Effects of quantum levels on transport through a Coulomb island


Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

S. J. Wind

IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

(Received 16 December 1992)

Transport measurements are used to study the quantized energy levels in a small electron gas, referred to as a Coulomb island. First, large-bias measurements reveal the excitation spectrum of the island. Second, small-bias measurements probe the intrinsic line shape of energy levels. Line shapes are generally fit well by thermally broadened Lorentzians. In addition, a decrease in the charging energy of the island is observed as the coupling to its leads is increased. This effect is shown to be consistent with a dramatic increase in one of the island-lead capacitances.

Conductance studies have demonstrated that transport through a small electron gas is strongly regulated by Coulomb interactions.1 Initially, much of this behavior was interpreted through the theory of the Coulomb blockade,2 a classical theory in which the discreteness of the energy spectrum of the electron gas is ignored. In metals, where the separation between energy levels is generally negligible, this has proven to be an excellent approximation. In submicrometer semiconductor systems, however, the spacing between energy levels can be orders of magnitude larger and must be taken into account. Consequently, several models have recently been introduced which extend the Coulomb-blockade model to include the effects of quantized energy levels in the limit that $kT$ is much greater than the intrinsic energy width of the levels.3–6 Some of the predictions of these models have been verified by experiments in small, two-dimensional electron gases (2DEG), which we refer to as Coulomb islands, when the Coulomb energy is larger than the level spacing.7–9

In this paper we report effects of quantized energy levels on transport through a Coulomb island. First, large-bias measurements are employed to map the discrete excitation spectrum of a Coulomb island. Specifically, this is the spectrum of excited states of the island when the number of charges on the island is constant. This is in contrast to previous experiments which measured the spectrum of states for adding additional charge to the island.8 Second, low-bias measurements are used to probe the intrinsic line shape of transmission resonances through the island; the line shapes are in general fit well by thermally broadened Lorentzians. Last, we observe a vanishing of the Coulomb charging energy as the coupling through one of the tunnel barriers is increased. This behavior is shown to be consistent with a dramatic increase in the measured value of the capacitance between the island and one of its leads.

The device studied is shown schematically in the inset to Fig. 1(a) and has been discussed in detail elsewhere.10 Briefly, the density of the 2DEG in an inverted GaAs/Al$_x$Ga$_{1-x}$As heterostructure is varied by a voltage $V_g$ on the substrate of the device. The density of the 2DEG is $\sim 2 \times 10^{15}$ m$^{-2}$ and varying $V_g$ by 1 mV changes the density by $\sim 1 \times 10^{13}$ m$^{-2}$. A negative bias $V_e$ applied to an electrode on the top of the device

![Figure 1](image-url)

**FIG. 1.** (a) The current as a function of gate voltage $V_g$ for drain-source biases $V_{ds}$ ranging from 50 to 500 µV in increments of 25 µV. Inset: A schematic top view of the device (not to scale) showing e-beam patterned gates (opaque) and 2DEG (shaded). (b) The current, calculated from the model described in the text, in units of $I/\gamma_{min}$ where $\gamma_{min}$ is the tunneling rate of the lowest active level through the right barrier. The notation $n/m$ labeling the extrema in the overstructure denotes that electrons can tunnel into n single-particle levels in the island and out from m levels. (c) A schematic of the “2/1” tunneling processes. Only the levels active in transport are shown. Increasing $V_g$ lowers the energy of levels relative to $\mu_v$, ultimately changing the set of active levels.
confines electrons to the island geometry with lithographic dimensions $450 \times 500$ nm$^2$. In this configuration the island is separated from its leads by tunnel barriers. The current $I$, which flows through the island in response to a bias between the right and left leads $V_{ds}$, is typically measured, with a high-precision, low-noise current amplifier. The output of the current amplifier was measured by a lock-in amplifier, which also provided the ac drain bias. The device is cooled in a 12-mK base-temperature dilution refrigerator. Due to residual electron heating, the resulting electron temperature, as determined from the temperature evolution of conductance peaks, is 60 mK at low bias.$^{11}$

