Exponential protection of zero modes in Majorana islands

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Motivation

• Majorana zero mode
• Experimental evidences of the Majorana

Not only Zero Bias Conductance Peak (ZBCP)

Devices

$L = 400 \text{ nm}$

$L = 790 \text{ nm}$

<table>
<thead>
<tr>
<th>$L$ [nm]</th>
<th>$E_C$ [meV]</th>
<th>$\eta$ [eV/V]</th>
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<tbody>
<tr>
<td>330</td>
<td>1.6</td>
<td>0.048</td>
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<td>400</td>
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<td>790</td>
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<td>1540</td>
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[Diagram of Devices with labels for InAs, Al, and Au]
Electron Teleportation via Majorana Bound States in a Mesoscopic Superconductor

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Zero-energy Majorana bound states in superconductors have been proposed to be potential building blocks of a topological quantum computer, because quantum information can be encoded nonlocally in the fermion occupation of a pair of spatially separated Majorana bound states. However, despite intensive efforts, nonlocal signatures of Majorana bound states have not been found in charge transport. In this work, we predict a striking nonlocal phase-coherent electron transfer process by virtue of tunneling in and out of a pair of Majorana bound states. This teleportation phenomenon only exists in a mesoscopic superconductor because of an all-important but previously overlooked charging energy. We propose an experimental setup to detect this phenomenon in a superconductor–quantum-spin-Hall-insulator–magnetic-insulator hybrid system.
Majorana island

\[ \mathcal{E}_N = \mathcal{E}_C N_G - N_G^2 + n_N \mathcal{E}_0 \]

The ground state has even parity

Coulomb peaks have 2e periodicity

For high enough B field

Transition from 2e periodicity to 1e periodicity

\[ \mathcal{E}_0 > \mathcal{E}_C \]
Coulomb Peak spacing

\[ <S_{e,o} > \] averaging even(odd)
Coulomb peak spacing

\[ <S_{e,o} > \] is oscillating around 1e periodicity

Oscillation is consistent with the Hybridization of the Majorana modes

The amplitude is exponential suppressed for long wire
Exponential protection

\[ \Psi_l(x) \propto e^{-x/\xi} e^{\pm ik_{F,\text{eff}}x} \]

\[ \xi = \text{effective coherence length} \]

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<tr>
<th>L [nm]</th>
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<th>(\eta) [eV/V]</th>
<th>(A) [(\mu)eV]</th>
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Das Sarma. et al PRB, 86, 220506(2012)

Overlap between \(\Psi_1\) and \(\Psi_2\) produce the oscillation of the Coulomb peaks spacing

Long wires -> less overlap -> decreasing oscillation amplitude
The discrete state moves linearly as function of B passing through zero and merging with the continuum at $100 \mu eV$.

Short-length limit; the bound state are not anymore topological protected and the bound state can merge with the continuum.

Long-length limit

Zero-Majorana mode extend for 120 mT and $\Delta_T \approx 30 \mu eV$.

The Majorana states are well separated and wave functions don’t overlap.

$E_0(B=0) \approx 50-160 \mu eV$
Coulomb peak height

B<B* -> 2e periodicity -> peak height is uniform.

B*<B<B** -> peak height decreases

B>B**-> peak height recovers -> uniform spacing -> localized zero-energy states at the ends of the wire

Electron teleportation through 0 energy Majorana mode

Majorana mode move to the ends of the wire
Conclusion

- Oscillating energy splitting
- Oscillation are exponential suppressed with the length of the wire. \((\xi=260 \text{ nm})\)
- Transport through discrete zero-energy state
- \(\Delta T = 30 \mu\text{eV}\) for long wire
-localized zero-energy states at the ends of the wire
- Electron teleportation