T-sensors / T-ctrl / PID
1. different types of T-sensors
   - for different T-ranges
   - with according techniques

2. controlling temperature
   - cybernetic model
   - T-ctrl on MCK-50
sensing temperature

**primary thermometer:**
- precise law for $X=X(T)$
- accurate measure without calibration
- difficult to use

**secondary thermometer:**
- needs calibration with primary sensor or fixed points
- high sensitivity
- convenient to use

**requirements for T-sensor:**
- simple and reliable T-dependence
- fast equillibration (high therm.-conductivity, small heat capacitity)
- reasonable sensitivity
- no self-heating and heating of environment
- insensitive to changes of others than temperature (B-field etc.)
- and all that for the full temperature range
Gas thermometer

\[ PV = nRT \]

primary sensor, but
- tricky for precise measurement of \( T \)
- virial coeff., therm. change of volumes,
  adsorption/desorption in walls
vapour pressure of cryo-liquids

Clausius-Clapeyron:

$$\left[ \frac{dP}{dT} \right]_{\text{vap}} = \frac{S_{\text{gas}} - S_{\text{liq}}}{V_{m,\text{gas}} - V_{m,\text{liq}}}$$

measure vapour pressure with calibrated manometer
thermoelectricity

thermoelectric force induced by a thermal gradient in a metal

\[ S = \frac{\Delta U}{\Delta T} \]  (Seebeck effect)

easy for \( T > 10K \)

vanishing \( V \) at low \( T \)
resistance thermometry

- simplest/most used T-sensor, but:
- residual resistance:
  no T-dependence in pure metals below 5K

- semiconductors:

  ideal: \( R(T) = \alpha \exp\left( \frac{\Delta E}{2k_B T} \right) \)

  realistic \( R(T) \) described by:

  \[ \ln R = \sum_{n=0}^{m} \alpha_n (\ln T)^n \]

  typical: Ge, Carbon, RuO\(_2\), Cernox....
Coulomb blockade thermometer

- primary sensor of electron (!) temperature
- use T-dependence of tunneling conductance

\[ \frac{G(V)}{G_T} = 1 - \left( \frac{E_C}{k_B T} \right) g(eV/Nk_B T) \]

with
\[ g(x) = \frac{(x \sinh x - 4 \sinh^2 x/2)}{(8 \sinh^4 x/2)} \]

\[ (k_B T \approx E_C) \]

1st order in \( E_C / k_B T \):
\[ \Delta G / G_T = E_C / 6k_B T \]

\[ V_{1/2} = 5.439 N k_B T / e \]
Curie-Weiss thermometer

- magnetization/ susceptibility of electronic paramagnets

\[ \chi = \chi_0 + \frac{\lambda}{T - \Delta} \]  
(Curie-Weiss law)

- well suitable for \( T_c < T < 1K \)
- paramagnet with 3d or 4f shell partly filled:

\[ \text{CMN: } 2\text{Ce}^{3+}(\text{NO}_3)_3 \cdot 3\text{Mg(NO}_3)_2 \cdot 24\text{H}_2\text{O} \]
\[ T_c \simeq 0.002 \text{ K} \]

- calibration of Curie- and Weiss-constant necessary for each sensor (secondary sensor)
## summary T-sensors

<table>
<thead>
<tr>
<th>Measured property</th>
<th>Function</th>
<th>Material</th>
<th>Temperature range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical resistance</td>
<td>$\ln R = \sum_{n=0}^{m} \alpha_n (\ln T)^n$</td>
<td>Composites with negative R–T characteristics</td>
<td>$&gt; 20 \text{ mK}$</td>
</tr>
<tr>
<td>Electronic param. suscept.</td>
<td>$\chi_e = \chi_0 + \lambda_e/(T_e - \Delta)$</td>
<td>CMN; $PdFe$</td>
<td>$&gt; 3 \text{ mK}$</td>
</tr>
<tr>
<td>Nuclear paramag. suscept.</td>
<td>$\chi_n = \chi_0 + \lambda_n/T_n$</td>
<td>Pt</td>
<td>$1 \mu K - 0.1 \text{ K}$</td>
</tr>
</tbody>
</table>

### less important methods:

- noise thermometers
- He melting pressure
- capacitance thermometry
- anisotropy of $\gamma$-rays
**Closed Loop Control Circuit**

- Sensor reads the actual output $y$
- This is fed back to the reference $r$
- Controller reads the error $e = r - y$
- Update of input $u$ by the controller

**Typical Ctrl Modes:**

- **Proportional**
  \[ u = K_p e \]

- **Integral**
  \[ u = K_i \int e \, dt \]

- **Differential**
  \[ u = K_d \frac{de}{dt} \]
P and PI circuit

\[ u = K_p e \]

\[ u = K_p e + K_i \int e \, dt \]

- const. offset
- reference value never reached!
- integration gives infinite gain
- no remaining offset to reference!
**PD and PID controller**

\[ u = K_p e + K_d \frac{de}{dt} \]

\[ u = K_p e + K_i \int edt + K_d \frac{de}{dt} \]

- faster, anticipating response
- const. offset

- linear combination of P, I and D
- fast and converging control
tuning the PID

- **Ziegler Nichols method I**

  open-loop response of unit step

  \[ G(s) = \frac{Ke^{-\tau_s}}{\tau + 1} \]

  \[ \text{Slope } R = \frac{K}{\tau} \]

  \[ \text{Time delay } L = \tau_d \]

- **P controller**
  - \( K_p = \frac{1}{RL} \)

- **PI controller**
  - \( K_p = \frac{0.9}{RL}, K_i = \frac{0.27}{RL^2} \)

- **PID controller**
  - \( K_p = \frac{1.2}{RL}, K_i = \frac{0.6}{RL^2}, K_d = \frac{0.6}{R} \)
tuning the PID

- Ziegler Nichols method II
  - Increase a pure gain $K_u$ of a closed-loop system until the system is marginally stable
  - Measure the period of oscillation $P_u$ (unit is second)

- P controller
  - $K_p = 0.5K_u$

- PI controller
  - $K_p = 0.45K_u, K_i = 0.54K_u/P_u$

- PID controller
  - $K_p = 0.6K_u, K_i = 1.2K_u/P_u, K_d = 0.075K_uP_u$
T-ctrl at MCK-50

T-ctrl. for T<4K:

- PID: picowatt controller
- sensor: RuO₂ on CF
- heater: MC- (or CF-)heater
- mostly heating the still also for sufficient cooling power

range: base-temp. – 4K(?)

T-ctrl. for T>4K:

- PID: picowatt controller+ HV-amplifier at output
- sensor: Cernox on CF (calibrated for 4.5< T <RT)
- heater: 1kOhm resistor on CF
- IVC thoroughly pumped at T~20K
- X-gas in MC (~5-10mbar He4)

range: 4K – 60K (tested)