the broad regions has a minimum.

Figure 1 shows the conductance $G_c$ obtained from the measured resistance after subtraction of the following background resistances: 0 T, 4352 $\Omega$; 0.7 T, 3477 $\Omega$; 1.0 T, 3836 $\Omega$; 1.8 T, 5269 $\Omega$; 2.5 T, 7859 $\Omega$. These values agree within 10% with the values for the Ohmic contact resistances determined from measurements of the two-terminal quantum Hall resistance at $V_g = 0.12$. As seen in Fig. 1, after subtraction of a constant background resistance a sequence of quantized plateaus is obtained for each value of magnetic field. For $B = 0$ we observe the conductance plateaus at integer multiples of $2e^2/h$ associated with electrical subbands reported in Ref. 6. As is evident from Fig. 1, the conductance quantization is conserved in a magnetic field. The effect of a magnetic field is to reduce the number of plateaus in a given gate voltage interval. As we shall demonstrate, this provides a direct measurement of the depopulation of one-dimensional subbands by a magnetic field.

At high magnetic fields spin-splitting gives rise to additional plateaus at odd multiples of $e^2/h$. The reason that the corresponding plateaus are much less well defined is that the ratio of plateau widths for odd and even values of $e^2/h$ is determined by the ratio of the Zeeman energy and the 1D subband splitting, which in our experiment is much smaller than 1.

In zero field a finite number $N_c$ of one-dimensional subbands in the constriction is occupied. This number can be changed in two ways. Decreasing the gate voltage reduces both the width and the carrier concentration in the constriction. This reduces the number of occupied subbands. In a magnetic field hybrid magnetoelastic subbands are formed, the number $N_c(B)$ of which decreases with increasing field.\[10,11\] The fundamental relation in ballistic transport between the number of occupied subbands in the constriction and the conductance is expressed by the Landauer formula\[13-16\]

\[ G_c = \frac{2e^2}{h} N_c(B), \]

which is valid if no backscattering occurs in the constriction\[14\] and $kT$ is less than the energy separation between subbands. Note that this relation holds irrespective of the nature of the subbands. For zero field the subbands are due to electric confinement only, for low fields the subbands are hybrid magnetoelastic subbands, whereas for high fields $N_c$ is equal to the number of occupied Landau levels.

To determine quantitatively the number of occupied subbands as a function of gate voltage and magnetic field, the shape of the electric potential well confining the electrons has to be specified. For simplicity we assume a square well of width $W$. We determine the number of occupied subbands $N_c$ from the semiclassical Bohr-Sommerfeld quantization rule\[17\]

\[ N_c(B) = \int \left( \frac{k_F l_c}{2} \right)^2 \frac{1}{2} \] if $W > 2l_c$, 

\[ N_c(B) = \int \left( \frac{k_F l_c}{2} + \frac{1}{2} \right) \] if $W < 2l_c$, 

between the gate electrodes. Decreasing the gate voltage further will cause the depletion region around the gates to increase, thus reducing the width. At $V_g = -2.2$ V the point contact is pinched off. We note that the gate voltage affects both the carrier concentration and the width of the point contact. Because of the rounding of the depletion region (inset Fig. 1) the point contact consists of a narrow and short constriction which gradually widens to make contact with broad 2D EG regions. Ohmic contacts are attached to these broad regions on either side of the constriction.

The two-terminal conductance of the constriction is defined as $G_c = I/(\mu_1 - \mu_2)$, in which $I$ is the net current through the constriction and $\mu_1$, $\mu_2$ are the electrochemical potentials of the carriers entering the constriction from either side. In the experiment the two-terminal resistance between the Ohmic contacts is measured as a function of gate voltage for several values of the magnetic field at 0.6 K. The measured resistance is the sum of the constriction resistance $1/G_c$ and a series background resistance, which does not depend on the gate voltage, once the point contact is defined at $V_g = -0.6$ V. The background resistance is dominated by the resistance of the Ohmic contacts, which in our samples depends on magnetic field. To minimize the contribution to the background resistance by the broad 2D EG regions, the measurements have been
in which \( \text{int}(x) \) denotes truncation to an integer. Here \( l_c \)
denotes the cyclotron radius \( l_c = h k_F/eB \). The gate voltage changes the number of subbands by its action on \( k_F \)
(through the electron density \( n_e = k_F^2/(2\pi) \)) and the width \( W \), while the magnetic field affects \( N_e \) through the
cyclotron radius \( l_c \).

To improve upon the semiclassical expression (2) one should take into account the penetration of the wave function beyond the classical orbit (for a smooth transition between the regimes \( W < 2l_c \) and \( W > 2l_c \)), and also possible oscillations with \( B \) of the Fermi energy in the constriction (in the bulk of the 2D EG \( E_F \) is pinned at Landau levels). As a result of these effects, Eq. (2) has a limited accuracy of \( \pm 1 \), which is sufficient for the purpose of Fig. 2 (see below).

The number of occupied subbands has been determined as a function of magnetic field for several values of the gate voltage. In contrast with previous studies, \(^{10,11}\) where the number of subbands is inferred from deviations of the periodicity of Shubnikov–de Haas oscillations in narrow channels, \( N_e \) in our experiment directly follows from the quantized conductance according to Eq. (1). The experimental data for \( N_e(B) \) are plotted in Fig. 2. Also shown are the theoretical curves according to Eq. (2). The unknown parameters \( k_F \) and \( W \) are determined, respectively, from the high-field conductance plateaus and the zero-field conductance (see inset of Fig. 2).

The agreement found confirms our expectation that the quantized conductance is exclusively determined by the number of occupied hybrid magnetoelectric subbands according to Eq. (1). To our knowledge this is first direct observation of magnetic and electric depopulation of subbands in a single wire. In addition, our experiment illustrates the fundamental significance of the Landauer conduction formula.

**Note added:** After submission of this manuscript a paper appeared by Wharam et al., \(^{18}\) in which similar experimental results were reported, including also the observa-

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