Introduction

- Standard circuit theory fails in solving RF/microwave network problems because of small wavelength.
Introduction

- Phase of voltage or current changes significantly over the physical length of the device
- In principle: solve Maxwell equations
- But: we only need quantities like power, impedance, voltage or current ➔ circuit theory concept, but more complex for RF/microwaves
Transmission Line Theory

- Transmission line is a two-wire line
- Can be modeled as an equivalent circuit with quantities such as R, L, G, C
Transmission Line Theory

- Apply Kirchhoff laws for current and voltage (telegrapher equations):
  \[
  \frac{\partial v(z,t)}{\partial z} = -R i(z,t) - L \frac{\partial i(z,t)}{\partial t} \\
  \frac{\partial i(z,t)}{\partial z} = -G v(z,t) - C \frac{\partial v(z,t)}{\partial t}
  \]

- Fourier-transform solution (traveling waves):
  \[
  V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \\
  I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z} \\
  \gamma = \sqrt{(R + i\omega L)(G + i\omega C)}
  \]
Transmission Line Theory

- $\gamma$ is the complex propagation constant
- The characteristic impedance can be defined as:

$$Z_0 = \frac{R + i\omega L}{\gamma} = \sqrt{\frac{R + i\omega L}{G + i\omega C}}$$

- Relates the voltage and the current of the line as:

$$\frac{V_0^+}{I_0^+} = Z_0 = \frac{-V_0^-}{I_0^-}$$

- Transmission line parameters can be calculated for several lines (Co-Ax, two-wire, parallel plates)
Transmission Line Theory

- Terminated lossless transmission line
  - lossless: \( R = G = 0 \)

- Incident wave (source at \( z < 0 \)):
  \[ V_0^+ e^{-i\beta z} \]

- Ratio of current to voltage given by \( Z_0 \)

- Line terminated \( Z_L \neq Z_0 \) => ratio of voltage to current at the load must be \( Z_L \) => a reflected wave has to be excited with an appropriate amplitude:
  \[ V(z) = V_0^+ e^{-i\beta z} + V_0^- e^{i\beta z} \]
Reflection coefficient is given by:

\[ \Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} \]

→ In order to have no reflection \((\Gamma = 0)\), \(Z_L = Z_0\)

(load has to match the line)

For \(\Gamma = 0\), maximum power is delivered to the load, no power for \(\Gamma = 1\)
Smith Charts

- Goal: Given the reflection coefficient, find the corresponding impedance $\rightarrow$ Smith chart provides a graphical method to do this (P. Smith 1939)
- Reflection coefficient can be plotted using a polar display
- Impedance can be plotted on a rectilinear plane
  $\rightarrow$ The Smith Chart is the result of mapping one plane onto the other (Moebius transformation, conformal map)
Smith Charts

- Constant resistance $R$ maps to a circle, constant reactance $X$ maps to arcs
- In general, Smith Charts are normalized to $Z_0$
Network Analyzers
Network Analyzers

- Test devices (DUT) such as cables, adapters, filters, attenuators, amplifiers, modulators, etc.
- Network analysis is concerned with the accurate measurement of the ratios between the reflected / transmitted signal to the incident signal
- We are interested in distortionless transmission of signals within the desired bandwidth
Network Analyzers

- Example: linear vs. non-linear distortion

We need to be able to measure the magnitude and phase.
S-Parameters

- S-parameters are a convenient way of characterizing high-frequency networks.
- Measure the S-parameters by voltage traveling waves (magnitude and phase).
S-Parameters

- Determine $S_{11}$ and $S_{21}$ while the output is terminated in a perfect $Z_0$ ($a_2 = 0$, no reflection)
- Measure $S_{22}$ and $S_{21}$ likewise ($a_1 = 0$)
- The accuracy of the measurement is critically dependent on how good the termination is
  - if $a_1, a_2 \neq 0$, the definition of S-Parameters is not fulfilled

$\Rightarrow$ Need for error correction (calibration)
S-Parameters

- $S_{11} =$ forward reflection coefficient (input match)
- $S_{22} =$ reverse reflection coefficient (output match)
- $S_{21} =$ forward transmission coefficient (gain or loss)
- $S_{12} =$ reverse transmission coefficient (isolation)

$\Rightarrow$ Inherently complex
Error Correction

- Systematic errors are characterized by the calibration and removed by an algorithm during the measurements.
Error Correction

- There are different types of error correction, response and vector corrections

- **response (normalization)**
  - simple to perform
  - only corrects for tracking errors
  - stores reference trace in memory, then does data divided by memory

- **vector**
  - requires more standards
  - requires an analyzer that can measure phase
  - accounts for all major sources of systematic error
Cal-Kit

- Use a cal-kit to perform the calibration
- A cal-lit provides a set of known standards (be careful)
- The cal-kit definition must match the actual cal-kit used (guided calibration)