Current and voltage are crucial quantities

Current law: \[ \sum_{k=1}^{n} I_k = 0 \]

Mesh rule: \[ \sum_{k=1}^{n} V_k = 0 \]

V and I are connected via Ohm’s law:

\[ V = RI \]
1. Ohm’s law

- Current and voltage are crucial quantities

- Current law: \[ \sum_{k=1}^{n} I_k = 0 \]

- Mesh rule: \[ \sum_{k=1}^{n} V_k = 0 \]

- \( V \) and \( I \) are connected via Ohm’s law:

\[ V = RI \]
Basics

- Defined by Ohm’s law: 
  \[ R = \frac{V}{I} \]

- Symbol: 
  \[ \text{\includegraphics[width=0.2\textwidth]{resistor}} \]

- In series: 
  \[ R_{\text{tot}} = \sum_{i} R_i \]

- In parallel: 
  \[ \frac{1}{R_{\text{tot}}} = \sum_{i} \frac{1}{R_i} \]

- Power dissipation: 
  \[ P = IV = I^2R = \frac{V^2}{R} \]
2. Resistors

- Defined by Ohm’s law:
  \[ R = \frac{V}{I} \quad [R] = \frac{kg \ m^2}{A^2 \ s^3} = \Omega \]

- Symbol:

- In series:
  \[ R_{tot} = \sum_{i}^{n} R_i \]

- In parallel:
  \[ \frac{1}{R_{tot}} = \sum_{i}^{n} \frac{1}{R_i} \]

- Power dissipation:
  \[ P = IV = I^2R = \frac{V^2}{R} \]
Basics

- Defined by: \( Q = CV \) or \( I = C \frac{dV}{dt} \)  
  \[
  [C] = \frac{C}{V} = \frac{A^2 s^4}{kg \ m^2} = F
  \]
- Symbol: 
  
- In series: \( \frac{1}{C_{tot}} = \sum_{i}^{n} \frac{1}{C_i} \)
- In parallel: \( C_{tot} = \sum_{i}^{n} C_i \)
- Cannot dissipate power
3. Capacitors

- Defined by: \( Q = CV \) or \( I = C \frac{dV}{dt} \)

- [C] = \( \frac{C}{V} = \frac{A^2 s^4}{kg\ m^2} = F \)

- Symbol:

- In series: \( \frac{1}{C_{tot}} = \sum_{i}^{n} \frac{1}{C_i} \)

- In parallel: \( C_{tot} = \sum_{i}^{n} C_i \)

- Cannot dissipate power
Basics

- Defined by: \( V = L \frac{dI}{dt} \)
- Symbol: \( \square \)
- In series: \( L_{tot} = \sum_{i}^{n} L_i \)
- In parallel: \( \frac{1}{L_{tot}} = \sum_{i}^{n} \frac{1}{L_i} \)
- Stored energy: \( E = \frac{1}{2} LI^2 \)

\[ [L] = \frac{kg \ m^2}{A^2 \ s^2} = H \]
4. Inductors

- Defined by: \( V = L \frac{dI}{dt} \)

- Symbol: ⬤

- In series: \( L_{tot} = \sum_{i}^{n} L_{i} \)

- In parallel: \( \frac{1}{L_{tot}} = \sum_{i}^{n} \frac{1}{L_{i}} \)

- Stored energy: \( E = \frac{1}{2} LI^2 \)

\([L] = \frac{kg \ m^2}{A^2 \ s^2} = H\)
Perfect voltage source:

Maintains fixed voltage droop independent from load

Symbol:  

Real voltage source:

Can only supply a finite current = perfect voltage source with a small resistor in series
5. Current - and voltage sources

- Perfect voltage source:

  *Maintains fixed voltage droop independent from load*

- Symbol: ±

- Real voltage source:

  *Can only supply a finite current = perfect voltage source with a small resistor in series*
Perfect current source:

_Maintains constant current regardless of load resistance_

Symbol: 

Real voltage source:

_Has maximum voltage it can supply = perfect current source with big resistor in parallel_
5. Current - and voltage sources

- Perfect current source:
  
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- Real voltage source:
  
  Has maximum voltage it can supply = perfect current source with big resistor in parallel
How can one compare the relative amplitudes/strengths of two signals?

