Low Temperature Methods
(1 K ~ 1 mK)

A. Abdelrahaman
Zumbühl Group
Department of Physics
University of Basel
• Physical properties He-3
  Artificial Production,
  Latent Heat of Evaporation and Vapour Pressure.

• He-3 and He-4 Mixtures
  Physical Properties,
  The Cooling Power,
  He-3 and He-4 Mixtures as Fermi Liquid,
  Finite Solubility of He3 in He4.

• Thermal Conductivity
  The lattice conductivity,
  Electronic Thermal Conductivity,
  Thermal and Electrical Conductivity: The Wiedeman-Franz Law,
  Influence of Impurities on Conductivity.

• Dilution Refrigerator

• Reference:
  Matter and Methods at Low Temperatures, 2edE, F.Pobell
$^4$He Phase Diagram

![Phase Diagram](image)

- Solid $^4$He
- Superfluid $^4$He
- Normal liquid $^4$He
- Melting curve
- $\lambda$-line
- Evaporation
- Vapour

Temperature $T$ [K] vs. Pressure $P$ [bar]
• **Production of He-3**
• Tritium Decay.
• D-D Fusion Reaction.
• The p+Li6 Reaction for Breeding He-3.

3He Phase Diagram
Cooling Power of evaporative cooling

\[ \frac{dP}{dT} = \frac{S_{gas} - S_{liq}}{V_{gas} - V_{liq}} \sim \frac{L}{TV_{gas}} = \frac{LP}{RT^2} \]  (1)

assuming \( V_{gas} \gg V_{liq} \) and using \( L \sim T\Delta S \)

latent heat \( L \sim \) independent of temperature

then from (1) it follows:

\[ \frac{dP}{P} \sim \frac{L}{R} \frac{dT}{T^2} \]  (2)

with simple solution

\[ P \propto \exp\left(-\frac{L}{RT}\right) \]  (3)

i.e. cooling power is proportional to vapor press.

which is \textit{exponentially small with temperature}
Cooling Power proportional to Vapour Pressure

\[ P \propto \exp \left( -\frac{L}{RT} \right) \]

pumping on \(^4\text{He}: \sim 1 \text{ K}

pumping on \(^3\text{He}: \sim 0.25 \text{ K}\)
3He refrigerator
Cooling Power proportional to Vapour Pressure

\[ P \propto \exp\left(-\frac{L}{RT}\right) \]

How cool below 0.2 K?
How can exponentially small vapor pressure be overcome?

pumping on \(^4\text{He}\): ~ 1 K
pumping on \(^3\text{He}\): ~ 0.25 K

Fig. 2.7. Vapour pressures of liquid \(^3\text{He}\) and liquid \(^4\text{He}\)
He3-He4 mixture

the working fluid mixture of the dilution refrigerator:
Phase separation into $^3\text{He}$ rich and $^3\text{He}$ poor phase below $T \sim 800 \text{ mK}$
He-3 and He-4 Mixtures

- **The Cooling Power:**
  - The cooling capacity is the heat mixing of the two isotopes. The cooling power of an evaporating cryogenic liquid:
    \[ Q = n \Delta H = n L \]
  - Make use of the latent heat \( L \) of evaporation, pumping with a pump of constant volume rate \( V \) on He3 and He4 bath with vapour pressure \( P \):
    \[ \dot{Q} = V P(T) L(T) \]
  - He3-He4 dilution refrigeration: Use the difference of the specific heats of the two phases (the enthalpy of mixing):
    \[ \Delta H \propto \int \Delta C dT \]
    \[ \Rightarrow \dot{Q} \propto x \Delta H \propto T^2 \]

**dilution refrigerator:**
**cooling power:** \(~ T^2\)
• **He3-He4 Dilution Refrigerator:**
  - Dilution refrigerator can be understood if we compare the cooling process with cooling which occurs when liquid is evaporated.
  - In evaporation we rely on the classical heat of evaporation for cooling.
  - In dilution refrigeration we rely on the enthalpy of mixing of two quantum liquid: the different zero-point motions of the two helium isotopes and the different statistics.
• **He-3 and He-4 Mixtures as Fermi Liquid**
  • He4: Nuclear spin=0, Bose static, At low T Bose liquid undergo Bose condensation in momentum space (correspond to transition to superliquid for He4).
  • At T<0.5 K He4 condensed into quantum mechanical ground state, no excitation (phonon).
  • In mixture: He4 acts as inert superfluid background contributes to the volume and to the dissolved isotope He3.
  • He3: Nuclear spin ½ is a Fermi particle, Fermi static and Pauli priniciple.
  • In analogy to conduction elelctrons, the specific heat of liquid He3 behaves as:

\[
C_3 = \left[ \frac{\pi^2}{2} \right] \frac{T}{T_F} R \rightarrow at \rightarrow T \ll T_F
\]

Fermi degenerate:

or

\[
C_3 = \left[ \frac{5}{2} \right] R \rightarrow at \rightarrow T \gg T_F \rightarrow P = \text{cons} \ \tan t
\]

Classical:

• Behaviour is classical-gas-like at: \( T \gtrsim 1K \)
  • Behaviour is Fermi-gas-like at: \( T \ll 0.1K \)
  • He3-He4 mixture can be described by the law of an interacting Fermi gas
• **Finite Solubility of He3 in He4**

- **He3 in Pure He3:** The chemical potential of pure liquid He3 is given by the latent heat of evaporation, corresponding to the binding energy.

