Quantum Dots

vertical dot

MBE grown

lateral

metal grain

self assembled

metallic SET

nanotube
lateral vs. vertical

an electrical engineers point of view
Lateral Dots: Formed in GaAs/AlGaAs 2DEG

Electrons travel in sub-surface layer:

- CONTACT
- GATE
- CONTACT

GaAs

Fermi energy

Surface

Al$_{0.3}$Ga$_{0.7}$As

GaAs

Only lowest sub-band occupied at low T

Negative voltage on gates depletes underlying electrons & defines dot cavity

I (pA)
A, B, C : control quantum point contacts transmission to reservoirs

1, 2, 3: control confinement potential / energy levels only

X control dot-internal tunneling rate
Quantum Point Contact Leads

Not all QPCs are perfect:

slide from A. Huibers, Thesis (1999)
Open vs. Closed

Open Dot

- $V_{\text{gate}}$ set to allow $\geq 2e^2/h$ conductance through each point contact
- Dot is well-connected to reservoirs
- Transport measurements exhibit CF and Weak Localization

Closed Dot

- $V_{\text{gate}}$ set to require tunnelling across point contacts
- Dot is isolated from reservoirs, contains discrete energy levels
- Transport measurements exhibit Coulomb Blockade
Finite energy $E_c = \frac{e^2}{C_{dot}}$ is needed to add an additional electron to the dot. When $kT \ll E_c$ charging blocks conduction in valleys.

Coulomb blockade peaks: resonant transport through dot levels

$1 \mu m^2$
Electrostatic Energy

apply voltages

what is potential on dot?

voltage divider...

\[ C_\Sigma = C_{sd} + C_{dd} + C_{g1} + C_{g2} + \ldots \]

\[ V_{dot} = \sum_i \alpha_i V_i \]

can use \( V_g \) to shift dot energy!!
Charging Energy

capacitance of dot to world = $C$

energy stored in capacitor

charging energy

$E_C = \frac{e^2}{C \Sigma}$

can range from ~0 to many meV

$C \Sigma \gtrsim 10 \text{ aF}$

Classical Effect, NOT quantum
Confinement Energy

harmonic potential

\[ E_n = \left( n + \frac{1}{2} \right) \hbar \omega \]

\( \mu eV \text{ to } meV \)

quantum mechanical effect!!

complicated potential

\[ \Delta = \frac{2\pi \hbar^2}{m^* A} \]
Capacitor Model

\[ E(N) = \frac{\left[ Q_{tot} \right]^2}{(2C_\Sigma)} + \sum_{k=1}^{N} \epsilon_k \]

total dot energy

\[ E(N) = \left[ e(N - N_0) - \sum_{k=1}^{N} C_k V_k \right]^2 \left/ (2C_\Sigma) \right. + \sum_{k=1}^{N} \epsilon_k \]

offset charge
Constant Interaction Model

\[ E_i = \sum_{k=1}^{N} q_k \phi_k \]

\[ q_k = -e \]

\[ \phi_k : \text{interaction of electron } k \text{ with rest constant interaction: model } \phi_k \text{ with } C_\Sigma \]

\[ \phi_k = -(k - 1)e/C_\Sigma \]

\[ E_i = \frac{e^2}{C_\Sigma} \sum_{k=1}^{N} (k - 1) \]

\[ = \frac{N(N - 1)e^2}{2C_\Sigma} \]

\[ E(N) = E_{QM} + E_i + E_e \quad \text{total dot energy} \]

\[ = \sum_{n=1}^{N} \epsilon_n + \frac{N(N - 1)e^2}{2C_\Sigma} - Ne \sum_{i=1}^{6} \alpha_i V_i \]
Chemical Potential / Addition Energy

$$\mu_{\text{dot}}(N) \equiv E(N) - E(N - 1)$$

energy to add one more electron

$$\mu = 0: \text{change } N \text{ current flows}$$

constant interaction model:

$$\mu_{\text{dot}}(N) = \epsilon_N + (N - 1) \frac{e^2}{C} - e \sum_i \alpha_i V_i$$

addition energy

$$\left( \mu_{\text{dot}}(N + 1) - \mu_{\text{dot}}(N) \right) \bigg|_{\text{fixed } V_i} = \epsilon_{N+1} - \epsilon_N + \frac{e^2}{C \Sigma}$$

$$\equiv \Delta \epsilon_{N \to N+1} + U$$
For \( kT < \Delta \), each peak describes tunnelling into a single eigenstate. Wavefunction amplitude fluctuations lead to peak height fluctuations.