Introduce the unit „(deci)Bel“:

\[ dB = 20 \cdot \log \left( \frac{A_1}{A_2} \right) \]

Note: \( A_2 = 2A_1 \rightarrow 6\text{dB} \)

In terms of power levels: \( dB = 10 \cdot \log \frac{P_1}{P_2} \)

Reference signal needed
6. The decibel scale

- How can one compare the relative amplitudes/strengths of two signals?

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- Reference signal needed
Output voltage is a certain fraction of the input voltage.

Without load, current is the same everywhere.

\[ I = \frac{V_{in}}{R_1 + R_2} \]

\[ V_{out} = \frac{R_2}{R_1 + R_2} V_{in} \]
Simple applications

1. Voltage dividers

- Output voltage is a certain fraction of the input voltage
- Without load, current is the same everywhere

\[
I = \frac{V_{in}}{R_1 + R_2}
\]

\[
V_{out} = \frac{R_2}{R_1 + R_2} V_{in}
\]
More sophisticated dividers use potentiometer

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Simple applications

1. Voltage dividers

More sophisticated dividers use potentiometer

\[ V_{out} = \frac{R_2}{R_1 + R_2} V_{in} \]
Real divider has a load attached

\[ R_{tot} = \frac{R_2 R_L}{R_2 + R_L} + R_1 \]

\[ I = \frac{V_{in}}{R_{tot}} \]

\[ V_L = I \frac{R_2 R_L}{R_2 + R_L} \]

\[ V_L = V_{in} \frac{R_2 R_L}{R_2 R_L + R_1 R_L + R_1 R_2} \]
1. Voltage dividers

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V_L = V_{in} \frac{R_2 R_L}{R_2 R_L + R_1 R_L + R_1 R_2}
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Simple applications

๏ Some circuits „perform“ calculations

\[ I = C \frac{d}{dt} (V_{in} - V_{out}) = \frac{V}{R} \]

\[ \frac{dV_{out}}{dt} \ll \frac{dV_{in}}{dt} \]

\[ \Rightarrow V(t) = RC \frac{d}{dt} V_{in}(t) \]

๏ Output is prop. to rate of change/derivative of input
2. Differentiators

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- Output is prop. to rate of change/derivative of input
Simple applications

example:
2. Differentiators

(example: [Diagram of V and V_in signals])
Simple applications

Exchange R and C in the differentiator circuit

\[ I = C \frac{dV}{dt} = \frac{V_{in} - V_{out}}{R} \]

*keep RC large → V_{out} ≪ V_{in}*

\[ \Rightarrow C \frac{dV}{dt} \approx \frac{V_{in}}{R} \]

\[ V(t) = \frac{1}{RC} \int_{0}^{t} V_{in}(t) \, dt + \text{const.} \]
3. Integrators

- Exchange R and C in the differentiator circuit.

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*keep RC large → \( V_{out} \ll V_{in} \)*

\[ \Rightarrow C \frac{dV}{dt} \approx \frac{V_{in}}{R} \]

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Every two terminal network of resistors and voltage sources can equivalently be represented by a single resistor $R_{th}$ in series with a single voltage source $V_{th}$. 
I. Thévenin’s theorem:

Every two terminal network of resistors and voltage sources can equivalently be represented by a single resistor \( R_{th} \) in series with a single voltage source \( V_{th} \).
How can one figure out $R_{th}$ and $V_{th}$?

