Piezoelectricity: Basics and applications

Friday Morning Meeting, 30.07.2010
Technical Talk
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Overview

-A simple molecular model
-Mathematical modelling
-Some general notes
-Overview Motors
-Slip-stick motion
-Few calculation regarding Slip-stick motion motors
Definition: Piezoelectricity

Piezoelectricity is the ability of some materials to generate an electric charge in response to applied mechanical stress.

The piezoelectric effect is reversible:

- **Direct** piezoelectric effect: charge separation due to stress
- **Converse** piezoelectric effect: occurrence of stress and strain when electric field is applied
A simple molecular model

- Only insulating materials
- Insulating Ferroelectrica and materials with a permanent dipol
- In crystals: only crystals without symmetry centre
  → 20 point groups
A simple molecular model

Without any external stress:
- Centers of charges coincide
- Charges are reciprocally cancelled
- Electrical neutral unit cell

Lecture Notes, Tomasz G. Zielinski, Warsaw, Poland
A simple molecular model

Applied external stress:
- Internal structure is deformed
- Separation of charge centers
- Dipols are generated

small dipole
A simple molecular model

Poles inside material are mutually cancelled

Charge occurs on surface polarization of material
Mathematical modeling

Piezoelectricity is the combination of:

The materials electrical behavior: \( D = \varepsilon E \)

And Hook’s law: \( S = sT \)

D: electric displacement, \( \varepsilon \): permittivity, E: electric field strength
S: strain, s: compliance, T: stress

The coupled strain-voltage equation:

\[
S = s^E T + d^t E \quad \text{converse piezoelectric effect}
\]

\[
D = \varepsilon^T E + dT \quad \text{direct piezoelectric effect}
\]

\[
d_{ij,k} = \frac{\partial S_{ij}}{\partial E_k} \quad \text{piezoelectric coefficient}
\]
Mathematical Modeling

\[
\begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
S_{21} & S_{22} & S_{23} \\
S_{31} & S_{32} & S_{33}
\end{bmatrix}
= s^E_{\alpha\beta\gamma\chi}
\begin{bmatrix}
T_{11} & T_{12} & T_{13} \\
T_{21} & T_{22} & T_{23} \\
T_{31} & T_{32} & T_{33}
\end{bmatrix}
\]

\[S_{\alpha\beta} = \sum_{\chi=1}^{3} \sum_{\gamma=1}^{3} s^E_{\alpha\beta\gamma\chi} T_{\gamma\chi}\]

\(S_{\alpha\beta}\): strain of the \(\beta\)-normal in \(\alpha\)-direction

\(T_{\gamma\chi}\): stress action in \(\gamma\)-direction on plane with \(\chi\)-normal

\[
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix}
= \begin{bmatrix}
\epsilon_{11} & 0 & 0 \\
0 & \epsilon_{22} & 0 \\
0 & 0 & \epsilon_{33}
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\]
Mathematical Modeling

Piezoelectric body

Stress & strain are symmetric tensors:

Voigt Notation

\[ 11 \rightarrow 1; 22 \rightarrow 2; 33 \rightarrow 3; 23 \rightarrow 4; 13 \rightarrow 5; 12 \rightarrow 6 \]

\[
\begin{bmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
S_5 \\
S_6
\end{bmatrix} = \begin{bmatrix}
\sigma_{11}^E & \sigma_{12}^E & \sigma_{13}^E & 0 & 0 & 0 \\
\sigma_{21}^E & \sigma_{22}^E & \sigma_{23}^E & 0 & 0 & 0 \\
\sigma_{31}^E & \sigma_{32}^E & \sigma_{33}^E & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_{44}^E & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_{55}^E & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_{66}^E
\end{bmatrix} = \begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_5 \\
T_6
\end{bmatrix} + \begin{bmatrix}
0 & 0 & d_{31} \\
0 & 0 & d_{32} \\
0 & 0 & d_{33} \\
0 & d_{24} & 0 \\
d_{15} & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\]

\[
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & 0 & d_{15} & 0 \\
0 & 0 & 0 & d_{24} & 0 & 0 \\
d_{31} & d_{32} & d_{33} & 0 & 0 & 0
\end{bmatrix} = \begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_5 \\
T_6
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{11} & 0 & 0 \\
0 & \varepsilon_{22} & 0 \\
0 & 0 & \varepsilon_{33}
\end{bmatrix} \begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\]
An example

A proper voltage is applied over a free standing piezoelectric element to create an electrical field of \( E = (4, 3, 2) \) V/m. The dimensions of the element are \( L = (1, 1, 5) \) mm. The constants are: \( d_{31} = 4 \) pm/V, \( d_{33} = 12 \) pm/V, \( d_{15} = 0 \) pm/V

What is the strain?

\[
\begin{bmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
S_5 \\
S_6
\end{bmatrix} = \begin{bmatrix} 0 & 0 & d_{31} \\
0 & 0 & d_{31} \\
0 & 0 & d_{33} \\
0 & d_{15} & 0 \\
d_{15} & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix} \begin{bmatrix} E_1 \\
E_2 \\
E_3 \\
\end{bmatrix} = \begin{bmatrix} 0 & 0 & 4E - 12m/V \\
0 & 0 & 4E - 12m/V \\
0 & 0 & 12E - 12m/V \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix} \begin{bmatrix} 4 \\
3 \\
23 \\
\end{bmatrix} = \begin{bmatrix} 8E - 12 \\
8E - 12 \\
24E - 12 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

