Nuclear Magnetism and Electronic Order in $^{13}$C Nanotubes

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(Dated: August 12, 2008)

arXiv:0808.1685v1

\begin{itemize}
  \item pure $^{13}$C SWNT, \textbf{metallic}
  \item Stable isotopes: $^{12}$C \textit{S} = 0 \quad 98.9 \% abundance
  $^{13}$C \textit{S} = $1/2$ \quad 1.1 \%
  \item hyperfine interaction couple electrons \& nuclei, $A =$ ?
    0.1 to 1 $\mu$eV dep. on curvature, Pennington et al., RMP 1996
    100 $\mu$eV Churchill et al., arXiv0811.3236v1
\end{itemize}
• electrons mediate interactions between nuclei (RKKY)
• 1D system (analogous, different, stronger compared to 2,3D)
• Electron correlations -> Tomonaga-Luttinger Liquid (interacting electrons)
• direct nuclear dipole-dipole interaction very weak (~10 peV), neglected

• single electronic mode

• circular cross section: identical overlap, identical spin alignment, i.e. ferromagnetic

• treat spins as 1D nuclear spin chain $I >> \frac{1}{2}$ spins (excludes Kondo spin $\frac{1}{2}$ physics)

$$\tilde{H} = H_{el} + A \sum_i \hat{S}_i \cdot \hat{I}_i$$

$$\hat{I}_i = (\hat{I}_i^x, \hat{I}_i^y, \hat{I}_i^z)$$ effective nuclear spin operator

$$\hat{S}_i = (\hat{S}_i^x, \hat{S}_i^y, \hat{S}_i^z)$$ electron spin operator
• hyperfine interaction couple electrons & nuclei, $A = ?$
  0.1 to 1 meV dep. on curvature, Pennington et al., RMP 1996
  100 meV Churchill et al., arXiv0811.3236v1

• $E_F = v_F k_F / 2$  meV to eV

  $k_F / \pi = n_{el}$
  $v_F \approx 8 \times 10^5$ m/s in SWNTs

• electron and nuclear time scales decouple $A/E_F << 1$
1D Physics in Armchair Carbon Nanotubes

1D conductor, with electron-electron interactions driven to Luttinger liquid

Interacting system of right- and left-moving electrons:

\[
H_{el} = \int dr \sum_{\sigma=\uparrow, \downarrow} \left[ \psi_{R\sigma}^\dagger(r)(-i v_F \nabla) \psi_{R\sigma}(r) + \psi_{L\sigma}^\dagger(r)(+i v_F \nabla) \psi_{L\sigma}(r) \right]
\]

\[
+ \int dr dr' \sum_{\sigma \sigma' = \uparrow, \downarrow} \sum_{\nu \nu' = L, R} V(r - r') \psi_{\nu\sigma}(r) \psi_{\nu'\sigma'}(r') \psi_{\nu'\sigma'}(r') \psi_{\nu\sigma}(r)
\]

allows diagonalization through bosonization

Note: Absence of curvature in dispersion & high electron density → no Wigner crystal or incoherent Luttinger liquid
1D Physics in Armchair Carbon Nanotubes

After **bosonization** (single band notation here)

\[
H_{\text{el}} = \int \frac{dr}{2\pi} \sum_{\kappa=c,s} \left[ \frac{\nu_\kappa}{K_\kappa} \left( \nabla \phi_\kappa(r) \right)^2 + \nu_\kappa K_\kappa \left( \nabla \theta_\kappa(r) \right)^2 \right]
\]

Luttinger parameter and renomalized charge / spin velocity

charge / spin density fluctuations

conjugated field

Typical values for nanotubes:

\[
K_c \approx 0.2 \quad K_s = 1 \quad v_{c,s} = \frac{v_F}{K_{c,s}} \quad v_F \sim 8 \times 10^5 \text{ m/s}
\]

\[
E_F = \frac{\hbar k_F v_F}{2} \sim 0.1 \text{ eV} \quad \text{tunable through bias}
\]
Effective Hamiltonian

\[ H_{n}^{\text{eff}} = \frac{1}{2} \sum_{ij\alpha} J_{ij}^{\alpha} \hat{I}_{i}^{\alpha} \hat{I}_{j}^{\alpha} = \sum_{\alpha} \int_{0}^{\pi/a} dq \frac{d}{2\pi} J_{q}^{\alpha} \hat{I}_{-q}^{\alpha} \hat{I}_{q}^{\alpha} , \]
Single energy scale

Derivation of effective model is similar to

RKKY interaction

\[ J_q = \frac{A^2a}{2} |\chi_s(q)| = -C T^{2g-2} |\Gamma(\kappa)/\Gamma(\kappa + 1 - g)|^2 \]

\[ C = A^2 a \sin(\pi g) \Gamma^2(1 - g)(2\pi ak_B/v_F)^{2g-2}/4\pi^2 v_F \]

\[ \kappa = g/2 - i\lambda_T(q - 2k_F)/4\pi \]

\[ g = (K_c + 1/K_s)/2 \]

Theory depends on a single energy scale only

\[ T^* = J_{2k_F}(T^*) \]

\[ \lambda_T = v_F/k_BT \]
RKKY Interaction

Two consequences

1. Height and width of $J_q$ are function of temperature curve characterized by a single parameter: $T$

Theory depends on a single energy scale only

$$T^* = J_{2k_F}(T^*)$$

2. max. of $|J_q|$ reached at $q = 2 k_F$: helical order favoured
Nuclear spin order

Helical order of nuclear spins

- 1D spin lattice of large effective spins
- no Kondo physics

Stable only in systems of finite length (quantum dot).

For typical nanotubes: \( T^* \sim 10 \, \mu K \)
Feedback is essential!

Nuclear magnetic (Overhauser) field

\[ \langle I_i \rangle = \text{Im}(2k_F r_i) [e_x \cos(2k_F r_i) + e_y \sin(2k_F r_i)] \]

magnetization

Feedback on electrons

\[ H_{Ov} = \sum_i A \langle I_i \rangle \cdot S_i = \sum_i A \text{Im}(2k_F) \cos(\sqrt{2}[\phi_c(r_i) + \theta_s(r_i)]) \]

Bosonization treatment; relevant sine-Gordon interaction

Opening of mass gap for \( \phi_+ \propto (\phi_c + \theta_s) \)
But a gapless field \( \phi_- \) remains!
Consequences

1. Modified RKKY interaction, $T^*$

- same shape
- modified exponents
  \[ g \approx 0.7 \rightarrow g' \approx 0.3 \]

\[ T^* \sim mK \]
Consequences

2. Anisotropy in electron spin susceptibility

Overhauser field defines spin \((x,y)\) easy-plane

\[
\chi^{x,y}(q) \sim \left| \frac{1}{q - 2k_F} \right|^{1 \over 2-g'} \quad g' \approx 0.33
\]

\[
\chi^{z}(q) \sim \left| \frac{1}{q - 2k_F} \right|^{1 \over 2-g''} \quad g'' \approx 0.17
\]
Consequences

3. Reduction of conductivity

Luttinger liquid connected to metallic leads

Conductance: \[ G = T \frac{e^2}{h} n \]

number of conducting channels

Nanotube: \( n = 4 \) (2: spins; 2: unit cell has two C atoms)

With the feedback: \( \phi_+ \) field is pinned

Reduction to \[ G = T \frac{e^2}{h} 2 \]

i.e. universal reduction of conduction in ordered phase