$V_{th}$ is the open circuit voltage

$R_{th}$ can be calculated by using the short circuit current:

$$R_{th} = \frac{V(\text{open circuit})}{I(\text{short circuit})}$$
Open circuit voltage is given by

\[ V = V_{in} \frac{R_2}{R_1 + R_2} \]

Short circuit current:

\[ V_{in}/R_1 \]

\( R_{th} \) is then given by:

\[ R_{th} = \frac{R_1 R_2}{R_1 + R_2} \]
Example: Voltage divider

- **Open circuit voltage** is given by
  \[ V = V_{in} \frac{R_2}{R_1 + R_2} \]

- **Short circuit current**:
  \[ V_{in}/R_1 \]

- **\( R_{th} \)** is then given by:
  \[ R_{th} = \frac{R_1 R_2}{R_1 + R_2} \]
- C, L are linear: output changes prop. to input
- Circuits with C and L have frequency dependence
- C, L change phase of input signal

\[ \Rightarrow \text{Impedance extends the concept of resistance to AC circuits, relative phase is taken into account} \]
1. Problem

- C, L are linear: output changes prop. to input
- Circuits with C and L have frequency dependence
- C, L change phase of input signal

⇒ Impedance extends the concept of resistance to AC circuits, relative phase is taken into account
Take relative phase into account by using complex numbers

Now \( V, I \in \mathbb{C} \) and \( V(t) = \text{Re}(Ve^{i\omega t}) \)

The Impedance is then defined as:

\[
Z = |Z|e^{i\omega t} \quad |Z| = R + iX
\]
2. Solution

- Take relative phase into account by using complex numbers

- Now \( V, I \in \mathbb{C} \) and \( V(t) = \text{Re}(Ve^{i\omega t}) \)

- The Impedance is then defined as:

\[
Z = |Z|e^{i\omega t} \quad |Z| = R + iX
\]
Impedance of R, C and L are given by:

\[ Z_R = R \quad Z_C = \frac{1}{i\omega C} \quad Z_L = i\omega L \]

Ohm's law is still valid: \[ V = IZ \]

\[
Z_{tot} = \sum_{i}^{n} Z_i \quad \text{(series)} \quad \frac{1}{Z_{tot}} = \sum_{i}^{n} \frac{1}{Z_i} \quad \text{(parallel)}
\]
3. Impedance $Z_R$, $Z_L$, $Z_C$ and Ohm’s law

- Impedance of $R, C$ and $L$ are given by:

$$
Z_R = R \quad Z_C = \frac{1}{i\omega C} \quad Z_L = i\omega L
$$

- Ohm’s law is still valid: $V = IZ$

$$
Z_{tot} = \sum_{i}^{n} Z_i \quad \text{(series)} \quad \frac{1}{Z_{tot}} = \sum_{i}^{n} \frac{1}{Z_i} \quad \text{(parallel)}
$$
- Impedance of the output of a circuit = int. resistance
- Limits the current that can be supplied to a load
- Very important for power supplies
- Cannot be measured with Ohmmeter
- Indirect measurement necessary
Output Impedance

1. Definition

- Impedance of the output of a circuit = int. resistance
- Limits the current that can be supplied to a load
- Very important for power supplies
- Cannot be measured with Ohmmeter
- Indirect measurement necessary
Output Impedance

- Output Impedance = $Z_i$
- Measure Output voltage $V$ without a load
- Use known load $R_L$, measure the voltage $V_L$ over $R_L$:

\[ V_L = R_L I \text{ and } I = \frac{V}{Z_i + R_L} \quad \Rightarrow \quad Z_I = R_L \left( \frac{V}{V_L} - 1 \right) \]
2. Measure $Z_i$

- Output Impedance $= Z_i$

- Measure Output voltage $V$ without a load

- Use known load $R_L$, measure the voltage $V_L$ over $R_L$:

$$V_L = R_L I \text{ and } I = \frac{V}{Z_i + R_L} \Rightarrow Z_I = R_L \left( \frac{V}{V_L} - 1 \right)$$
Used to maximize the transferred power

Design input impedance of load/output impedance of source

\[ Z_L = Z_S^* \]
2. Impedance Matching

- Used to maximize the transferred power
- Design input impedance of load/output impedance of source

\[ Z_L = Z_S^* \]
RC-circuits

- Frequency dependent „voltage dividers“
- Ability to pass frequencies of interest while rejecting all other frequencies
- Usually one speaks of RC-filters
- Plenty of applications, therefore very important
Most common RC circuit, used e.g. in audio amp.

