1. Open Dot Experiments

2. Kondo effect

3. Few Electron Dots

4. Double Quantum Dots

van der Wiel et al., RMP75, 1 (2003)
Double Quantum Dots

mutual charging energy

\[ E_m = \frac{e^2}{C_m} \left( \frac{C_1 C_2}{C_m^2} - 1 \right)^{-1} \]

interdot tunneling \( t \)

\[ G_m = 4\pi \frac{e^2}{\hbar} \left( \frac{t}{\Delta} \right)^2 \]

\( t < \Delta \) well localized electrons

individual charging energies

\[ E_{c1(2)} = \frac{e^2}{C_{1(2)}^*} \left( 1 - \frac{C_m^2}{C_1 C_2} \right)^{-1} \]
Double Quantum Dots: Quadruple Points

$E_m = \frac{e^2}{C_m \left( \frac{C_1 C_2}{C_m^2} - 1 \right)^{-1}} \rightarrow 0$

costs zero energy to add a 2nd electron to other dot if one electron is already present

$E_{C1(2)} = \frac{e^2}{C_{1(2)}}$  individual charging energies

assume well localized electrons (weak tunneling, but large enough to measure a current)

- quadruple points
degeneracy of four charge states
Double Quantum Dots: Triple Points and Honeycombs

\[ 0 < C_m < C_{1,2} \quad 0 < E_m < E_{C_1,C_2} \]

(1,1) – (0,0) degeneracy lifted

quadruple points split into two triple points

hole like process

electron like process
Double Quantum Dots: Single Dot Limit

\[ 0 < C_m \sim C_{1,2} \]

\[ E_m \sim E_{C_{1,2}} \]

double dot behaves like a single dot with two plunger gates
Double Quantum Dots
Double Dot Experiment

van der Wiel et al., RMP75, 1 (2003)
Double Dot Hamiltonian

\[ H_{DDQD} = \frac{E_{c1}}{2} N (N - 1) - \frac{N E_{c1} + M E_m}{e} (C_{g1} V_{g1} + C_s V_s) + \sum_{i,\sigma} N_{i\sigma} \epsilon_{i\sigma} \]

\[ + \frac{E_{c2}}{2} M (M - 1) - \frac{M E_{c2} + N E_m}{e} (C_{g2} V_{g2} + C_d V_d) + \sum_{j,\sigma} M_{j\sigma} \epsilon_{j\sigma} \]

\[ + E_m N M + \sum_{i,j,\sigma} t_{ij\sigma} (c_{i\sigma}^+ e_{j\sigma} + h.c.) + \text{lead tunneling} \] (3.11)

\[ \text{mutual charging} \quad \text{inter-dot tunneling} \]

\[ C_s R_s \quad N \quad C_m R_m \quad M \quad C_d R_d \]

\[ C_{g1} \quad QD1 \quad C_{g2} \quad QD2 \]

\[ V_s \quad V_{g1} \quad V_{g2} \quad V_s \]

electrons well localized

\[ G_m < e^2/h \]
Double Dot Capacitances in the Honeycombs

\[ \Delta V_{g1} = \frac{|e|}{C_{g1}} \]

\[ \Delta V_{g2} = \frac{|e|}{C_{g2}} \]

\[ \Delta V^m_{g1} = \frac{|e| C_m}{C_{g1} C_2} = \Delta V_{g1} \frac{C_m}{C_2} \]

\[ \Delta V^m_{g2} = \frac{|e| C_m}{C_{g2} C_1} = \Delta V_{g2} \frac{C_m}{C_1} \]

\( \Delta V \): detuning controls energy difference \( \Delta \) between the dot levels keeping constant the total dot occupation \( N + M \).
Double Dot Transport

triple points: sequential tunneling

honey comb lines: cotunneling
Double Dot Experiment

finite bias: nonlinear transport

van der Wiel et al., RMP75, 1 (2003)
Double Dot at finite bias: Excited State Spectroscopy

 triple points expand into triangles obeying

\[ 0 \leq \mu_1 \leq \mu_2 \leq eV \]
Double Dot Experiment: Finite Bias

van der Wiel et al., RMP75, 1 (2003)
Interdot Tunneling: Anticrossing

\[ \mathbf{H}_0 | \phi_1 \rangle = E_1 | \phi_1 \rangle \]
\[ \mathbf{H}_0 | \phi_2 \rangle = E_2 | \phi_2 \rangle \]
\[ \mathbf{T} = \begin{pmatrix} 0 & t_{12} \\ t_{21} & 0 \end{pmatrix}, \quad t_{12} = t_{21}^*, \quad t_{21} = |t_{21}| e^{i \varphi} \]
\[ \mathbf{H} = \mathbf{H}_0 + \mathbf{T} \]