From J. Folk (2002)
Coulomb Diamonds

$kT \sim 1.5 \mu eV$

$\Delta \sim 1000 \mu eV$

$E_c \sim 2000 \mu eV$

differential conductance: peaks when current through dot is changing

g = \frac{dI}{dV}$

$\left(\text{e}^2/\text{n}\right)$

$d$
Coulomb Diamonds

peaks in $g$ appear when dot level aligned with either source or drain chemical potential.
Coulomb Diamonds, Sequential Tunneling Transport

Electrons tunnel on/off dot one by one (charging energy)

Electrons do not change energy when tunneling
dot energy unchanged
Coulomb Diamonds

two slopes, each associated with its respective dot-lead capacitance
Excited State Spectroscopy: Sequential Transport

lab to investigate quantum levels in device!!

quantum confinement energies
internal excitations (spin)

only one excess electron can be on dot (charging energy)
Cotunneling Transport

higher order process: two electrons tunnel and change energy

elastic cotunneling (blue circle)

inelastic cotunneling (white lines) dot energy changes only possible for $V_{SD} > \Delta$

before tunneling

dot energy unchanged

after

dot now in excited state
Temperature Regimes

\[ \Delta, \frac{e^2}{C} \ll kT \]

no charging effects, no Coulomb blockade

\[ g_\infty = \left( \frac{1}{g_L} + \frac{1}{g_R} \right)^{-1} \]

\[ \Gamma, \Delta \ll kT \ll \frac{e^2}{C} \]

classical Coulomb blockade (metallic CB)
temperature broadened
transport through several quantum dot energy levels

\[ g \sim \frac{g_\infty}{2} \cosh^{-2} \left( \frac{\epsilon}{2.5kT} \right) \]

peak conductance independent of T
FWHM \~ 4.35kT

\[ \Gamma = \Gamma_L + \Gamma_R \]

escape broadening (tunneling rates)
Temperature Regimes

\[ \Gamma \ll kT \ll \Delta \ll \frac{e^2}{C} \]

**quantum Coulomb blockade**

temperature broadended regime

**resonant tunneling**

transport through only one dot level

\[ g \sim \frac{e^2}{\hbar} \frac{\gamma}{4kT} \cosh^{-2} \left( \frac{\epsilon}{2kT} \right) \]

peak conductance \( 1/T \)

FWHM \( \sim 3.5kT \)

\[ \gamma = \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R} \]

\[ \Gamma_T \ll \Gamma, \Delta \ll \frac{e^2}{C} \]

**quantum Coulomb blockade**

lifetime broadended regime

transport through only one dot level

\[ g_{BW} \sim \frac{e^2}{\hbar} \frac{\gamma \Gamma}{(\epsilon/\hbar)^2 + (\Gamma/2)^2} \]

peak conductance \( e^2/\hbar \) indep. of \( T \)

FWHM \( \sim \Gamma \)
Temperature Dependence: Theory

\( \Delta = \frac{0.01 \, e^2}{C} \)

\( kT / e^2C \)

\( a \ 0.075 \)

\( b \ 0.15 \)

\( c \ 0.3 \)

\( d \ 0.4 \)

\( e \ 1 \)

\( f \ 2 \)

\( \Delta E = 0.01 \, e^2 / C \)

van Houten, Beenakker & Staring, NATO ASI Series
Temperature Dependence: Experiment

crossover 3.5 to 4.3kT peak width

peak g

1/T dependence: quantum regime

T independent: classical regime

Foxman et al., PRB50, 14193 (1994)
T-broadened lifetime broadened

Foxman et al., PRB47, 10020 (1993)
Charge Switching / Telegraph Noise

Elzermann, 2003