3-Axis Vector Magnet: Construction and Characterisation of Split Coils at RT

Semester Project
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Outline

• Field Calculation and Simulation
• Construction Details
• Field Calculations
• Characterization at RT
• Summary
A short repetition

Maxwell’s equations (SI units)

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} = 4\pi k \rho \]
\[ \nabla \cdot \vec{B} = 0 \]
\[ \nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} \]
\[ \nabla \times \vec{B} = \frac{\vec{J}}{\varepsilon_0 c^2} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \]

\[ \int \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} \]
\[ \int \vec{B} \cdot d\vec{A} = 0 \]
\[ \int \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \]
\[ \int \vec{B} \cdot d\vec{s} = \mu_0 i + \frac{1}{c^2} \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{A} \]

Right-hand rule
Magnetic field of a circular loop

**Biot-Savart Law**

\[
\mathbf{dB}(r) = \frac{\mu_0}{4\pi} I \mathbf{dl}(r') \times \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3}
\]

- \(\mu_0\): Permeability
- \(I\): Current in Ampere
- \(\mathbf{dl}(r')\): Infinitesimal length of the conductor at position \(r'\)
- \(\mathbf{r}\): Point where the field is computed

**Parametrization of the loop**

\[
\begin{align*}
\mathbf{r}' &= \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ r_Q \cos(\phi) \\ r_Q \sin(\phi) \end{pmatrix}
\end{align*}
\]

\[
B(r_P) = \frac{I\mu_0 N}{2} \frac{r_Q^2}{\left(r_Q^2 + r_P^2\right)^{3/2}}
\]

- \(N\): Number of windings
Curved circular loop: Parametrization I

\[ y = r_2 \sin(\theta) \]
\[ x = r_2 \cos(\theta) \]
\[ z = r_1 \cos(\varphi) \]

\[ \theta = \frac{r_1'}{r_2} = \frac{r_1 \sin(\varphi)}{r_2} \]
Curved circular loop: 
Parametrization II

\[
r(\varphi) = \begin{pmatrix} 
    r_2 \sin \left( \frac{r_1 \sin(\varphi)}{r_2} \right) \\
    r_2 \cos \left( \frac{r_1 \sin(\varphi)}{r_2} \right) \\
    r_1 \cos(\varphi) 
\end{pmatrix}
\]

\[r_1 = 0.017 \text{m}\]
\[r_2 = 0.022 \text{m}\]
Curved circular loop: Parametrization III

There are a lot of other possible parametrizations like:

\[ r(\varphi) = \begin{pmatrix} \rho \sin(\varphi) \\ \rho \cos(\varphi) \\ a \cos^{2n}(\varphi) \end{pmatrix} \]

with \( n = 1, 2, 3, \ldots \)

- \( \rho \): radius of the loop in x-y plane
- \( a \): difference between the highest and the lowest point of the loop

**Advantage**
- Calculations can be simplified
- The term \( a \cos^{2n}(\varphi) \) can be substituted for any symmetric and periodic function

**Disadvantage**
- The field is not calculated in the centre of the IVC, but in the centre of the curved loop -> need of an additional parameter
**Field Calculations**

Remember: Bio-Savart Law

\[
\frac{dB(r)}{4\pi} = \frac{\mu_0}{4\pi} I dl'(r') \times \frac{r - r'}{|r - r'|^3}
\]

\[
r(\varphi) = \begin{pmatrix}
    r_2 \sin \left( \frac{r_1 \sin(\varphi)}{r_2} \right) \\
    r_2 \cos \left( \frac{r_1 \sin(\varphi)}{r_2} \right) \\
    \frac{1}{\sqrt{r_2^2 + r_1^2 \cos^2(\varphi)}} \\
\end{pmatrix}
\]

How to proceed:

- \( r(\varphi) \) has to be normalized
- To get the infinitesimal length \( dl \) of the conductor take the derivative of \( r(\varphi) \):

\[
dl(\varphi) = \frac{dr(\varphi)}{d\varphi}
\]

- Calculate only the x-component of the cross-product

\[
dB_x = \frac{\mu_0 I}{4\pi} \frac{r_1^2 \cos^2(\varphi) \cos \left( \frac{r_1}{r_2} \sin(\varphi) \right) + r_1 r_2 \sin(\varphi) \sin \left( \frac{r_1}{r_2} \sin(\varphi) \right)}{\left( r_2^2 + r_1^2 \cos^2(\varphi) \right)^{3/2}}
\]
Integration of $dB_x$ not easy $\rightarrow$ Numerical approach

Real coils have a certain thickness $\rightarrow$ Separation of the curvature from distance in x-direction $\rightarrow$ Introduction of a new parameter $\xi$ which describes the distance in x-direction

$$
DB_x = \frac{\mu_0 I}{4\pi} \left[ \frac{r_1^2 \cos^2(\varphi) \cos \left( \frac{r_1}{r_2} \sin(\varphi) \right) + r_1 r_2 \sin(\varphi) \sin \left( \frac{r_1}{r_2} \sin(\varphi) \right)}{\xi^2 \cos^2 \left( \frac{r_1}{r_2} \sin(\varphi) \right) r_2^2 \sin^2 \left( \frac{r_1}{r_2} \sin(\varphi) \right) + r_1^2 \cos^2(\varphi)} \right]^{3/2}
$$
Field Simulation I