We begin by discussing large-bias transport measurements which reveal the discrete excitation spectrum of the island. The current as a function of gate voltage is plotted in Fig. 1(a), for $V_{ds}$ ranging from 50 to 500 $\mu$V in increments of 25 $\mu$V in zero magnetic field. The periodic spacing of the current peaks at low $V_{ds}$ reflects the quantization of charge on the island.$^{10}$ As $V_{ds}$ is increased, the low-bias peaks broaden, take on an asymmetric profile, and exhibit a modulated overstructure. This overstructure persists to $T \approx 250$ mK and is observable in both zero and finite magnetic fields.$^{12}$

Figure 1(b) shows the current versus gate voltage calculated from a generalized Coulomb-blockade model in which quantized levels have been included.$^{5,13}$ The levels are spaced by $\Delta \epsilon = 0.4U$, where $U$ is the Coulomb-blockade charging energy required to increase the charge of the island by $e$. To reflect the reduction of the tunnel barrier heights with increasing level energy, the tunneling rates across the barriers are increased in the calculation by a factor of 1.2 for each higher level. As shown in Fig. 1(c), the current rises (falls) when a level is pulled below the higher (lower) chemical potential and becomes accessible (inaccessible) for tunneling. This simple model is clearly consistent with the observed modulations of the current in Fig. 1(a). We therefore interpret the modulations in the data as arising from a discrete set of well-coupled, single-particle levels in the island. Furthermore, we point out that unlike previous experiments,$^{8}$ these data probe the quantized levels of the island in zero magnetic field. To match the experimental results, it is necessary to assume that for each level the tunneling rate through the barrier on the left is larger than that on the right by a factor of 10.

The excitation spectrum of the well-coupled, single-particle levels of the island is also manifest in $I$ versus $V_{ds}$ measurements at fixed $V_g$. The inset to Fig. 2(a) shows $|I|$ versus $V_{ds}$ for $V_g = 304.06$ mV and $B = 3.35$ T. The suppression of current in the neighborhood of $V_{ds} = 0$ is due to the Coulomb blockade and the large rise, in current at $V_{ds} = 1.1$ mV is a manifestation of the Coulomb staircase.$^{7}$ Additional fine structure, seen most clearly in the plot of $dI/dV_{ds}$ in Fig. 2(a), arises from the discrete excitation levels spectrum. A peak in $dI/dV_{ds}$ occurs each time a level in the island is aligned with the Fermi level in the biased lead.$^{14}$ Similar observations have been reported in vertical quantum-dot structures.$^{7}$ The $V_{ds}$ of peaks in $dI/dV_{ds}$, multiplied by a factor $e\beta$, are plotted as a function of $V_g$ in Fig. 2(b). The factor $\beta = -0.80$ relates $V_{ds}$ to the actual change in the Fermi energy of the biased lead relative to the island.$^{15}$ The lines in Fig. 2(b) show the single-particle excitation spectrum of the island as a function of $V_g$.$^{16}$ The Coulomb-blockade gap in the tunneling density of states and the downward shift in energy of excitation levels with increasing $V_g$ are clearly visible. The typical level spacing is seen to be $\Delta \epsilon \approx 0.1$ meV. Also evident is the reaignment of the Coulomb gap when the charge state of the island increases by $e$ and levels shift by $U$.

We now discuss measurements which probe the intrinsic line shapes of transmission resonances of the island. Figure 3(a) shows the low-bias ($V_{ds} = 2.5$ $\mu$V) conductance of the island as a function of $V_g$ at $B = 2.53$ T. At high $V_g$ the peaks broaden, the valley conductances increase, and the peak to valley ratios decrease from values greater than 10 at low $V_g$ to $\sim 10$ at higher $V_g$. Figure 3(b) is an expanded view of a low $V_g$ conductance peak from Fig. 3(a), for which the coupling to the island is still relatively small and hence so too is its intrinsic width. This peak is fit well with the line shape of a purely thermally broadened resonance$^{10}$ in the limit that the resonance width is much less than $kT$,