\[
I = \frac{V_{\text{in}}}{Z_{\text{tot}}} = \frac{V_{\text{in}}}{R + \frac{1}{i\omega C}}
\]

\[
= \frac{V_{\text{in}} \left(R + \frac{i}{\omega C}\right)}{R^2 + \frac{1}{\omega^2 C^2}}
\]

\[
V_{\text{out}} = \frac{V_{\text{in}} \left(R + \frac{i}{\omega C}\right) R}{R^2 + \frac{1}{\omega^2 C^2}}
\]
1. High-pass filter

Most common RC circuit, used e.g. in audio amp.

\[ I = \frac{V_{in}}{Z_{tot}} = \frac{V_{in}}{R + \frac{1}{i\omega C}} \]

\[ = \frac{V_{in} \left( R + \frac{i}{\omega C} \right)}{R^2 + \frac{1}{\omega^2 C^2}} \]

\[ V_{out} = \frac{V_{in} \left( R + \frac{i}{\omega C} \right) R}{R^2 + \frac{1}{\omega^2 C^2}} \]
Amplitude of output signal: \[ V_{out} = V_{in} \frac{R}{(R^2 + \frac{1}{\omega^2 C^2})^{1/2}} \]
1. High-pass filter

- Amplitude of output signal: \( V_{\text{out}} = V_{\text{in}} \frac{R}{(R^2 + \frac{1}{\omega^2 C^2})^{1/2}} \)
Exchange $R$ and $C$ in the High-pass filter

$$V_{out} = V_{in} \frac{1}{(1 + \omega^2 R^2 C^2)^{1/2}}$$
2. Low-pass filter

- Exchange R and C in the High-pass filter

![Diagram of RC-circuit]

\[ V_{out} = V_{in} \frac{1}{(1 + \omega^2 R^2 C^2)^{1/2}} \]
RC-circuits

- Exchange R and C in the High-pass filter
- Can be used e.g. to eliminate interference

\[ \omega_{3DB} = \frac{1}{\tau} \]

\[ \frac{V_{out}}{V_{in}} \]

Graph showing the relationship between \( V_{out} \) and \( V_{in} \) for different values of \( \omega \).
2. **Low-pass filter**

- Exchange R and C in the High-pass filter
- Can be used e.g. to eliminate interference

\[
\omega_{3DB} = \frac{1}{RC}
\]
RC-circuits

- Combinations of capacitors and inductors
- Very sharp frequency characteristic

\[ Z_{CL} = \frac{i}{\frac{1}{\omega L} - \omega C} \]

- \( Z_{CL} \) and \( Z_R \) act like a voltage divider
3. **Resonant circuit**

- Combinations of capacitors and inductors
- Very sharp frequency characteristic

\[
Z_{CL} = \frac{i}{\frac{1}{\omega L} - \omega C}
\]

- \(Z_{CL}\) and \(Z_R\) act like a voltage divider
Opposite behavior of C and L leads to: \[ \lim_{\omega \to \omega_R} Z_{CL} \to \infty \]

- Losses in C and L limit sharpness of the peak
- Used e.g. in radio amplifiers
3. Resonant circuit

- Opposite behavior of C and L leads to: \( \lim_{\omega \to \omega_R} Z_{CL} \to \infty \)

- Losses in C and L limit sharpness of the peak

- Used e.g. in radio amplifiers
Source-Drain Box

- Couple AC- and DC voltages
- Precise control over the amplitude of both AC and DC voltages
- Should not depend on frequency
1. Intention

- Couple AC- and DC voltages
- Precise control over the amplitude of both AC and DC voltages
- Should not depend on frequency
2. Circuit

[Diagram of a circuit with labels AC, DC, 6.8kΩ, 56Ω, 100Ω, 50kΩ, and 4:1 transformer connections to Source, Drain, and Ground.]
Voltage Divider 1: 1:122
Voltage Divider 2: 1:501
Transformer: 4:1
Output is 1:244857 of AC input
DC input is divided by 501
Simple to use voltage source (DC)

Adjustable over wide range of voltages (1mV - 9V)
I. Intention

- Simple to use voltage source (DC)
- Adjustable over wide range of voltages (1mV - 9V)
- Standard 9V battery
- Potentiometer to adjust voltage
- Voltage divider to switch between ranges