Quantum Dots

vertical dot

MBE grown

lateral

metal grain

self assembled

metallic SET

GaAs/InAs QDs

$\text{Al}_{0.7}\text{Ga}_{0.3}\text{As}$

GaAs

nanotube
lateral vs. vertical

a lateral quantum dot

b vertical quantum dot

an electrical engineers point of view
Lateral Dots: Formed in GaAs/AlGaAs 2DEG

Electrons travel in sub-surface layer:

CONTACT  GATE  CONTACT

GaAs

Fermi energy

Surface

Only lowest sub-band occupied at low T

Negative voltage on gates depletes underlying electrons & defines dot cavity

GaAs

Al_{0.3}Ga_{0.7}As
A, B, C : control quantum point contacts
transmission to reservoirs

1, 2, 3: control confinement potential / energy levels only

X control dot-internal tunneling rate
Quantum Point Contact Leads

Open vs. Closed

Open Dot

- $V_{\text{gate}}$ set to allow $\geq 2e^2/h$ conductance through each point contact
- Dot is well-connected to reservoirs
- Transport measurements exhibit CF and Weak Localization

Closed Dot

- $V_{\text{gate}}$ set to require tunnelling across point contacts
- Dot is isolated from reservoirs, contains discrete energy levels
- Transport measurements exhibit Coulomb Blockade
Finite energy $E_c = e^2/C_{dot}$ is needed to add an additional electron to the dot. When $kT \ll E_c$, charging blocks conduction in valleys.

Coulomb blockade peaks: resonant transport through dot levels

![Diagram of Coulomb blockade peaks with dot level transitions](image)
**Electrostatic Energy**

apply voltages

what is potential on dot?

voltage divider...

can use $V_g$ to shift dot energy!!

$$C_\Sigma = C_{sd} + C_{dd} + C_{g1} + C_{g2} + \ldots$$

$$V_{dot} = \sum_i \alpha_i V_i$$

$$\alpha_i = \frac{C_i}{C_\Sigma}$$
Charging Energy

capacitance of dot to world  =  \( C \)

energy stored in capacitor

charging energy  \( E_C = \frac{e^2}{C_{\Sigma}} \)

can range from \(~0\) to many meV

\[ C = \epsilon_0 \epsilon \frac{A}{d} \]

\[ U = \int_0^Q V \, dq = \int_0^Q \frac{q}{C'} \, dq = \frac{1}{2} \frac{Q^2}{C} \]

\[ C_{\Sigma} \gtrsim 10 \text{ aF} \]

Classical Effect, NOT quantum
Confinement Energy

harmonic potential

\[ E_n = \left[ n + \frac{1}{2} \right] \hbar \omega \]

\( \mu eV \) to \( meV \)

quantum mechanical effect!!
Capacitor Model

\[ E(N) = \frac{[Q_{tot}]^2}{2C_\Sigma} + \sum_{k=1}^{N} \epsilon_k \]

total dot energy

\[ E(N) = \left[ e(N - N_0) - \sum_{k=1}^{N} C_k V_k \right]^2 \frac{1}{2C_\Sigma} + \sum_{k=1}^{N} \epsilon_k \]

offset charge
Constant Interaction Model

\[
E_i = \sum_{k=1}^{N} q_k \phi_k
\]

\[
q_k = -e
\]

\[
\phi_k : \text{interaction of electron } k \text{ with rest}
\]

constant interaction: model \( \phi_k \) with \( C_{\Sigma} \)

\[
\phi_k = -(k - 1)e/C_{\Sigma}
\]

\[
E_i = \frac{e^2}{C_{\Sigma}} \sum_{k=1}^{N} (k - 1)
\]

\[
= N(N - 1)e^2
\]

\[
= \frac{N(N - 1)e^2}{2C_{\Sigma}}
\]

\[
E(N) = E_{QM} + E_i + E_e \quad \text{total dot energy}
\]

\[
= \sum_{n=1}^{N} \epsilon_n + \frac{N(N - 1)e^2}{2C_{\Sigma}} - Ne \sum_{i=1}^{6} \alpha_i V_i
\]
Chemical Potential / Addition Energy

\[ \mu_{\text{dot}}(N) \equiv E(N) - E(N - 1) \]