Grid 5mm x 5mm
2200 windings
B-field at 1 Ampere: 0.023 T; B-field at 50 Ampere: 1.16 T
Estimated wire length: 250.5 m
The Coil
Winding
Lorentz Force
\[ \vec{F} = I \int d\vec{l} \times \vec{B} \]

Right Hand Rule

Integration over the 2π → no net force
Integration over the semi circle (-\(\pi/2, \pi/2\))

\[ F_x \cap = 2 NIBr \]

\[ 33.6 \text{ kN} \approx 34 \times 10^3 \text{ chocolate bars} \]

B = 9T; I = 50A; r = 0.017m; N = 2200
For the curved circular loop:

\[ F_x = 2NIBr \sin \left( \frac{r_1}{r_2} \right) \]

The force over the whole circle is zero, but points in opposite direction over the two semi circles

\[ \rightarrow \text{Emergence of a torque} \]

30.4 kN

\[ I = 50A; \quad B = 9T; \]
\[ N = 2200; \quad r_1 = 0.017m \]
\[ r_2 = 0.022m \]

We just consider the flat case

\[ \vec{M} = \vec{\mu} \times \vec{B} \]
\[ \vec{\mu} = \vec{A}I \]
\[ \vec{A} = r^2 \pi \vec{n} \]

\[ M_y = 1244 Nm \]

712 Nm
Measurement setup
Measurements I

Spools 1 & 2

Spools 3 & 4

2.53 mT @ 100 mA (1&2)

2.38 mT @ 100 mA (3&4)

Comparison with simulation ($B_z(0)=23$ mT): Accuracy 3.4% - 8.9%
Measurements II

$B_z(z)$ @ 100 mA
Measurements III

\[ B_z(x) \at 100 \, mA \]

\[ B_z(y) \at 100 \, mA \]
# Measurements IV

<table>
<thead>
<tr>
<th></th>
<th>Coil 1</th>
<th>Coil 2</th>
<th>Coil 3</th>
<th>Coil 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of windings</strong></td>
<td>2200</td>
<td>2200</td>
<td>2202</td>
<td>2200</td>
</tr>
<tr>
<td><strong>Bz @ 100mA</strong></td>
<td>1.29mT</td>
<td>1.26mT</td>
<td>1.26mT</td>
<td>1.25mT</td>
</tr>
<tr>
<td><strong>Resistance R DC</strong></td>
<td>169.4Ω</td>
<td>163.4Ω</td>
<td>162.2Ω</td>
<td>163.6Ω</td>
</tr>
<tr>
<td><strong>Resistance R @ 120Hz</strong></td>
<td>184.1Ω</td>
<td>174.2Ω</td>
<td>175.5Ω</td>
<td>176.9Ω</td>
</tr>
<tr>
<td><strong>Resistance R @ 1kHz</strong></td>
<td>338.7Ω</td>
<td>322.8Ω</td>
<td>325.4Ω</td>
<td>328.6Ω</td>
</tr>
<tr>
<td><strong>Inductance L @ 120Hz</strong></td>
<td>107.8mH</td>
<td>101.9mH</td>
<td>104.6mH</td>
<td>104.6mH</td>
</tr>
<tr>
<td><strong>Inductance L @ 1kHz</strong></td>
<td>62.1mH</td>
<td>59.8mH</td>
<td>60.6mH</td>
<td>59.9mH</td>
</tr>
<tr>
<td><strong>Inductance L @ 10kHz</strong></td>
<td>41.5mH</td>
<td>40.5mH</td>
<td>41.2mH</td>
<td>40.5mH</td>
</tr>
<tr>
<td><strong>Phase θ @ 120Hz</strong></td>
<td>23.84°</td>
<td>23.83°</td>
<td>24.12°</td>
<td>24.03°</td>
</tr>
<tr>
<td><strong>Phase θ @ 1kHz</strong></td>
<td>49.03°</td>
<td>49.36°</td>
<td>49.42°</td>
<td>48.89°</td>
</tr>
<tr>
<td><strong>Phase θ @ 10kHz</strong></td>
<td>71.47°</td>
<td>70.74°</td>
<td>71.75°</td>
<td>71.59°</td>
</tr>
</tbody>
</table>
Next Steps

- Install the coils into the cage
- Insert and install the 2-axis split pair system into the solenoid
- Characterize the system at liquid helium temperature
Summary

- High forces and a high torque can act on the system
- The calculated magnetic field is comparable with the measured field
- In the dimension of the device the magnetic field is almost homogeneous
Basel Cryolab, 2:45pm, incredible 9 Tesla...
...and the look fits

3 Wetter Taft