And \( \Delta L \)?

\[
\Delta L = \begin{bmatrix} 8E - 15m \\
8E - 15m \\
120E - 15m \\
\end{bmatrix}
\]
Real behavior

Piezoelectric ceramics show hysteresis in polarization

And they show hysteresis in strain

http://www.americanpiezo.com/piezo_theory/, 07.28.2010
Piezoresistive effect

- Change in resistivity due to applied mechanical stress.
- But differs from Piezoelectric effect: It changes only resistivity and does not create an electric potential.
- Effect is mainly seen in semiconductors:

$$\rho_\sigma = \left( \frac{\partial \rho}{\rho} \right) \frac{S}{S}$$

$\rho_\sigma$: Piezoresistivity, $\rho$: original resistivity, $S$: strain

Mechanism:
- Change in inter-atomic spacing affects bandgaps
- Bandgaps might be shifted
- Shape might be affected -> change in effective mass
Electrostriction

- Change in shape by applying electrical field
- Proportional to the square of the field

\[ S_{ij} = \gamma_{ijkl} \times E_k \times E_l \]

\[ \gamma_{ijkl} = \frac{1}{2} \frac{\partial^2 S_{ij}}{\partial E_k \partial E_l} \]

- Is not reversible
- Occurs in all dielectric materials and in all 32 point groups
- Caused by randomly aligned electrical domains
  - Applied field aligns electrical domains
  - Opposite charges of domains attract each other
  - Material thickness is reduced along applied field
Some piezoelectric materials

Naturally occurring:
- Quartz
- Cane sugar
- Collagen
- Topaz
- Rochelle salt
- DNA
- Wood
- Many many others
- Tendon

Man-made crystals:
- Gallium orthophosphate (GaPO4), a quartz analogic crystal
- Langasite (La3Ga5SiO14), a quartz analogic crystal

Man-made ceramics:
- Barium titanate (BaTiO3), Barium titanate was the first piezoelectric ceramic discovered
- Lead zirconate titanate (Pb[ZrxTi1−x]O3 0<x<1)—more commonly known as PZT, lead zirconate titanate is the most common piezoelectric ceramic in use today
- Lithium niobate (LiNbO3)
Applications

Sensor
- Microphones, Pick-ups
- Pressure sensor
- Force sensor
- Strain gauge

Actuators
- Loudspeaker
- Piezoelectric motors
- Nanopositioning in AFM, STM
- Acousto-optic modulators
- Valves

High voltage and powersource
- Cigarette lighter
- Energy harvesting
- AC voltage multiplier

Frequency standard
Piezoelectric motors

- Traveling wave motor
- Inchworm motor
- Piezo ratchet motor
- Stepping motor using slip-stick motion
Traveling wave motor

1996 Smart Mater. Struct. 5 361
Inchworm Motor

Six Step Actuation Processes of the Piezo Inchworm Motor

Step 1.

Step 2.

Step 3.

Step 4.

Step 5.

Step 6.

http://en.wikipedia.org/wiki/Piezoelectric_motor, 07.28.2010
Piezo ratchet stepping motor
ANRv51/RES
Slip-Stick Inertial Motion

1 → 2: Slow rising flank of voltage, Rod and table move simultaneously

2 → 3: Fast decreasing voltage flank, piezo contracts fast and rod slips through table, inertia is overcome

Conversion of motion:
Signal is inverted in time, not in voltage
Currents

at 1000Hz slow rising flank or loading voltage: \( \tau_{\text{rise}} \approx 1\,\text{ms} \)
fast falling flank of discharge voltage: \( \tau_{\text{fall}} \approx 10\,\mu\text{s} \)

\[ I = \frac{CU}{\tau} \]

\( C_{P,RT} = 2.8\,\mu\text{F} \)
\( dU_{RT} = 30\,\text{V} \)
\( I_{\text{rise},RT} = 84\,\text{mA} \)
\( I_{\text{fall},RT} = 8.4\,\text{A} \)
\( |I|_{\text{average,RT}} = 167\,\text{mA} \)

\( C_{P,LT} = 0.2\,\mu\text{F} \)
\( dU_{LT} = 70\,\text{V} \)
\( I_{\text{rise},LT} = 14\,\text{mA} \)
\( I_{\text{fall},LT} = 1.4\,\text{A} \)
\( |I|_{\text{average,LT}} = 28\,\text{mA} \)
Effects of resistive wiring

70V sawtooth signal
1µF capacitance

RC time constant
\( \tau = RC \)

Cabling capacitance of up to 10nF has barely no effect
Heat dissipation

Assumption: $F_{\text{friction}} = 5\text{N}$, Step size: 100nm $\rightarrow$ 500nJ

At 1000Hz $\rightarrow$ 500$\mu$W

Electrical loss: $P = CU^2 f\tan(\delta)$, $\delta \approx 1^\circ$

$P = 17mW$

Rotator has 2 piezos $\rightarrow$ 2P, $t_{90^\circ} \approx 30s$

$U \approx 1.1J$
Absolut position encoder

Position is read out with a potentiometer

\[ R_{AW} \text{ depends on } T \]
\[ R_{AB} \text{ depends on } T \]

But, \( \frac{R_{AW}}{R_{AB}} \) is \( T \) independent in equilibrium

Absolut position

Encoder has a “blind spot” of about 40°
Did you really pay attention?

Some piezoelectric materials

Naturally occurring:
- Quartz
- Topaz
- Rochelle salt
- many many others

Man-made crystals
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