$$G = (e^2/h)(1/4kT)A \cosh^{-2}[(e\alpha V_g - E_{res})/2kT],$$

FIG. 2. (a) $dI/dV_{ds}$ as a function of $V_{ds}$ for $V_g = 304.06$ mV and $B = 3.35$ T. Inset: $|I|$ vs $V_{ds}$ for the data in (a). (b) The energy of peaks in $dI/dV_{ds}$ as a function of $V_g$. The $V_{ds}$ of a peak is converted to an energy via the factor $\beta$ described in the text. The lines represent the single-particle excitation spectrum of the island as a function of $V_g$. Note that the Coulomb gap decreases with $V_g$. 

$G = (e^2/h)(1/4kT)A \cosh^{-2}[(e\alpha V_g - E_{res})/2kT]$.
where $E_{\text{res}}$ is the energy of the transmission resonance and $A$ is the temperature-independent energy-integrated strength of the resonance. The factor $\alpha$, in Eq. (1), relates a change in $V_g$ to a shift in the energy of the island; $\alpha = U/e\Delta V_g$ where $\Delta V_g$ is the spacing between peaks.\textsuperscript{18} From the fit we obtain $\alpha = 0.52$ and hence $U = 0.61$ meV. Figure 3(c) shows a peak at higher $V_g$, at which the island is more strongly coupled to its leads. A striking feature of this peak is that its tails do not fall off exponentially. Clearly, this line shape cannot be fit by the expression in Eq. (1); the intrinsic line shape of the resonance is influencing the conductance peak profile. We have fit the data in Fig. 3(c) to a thermally broadened Lorentzian parametrized by a full width at half-maximum $\Gamma$,

\[
G = \frac{e^2}{h} \frac{1}{4kT} A \int_{-\infty}^{+\infty} \cosh^{-1}(E/2kT) \times \frac{(\Gamma/2)\pi}{(\Gamma/2)^2 + [(e\alpha V_g - E_{\text{res}} - E)^2]} dE,
\]

(2)

which describes resonant tunneling in noninteracting systems. The fit yields $\Gamma = 5.0 \mu$eV and $\alpha = 0.30$ giving $U = 0.35$ meV. As seen in Fig. 4(a) peaks often exhibit asymmetries and other features which underscore the limitations of broadly applying the Lorentzian line shape.\textsuperscript{11} Nevertheless, reasonable fits can be made with this line shape for many of the peaks observed. The success of the fit in Fig. 3(c) strongly indicates that transmission resonances in a Coulomb island have Lorentzian intrinsic line shapes despite the presence of interactions. In zero magnetic field, similar results are seen, though in general a larger percentage of peaks show deviations from the Lorentzian line shape.

As a final consideration, notice the different values of $U$ determined from the fits in Figs. 3(b) and 3(c), 0.61 and 0.35 meV, respectively. In order to understand this behavior we have fit a series of peaks, shown in Fig. 4(a), with the line shape of Eq. (2) and have extracted values of $U$ and $\Gamma$. Figure 4(b) shows that $\Gamma$ increases exponentially with $V_g$, consistent with tunneling through a saddle-point potential in a magnetic field.\textsuperscript{19} Figure 4(c) shows that $U$ (open triangles) diminishes with $V_g$. Within the context of the Coulomb-blockade model this implies that the total capacitance of the island, $C = e^2/U$, increases with $V_g$. We test this proposition by assuming that $C$ is the sum of four capacitances, the island-gate capacitance $C_{g}$, the island-electrode capacitance $C_{e}$, and the island-lead capacitances to both the left and right leads $C_{l}$ and
$C_r$, respectively. $C_g$ and $C_e$ are measured directly from $\Delta V_g$, the spacing between low-bias peaks.\textsuperscript{18,20} $C_l$ and $C_r$ are determined from the slope of the peak positions versus gate voltage, $dV_{ds}/dV_g$ in measurements similar to those in Fig. 2(b). The Coulomb-blockade model gives

$$dV_{ds}/dV_g = C_g/(C - C_{bias}),$$

where $C_{bias}$ is the capacitance of the island to the biased lead.\textsuperscript{1}

