Antilocalization of Coulomb Blockade in a Ge/Si Nanowire

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Motivation

• Spin Qubits
  – Electric field to magnetic field conversion (EDSR)

• Majorana Physics, proposed ingredients
  – (strong) Spin-Orbit Coupling
  – Superconductivity

Mourik et al. Science, 2012
Antilocalization Introduction

- Coherent backscattering → quantum interference effects
- Absence of Spin-Orbit → Weak localization, lifted by B-Field
- Strong Spin-Orbit → Antilocalization, lifted by B-Field
- Relevant length scales: $l, l_{SO} < l_\varphi$

Antilocalization in low D

1D: $l_{SO} \sim 20$ nm

Hao et al. Nanoletters, 2010

Zumbühl et al. PRL, 2002

$\epsilon_{SO} > \Delta$, smaller for bigger dots

Antilocalization has been observed in Ge/Si Nanowires, and in 0D, why bother?

→ Effects of Coulomb Blockade!
Coulomb Blockade

\[ \Gamma \ll kT \ll \Delta \ll \frac{e^2}{C} \]

\[ g \sim \frac{e^2}{h} \frac{\gamma}{4kT} \cosh^{-2} \left( \frac{\epsilon}{2kT} \right) \]

\[ g_p \sim \frac{1}{kT} \]

FWHM of peak \( \sim \) 3.5kT

\[ \gamma = \frac{\Gamma L \Gamma R}{\Gamma L + \Gamma R} \]

different for different levels!

Folk et al. PRL, 1996

Van Houten et al. Single Charge Tunneling, 1992
Coulomb Peak Statistics

- $g_{\text{max}}$ depends on coupling of the dot to the leads for every level.

- Random Matrix Theory (RMT) can describe the distribution of peak heights based on symmetries of wavefunctions.

$g_{\text{max}} = \frac{e^2}{h} \frac{\pi}{2kT} \frac{\Gamma_l \Gamma_r}{\Gamma_l + \Gamma_r} \equiv \frac{e^2}{h} \frac{\pi \bar{\Gamma}}{2kT} \alpha,$

$P_{(B=0)}(\alpha) = \sqrt{\frac{2}{\pi \alpha}} e^{-2\alpha},$

$P_{(B\neq0)}(\alpha) = 4\alpha [K_0(2\alpha) + K_1(2\alpha)] e^{-2\alpha}.$

Increased mean height of peaks at high field

Folk et al. PRL, 1996
RMT of strong SO Coupling

\[ \beta = 0: \text{No SO, time-reversal symmetry} \]
\[ \beta = 1: \text{time-reversal symmetry broken} \]
\[ \beta = 2: \text{broken spin rotation symmetry} \]

Introduce additional parameters because of Zeeman splitting and SO:

- \( s \) for Kramers degeneracy; \( s = 1 \) or \( s = 2 \)
- \( \Sigma \) for mixing of spin levels; \( \Sigma = 1 \) or \( \Sigma = 2 \)

\[ g_p = \frac{2e^2}{\hbar} \frac{\chi_s}{kT} \frac{\Gamma_l \Gamma_r}{\Gamma_l + \Gamma_r} = \frac{e^2}{\hbar} \frac{\tilde{\Gamma}}{2kT} \chi_s \alpha \]

\[ \chi_{s=2} = 3 - 2\sqrt{2} \quad \chi_{s=1} = 1/8 \]

For weak SO, \( B = 0 \):

\[ P_{\beta=1,\Sigma=1,s=2}(\alpha) = \sqrt{\frac{1}{\pi \alpha}} e^{-\alpha}, \quad \tilde{\alpha} = 1/2 \]

For strong SO, \( B = 0 \):

\[ P_{\beta=4,\Sigma=1,s=2}(\alpha) = 16\alpha^3 e^{-2\alpha} \left[ K_0(2\alpha) + \left(1 + \frac{1}{4\alpha}\right) K_1(2\alpha) \right], \quad \tilde{\alpha} = 4/5 \]

Equal to high field of weak SO

\[ P_{\beta=2,\Sigma=2,s=1}(\alpha) = P_{\beta=4,\Sigma=1,s=2}(\alpha) \quad \rightarrow \text{just scaling of strong SO case at high field} \]

Ahmadian et al. PRB, 2006
Device: Ge/Si Nanowire

- Core/Shell nanowire: 10 nm Ge core, 2 nm Si shell. \( \rightarrow \) hole gas
- Pre-patterned bottom gates
- Contacts after deposition of wire
- \( T \sim 100 \text{ mK}, V_{AC} \sim 100 \mu \text{V} \)
- Operable in both open \( (N_H \sim 1700) \) and blockated regime \( (N_H \sim 600) \)
- Elastic scattering length: 35-50 nm
- \( \mu \sim 800 \text{ cm}^2/\text{Vs} \); 4-6 modes occupied

Lu et al. PNAS, 2005
Temperature dependence

- Simple model: At $T\sim\Delta$ transport trough multiple levels:
  → suppression of fluctuations

- Deviation at high $T$ from theory due to correlations of the 50 neighbouring peaks?
Both the decreasing average peak height with B-field and distribution at B=0 are signatures of strong spin-orbit coupling.

Scaling factor between B=0T and B=6T distributions is larger than expected.
Open regime

- Magnetoconductance in open regime shows antilocalization
- Depending on the boundary scattering (diffusive or specular) fits can change.
- $l_e < 10 \mu m$
- $l_\varphi = 0.2-1.2 \mu m$
- $l_{SO} < 20 \text{ nm}$
- From dot AL

$$\epsilon_{SO} > \Delta \quad \epsilon_{SO} = \hbar^2 / (2m^* l_{SO}^2)$$

$$l_{SO} < 25 \text{ nm}$$
Conclusion

- Coulomb Peak height distribution and field dependence consistent with strong SO coupling in Ge/Si nanowires
- Magnitude of antilocalization larger than expected
- Results confirmed in the open regime
- Spin-Orbit length on the order of tens of nanometer