energy to add one more electron

\[ \mu = 0: \text{change } N \text{ current flows} \]

constant interaction model:

\[ \mu_{\text{dot}}(N) = \epsilon_N + (N - 1) \frac{e^2}{C} - e \sum_i \alpha_i V_i \]

addition energy

\[ (\mu_{\text{dot}}(N + 1) - \mu_{\text{dot}}(N)) \big|_{\text{fixed } V_i} = \epsilon_{N+1} - \epsilon_N + \frac{e^2}{C \Sigma} \]

\[ \equiv \Delta \epsilon_{N \rightarrow N+1} + U \]
Quantum Coulomb Blockade

For $kT < \Delta$, each peak describes tunnelling into a single eigenstate. Wavefunction amplitude fluctuations lead to peak height fluctuations.

$kT \sim 3\mu eV$
$\Delta \sim 10\mu eV$
$E_c \sim 200\mu eV$

slide from J. Folk (2002)
Coulomb Diamonds

\[ kT \sim 1.5 \mu\text{eV} \]
\[ \Delta \sim 1000 \mu\text{eV} \]
\[ E_c \sim 2000 \mu\text{eV} \]

Differential conductance: peaks when current through dot is changing
Coulomb Diamonds

peaks in $g$ appear when dot level aligned with either source or drain chemical potential.
Coulomb Diamonds, Sequential Tunneling Transport

Electrons tunnel on/off dot one by one (charging energy)

Electrons do not change energy when tunneling
Dot energy unchanged
Coulomb Diamonds

two slopes, each associated with its respective dot-lead capacitance
Excited State Spectroscopy: Sequential Transport

Only one excess electron can be on dot (charging energy)

Lab to investigate quantum levels in device!!

Quantum confinement energies

Internal excitations (spin)
Cotunneling Transport

Higher order process: two electrons tunnel and change energy

**inelastic cotunneling** (white lines)
dot energy changes only possible for $V_{SD} > \Delta$

**elastic cotunneling** (blue circle)
dot energy unchanged

dot now in excited state
Temperature Regimes

\[ \Delta, \frac{e^2}{C} \ll kT \]

no charging effects, no Coulomb blockade

\[ g_\infty = \left( \frac{1}{g_L} + \frac{1}{g_R} \right)^{-1} \]

\[ \Gamma, \Delta \ll kT \ll \frac{e^2}{C} \]

classical Coulomb blockade (metallic CB)

temperature broadened

transport through several quantum dot energy levels

\[ g \sim \frac{g_\infty}{2} \cosh^{-2} \left( \frac{\epsilon}{2.5kT} \right) \]

peak conductance independent of T

FWHM \sim 4.35kT

\[ \Gamma = \Gamma_L + \Gamma_R \]

escape broadening (tunneling rates)
Temperature Regimes

quantum Coulomb blockade

temperature broadended regime

resonant tunneling

transport through only one dot level

\[ g \sim \frac{e^2}{h} \frac{\gamma}{4kT} \cosh^{-2} \left( \frac{\epsilon}{2kT} \right) \]

peak conductance \( \frac{1}{T} \)

FWHM \( \sim 3.5kT \)

\[ \gamma = \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R} \]

quantum Coulomb blockade

lifetime broadended regime

transport through only one dot level

\[ GBW \sim \frac{e^2}{h} \frac{\gamma \Gamma}{(\epsilon/\hbar)^2 + (\Gamma/2)^2} \]

peak conductance \( e^2/h \) indep. of \( T \)

FWHM \( \sim \Gamma \)
Temperature Dependence: Theory

\[ \Delta = 0.01 \text{ e}^2/\text{C} \]
\[ kT / \text{e}^2\text{C} \]
\[ a \ 0.075 \]
\[ b \ 0.15 \]
\[ c \ 0.3 \]
\[ d \ 0.4 \]
\[ e \ 1 \]
\[ f \ 2 \]

van Houten, Beenakker & Staring, NATO ASI Series
Temperature Dependence: Experiment

- Crossover 3.5 to 4.3kT peak width
- Peak g
- 1/T dependence: quantum regime
- T independent: classical regime

Foxman et al., PRB50, 14193 (1994)
Line Shapes: Experiments

Foxman et al., PRB47, 10020 (1993)
Charge Switching / Telegraph Noise

Elzermann, 2003