The above procedure gives $C_g = 0.124$ fF, $C_e = 0.025$ fF, and $C_l = 0.045 \pm 0.005$ fF independent of $V_g$. Surprisingly, however, $C_l$ increases from $\sim 0.045$ to $> 0.300$ fF over the range of $V_g$ in Fig. 4. $U$ thus diminishes due to the increasing island-lead capacitance.\textsuperscript{21} Figure 4(d) shows that $C_r^{-1}$ goes to zero linearly in $V_g$. $U$, determined from the sum of $C_g$, $C_e$, $C_l$, and $C_r$, is plotted as the filled circles in Fig. 4(c). The agreement between this measure of $U$ and that determined from the fits confirms the accuracy of the above procedure for measuring the capacitances to the island.

In summary, the effects of quantized energy levels on transport through a Coulomb island have been characterized. Large-bias measurements reveal the quantized excitation spectrum of these levels, and low-bias measurements probe their intrinsic line shapes. In addition, a decrease in the charging energy of the island at larger island-lead couplings is consistent with an observed increase in the island-lead capacitance.

We recently received a copy of work by A. T. Johnson et al. of the Delft University of Technology which reports measurements similar to our results in Figs. 1(a) and 2(a).

We wish to thank A. Kumar, D. Chklovskii, L. I. Glazman, and P. A. Lee for useful discussions. This work was supported by NSF Grant No. ECS-8813250 and by the U.S. Joint Services Electronics Program under Contract No. DAALL03-89-C-001.

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\textsuperscript{1}Present address: Department of Physics, University of California at Berkeley, Berkeley, CA 94720.

\textsuperscript{2}Present address: Department of Nuclear Physics, Weizmann Institute of Science, Rehovot 76100, Israel.

\textsuperscript{3}Present address: NEC Research Institute, 4 Independence Way, Princeton, NJ 08540.

\textsuperscript{4}Present address: Department of Physics, University of California at Santa Barbara, Santa Barbara, CA 93106.


\textsuperscript{11}Bo Su, V. J. Goldman, and J. E. Cunningham, Science 255, 313 (1992).


\textsuperscript{15}E. B. Foxman et al. (unpublished).

\textsuperscript{16}In the device discussed here the modulated overstructure was not observed for fields $B \gtrsim 4$ T. It is not yet known if this is a general feature of these systems.

\textsuperscript{17}The division of the energy scales into separate Coulomb and single-particle pieces is an oversimplification. (See Ref. 17.) Nevertheless, the division is very illustrative and will be used exclusively in our discussion.

\textsuperscript{18}In principle, peaks may also result from levels being brought into alignment with the unbiased lead due to the finite island-to-biased-lead capacitance. However, given the relatively small size of this capacitance such peaks occur rarely in Fig. 3.

\textsuperscript{19}According to the Coulomb-blockade model $B = -(1-C_{bias}/C)$, where $C_{bias}$ and $C$ are as defined later in the text.

\textsuperscript{20}The results shown in Figs. 1 and 2 are from measurements on two different devices. This accounts for the different values of $U$ and $\Delta e$ in Figs. 1 and 2.


\textsuperscript{22}A small contribution to the spacing between peaks ($\sim 10\%$) is due to the single-particle energy to add an electron to the island. (See Ref. 5.) Therefore, the minimum observed spacing between peaks is used for $\Delta V_g$.


\textsuperscript{24}In a different study, a similar procedure was carried out except that the equivalents of $C_l$ and $C_r$ were assumed to be zero [L. P. Kouwenhoven et al., Z. Phys. B 85, 367 (1991)].

\textsuperscript{25}The island-lead capacitances probably account for the missing stray capacitance discussed in Refs. 8 and 10.