Adiabatic Cooling with Non-Abelian Anyons

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Thermopower of two-dimensional electrons at filling factors $\nu=3/2$ and $5/2$

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\textbf{FMM}

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intro

• interested in non-abelian anyons as intrinsically fault-tolerant systems for quantum information storage & processing
  – local fluctuations (e.g., precise location of quasiparticles) do not affect the topology of a particular quantum state

• goals for today
  – present short theory paper proposing a technique we can use to demonstrate non-abelian character of quasiparticles in the $\nu = 5/2$ state
  – review a related experiment that shows promise for investigating the $\nu = 5/2$ state ‘in the bulk’, in order to complement electron transport studies (‘at the edge’)

anyons

• exchanging two QM particles produces a phase factor in the wave function
  – boson: exp[i2πn] = 1
  – fermion: exp[iπ(2n+1)] = -1
  – anyon: exp[iθ] = anything because θ = anything

• anyons only occur in 2d (or not? PRL 104, 046401 (2010))
  – topology matters…2d systems with singularities are not simply connected
  – in the FQHE picture, particles encircling one another pick up a phase that depends on their charge and magnetic flux
abelian/non-abelian statistics

• abelian: particle exchanges commute
  \[ X_{12}X_{23}|abc> – X_{23}X_{12}|abc> = X_{12}|acb> – X_{23}|bac> \]
  \[ = \exp[i\theta_{23}]X_{12}|abc> – \exp[i\theta_{12}]X_{23}|abc> \]
  \[ = \exp[i\theta_{12}+i\theta_{23}]|abc> - \exp[i\theta_{12}+i\theta_{23}]|abc> = 0 \]

• non-abelian: particle exchanges don’t commute
  – degenerate set of states, represented by a vector, with particles at fixed positions
  – exchanges represented by matrices, which don’t commute
  \[
  \begin{pmatrix}
  0 & 1 & 0 \\
  1 & 0 & 0 \\
  0 & 0 & 1
  \end{pmatrix}
  \begin{pmatrix}
  a \\
  b \\
  c
  \end{pmatrix}
  \neq
  \begin{pmatrix}
  1 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & 1
  \end{pmatrix}
  \begin{pmatrix}
  a \\
  b \\
  c
  \end{pmatrix}
  \]

arXiv:0707.1889v2
the theory paper

• simple concept:
  – note: non-abelian quasiparticles, if present in the $\nu = 5/2$ state, produce a temperature-independent entropy (due to ground state degeneracy)

$$S_{tot} = S_D + S_n(T) = k_B \ln[D] + S_n(T) \sim N_q k_B \ln[d] + S_n(T)$$

  – step 1: isolate system
  – step 2: change $S_D$ by adiabatically tuning some parameter ($N_q$)
  – observation: $S_{tot}$ cannot change, so $\Delta S_D = -\Delta S_n(T) \Rightarrow T$ changes!

• tune $N_q$ with magnetic field or electron density

$$S_D = (e/e^*)(\Delta B/B_0)N_e k_B \ln[d]$$
the theory paper

• isolate system?
  – cooling 2DEGs in GaAs heterostructures is difficult below 50mK due to weak e-p coupling (and freezing out of phonons)
  – additionally, one can use a SC heat switch as part of the electrical leads

• tune $N_q$?
  – changing $B \rightarrow$ possible eddy current heating
  – perhaps changing $n_e$ is more promising

• significant change in $T$? depends on entropies
  – large $S_D$…$D$ only large for $T > T_0$
  – small $S_n(T)$…wigner crystal formed at $T < T_m$

\[ S_n(T) \approx \alpha N_e k_B \frac{e}{e^*} \frac{\Delta B}{B_0} \left[ \frac{k_B T}{E_0(\Delta B)} \right]^{4/3} \]

  – $S_n$ might be small at higher temperatures as well…$T < T^*$
the theory paper

\[
\frac{dT}{d\Delta B} = \frac{3T}{4\Delta B} \left\{ 1 - \frac{\ln d}{\alpha} \left[ \frac{E_0(\Delta B)}{k_B T} \right]^{4/3} \right\}
\]

- increase \( \Delta B \), decrease \( T \)

\[
\frac{dT}{d\Delta B} < 0
\]

- cooling power increases with decreasing \( T \)

\[
\frac{dT}{d\Delta B} \propto -T^{-1/3}
\]

- lastly, and most importantly, if quasiparticles obey abelian statistics...

\[
\frac{dT}{d\Delta B} = \frac{3T}{4\Delta B} > 0
\]
the theory paper

- cooling power estimate for sample & parameters in the PRB we will discuss
  - $T_0 \sim 10\text{mK}$, $T_m \sim 11\text{mK}$, $T^* \sim 75\text{mK}$
  - $\dot{Q}_{n-\text{Abelian}} \approx -0.1\text{fW}$, which can be larger than joule heating values(?)
the experimental paper

• measurement of 2DEG thermopower

• sample:
  – $n_e \sim 2.9 \times 10^{11}$ cm$^{-2}$
  – $\mu \sim 31 \times 10^6$ cm$^2$/Vs
  – $L = 12$ mm by $W = 3$ mm
  – 2 mesas, each 3x3mm$^2$

• protocol:
  – measure thermal conductance of wafer (not described today)
  – at different $T$, apply heat pulses (square wave) to establish time-dependent temperature gradient
  – voltage along 2DEG measured with dc nanovoltmeter
  – obtain average $T$ from integrated thermal conductance
  – plot $-S$ versus $T$
the experimental paper

- measurement of 2DEG thermopower
  - this property is closely related to the entropy (they are proportional)
  - states investigated: $B = 0$, $\nu = \frac{3}{2}$, $\nu = \frac{5}{2}$
- results for first two...note, seebeck coefficient:

$$S = -\frac{\Delta V}{\Delta T}$$

$$S^d = -\frac{\pi k_B^2}{3e N \hbar^2} m^*(1 + \alpha) T$$

$$S_{CF}^d = -\frac{\pi k_B^2}{6e N \hbar^2} (1 + \alpha) T$$

![Graphs showing thermopower versus temperature for different states](image1)

- $B = 0$, drude model
- $\nu = \frac{3}{2}$, fermi liquid of CFs
the experimental paper

- lastly, the $\nu = 5/2$ state ($B_0 \sim 4.8T$)
- energy gap at fermi level should strongly suppress the entropy at low $T$, perhaps $S_{5/2} \sim \exp[-\Delta_{5/2}/2T]$?
the experimental paper

- arrhenius plot yields $\Delta_{5/2} \sim 450\text{mK}$ for $R_{xx}$ and $\Delta_{5/2} \sim 370\text{mK}$ for $-S$
the experimental paper

- minimum $T \sim 82\text{mK}$, which is approaching the calculated $T^*$ in the PRL
  - it appears there is already a non-negligible entropy (proportional to y-intercept in graph) from the quasiparticles that is independent of $T$
conclusions

• sizeable entropy from non-abelian anyons predicted in ultra-clean 2DEGS for the \( \nu = 5/2 \) state

• could detect this in either an adiabatic cooling experiment, or via the thermopower
  – relevant T range: between 1 and 100mK

• evidence from thermopower measurements is already available, suggesting a finite \( S_D \)
  – T needs to be reduced in this experiment