- **One He3 Atome in Liquid He4:**
  Identical chemical structure of the He isotopes-van der Walls force. The liquid phase He4 atoms occupy a smaller volume than He3 atoms. Its binding energy – due to the smaller distance or larger density - is stronger if it is in He4 than it would be in He3.

- **Many He3 Atoms in Liquid He4:** Attractive interaction between the He3 atoms and in liquid He4, due to:
  ~ Magnetic interaction due to the nuclear magnetic moments of He3 as in pure He3
  ~ Density effect

- **Pauli principle:** The energy states up to the Fermi energy are filled with two He3 atoms of opposite nuclear spin.

\[
E_F = K_B T_F
\]

- **Result:** The binding energy of the He3 atoms has to decrease, due to their Fermi character, if their number is increased.
Thermal Conductivity - Introduction

- Introduction:
- Thermal conductivity is a transport property of matter.
- Transport theory gives the thermal conductivity:
  \[ \kappa = \frac{1}{3} \frac{C}{C_m} v \lambda \]
- \( \lambda \) - the mean free path, \( v \) - the velocity.
- The electrons involved in thermal transport can only be electrons with energy near the Fermi energy, and perform transition to higher non-occupied states, with Fermi velocity:
  \[ v_F = \frac{\hbar}{m_e} \left( \frac{3\pi^2 N_o}{V_m} \right)^{1/3} \]
- At low temperature \( T \), Fermi velocity is independent of \( T \), and the transport properties depend on the mean free path \( \lambda \), determined by the scattering process of the heat carriers.
- The main limiting scattering processes for thermal conductivity: phonon-phonon, electron-phonon, electron-impurity.
- **The lattice conductivity** : phonons
  \[ k_{ph} = \frac{1}{3} \left( \frac{C_{ph}}{V_m} \right) V_s \lambda_{ph} \propto T^3 \lambda_{ph} (T), \]

- At intermediate T: Thermal conductivity decrease with increasing T.
- At low T: Small number of thermally excited phonons, heat carrier scattered by crystal defects or boundaries only, the wave length is larger than the size of the lattice imperfections:
  \[ k_{ph} = C_{ph} \propto T^3 \]
- Result: Thermal conductivity due to phonon transport goes through maximum.
- **Electronic Thermal Conductivity**: In metal, it is larger than the lattice thermal conductivity, the Fermi velocity of the conduction electrons is larger than the sound velocity of the photons.

\[
k_e = \frac{1}{3} \left( \frac{C_e}{V_m} \right) v_F \lambda_e \propto T \lambda_e(T),
\]

- High T: The number of thermally excited phonons increase with T, the electronic thermal conductivity in the electron-phonon scattering region decreases with increasing T.
- Low T: The scattering of electrons from defects and impurities dominates, small number of phonons:

\[
k_e \propto C_e \propto T
\]

- Result: The electronic contribution to the thermal conductivity goes through a maximum.
- The value and the position of this maximum strongly depend on the perfection of the metal.
- In disordered alloy the scattering of electrons by the varying potential can become so strong that electronic and lattice conductivities become comparable.
Thermal and Electrical Conductivity: The Wiedeman-Franz Law

- Metal at low $T$: in the defect scattering limit, $\lambda$ -the mean free path is constant
- Electrical conductivity: the electrons conduct charge, temperature independent.
- In other words: in the defect scattering limit or the residual resistivity range, the electrical conductivity is $T$ independent.
- Thermal conductivity: proportional to temperature in the RR range since the electrons carry specific heat.
- Result: The ratio of the thermal conductivity to electrical conductivity is proportional to the temperature

$$\frac{k}{\sigma} = L_o T \Rightarrow k = \sigma L_o T \Leftrightarrow L_o = 3\left(\frac{\pi K_B}{9e}\right)^2 \equiv \text{Lorenz}$$
• **Wiedeman-Franz Law**: For the temperature range where the conductivity due to electron transport is limited by large-angle elastic electron-phonon scattering, the same result:

• **Residual Resisitivity Ratio RRR**: A measure of the electrical conductivity at low temperature (boiling point of liquid helium), to the electrical conductivity at room temperature:

• RRR is direct measure of the limiting defect scattering

\[
RRR = \frac{\sigma_{4.2\,K}}{\sigma_{300\,K}} = \frac{\rho_{300\,K}}{\rho_{4.2\,K}}
\]
• **Influence of Impurities on Conductivity:**
• Magnetic and non magnetic impurities.
• 1-Electron scattering by non-magnetic impurity atoms: The increase of the electrical resistance is not very large, and for small impurities concentrations is given by Linde rule:

\[ \Delta \rho_{nm} = a + b(\Delta Z)^2 \]

• 2-Scattering of electrons by magnetic impurities atoms: magnetic or spin-flip scattering of the conduction electrons can occur at localized moments of magnetic impurities, the increase of the resistance can be much larger.
• The strength of the scattering and the resulting resistance increase depend strongly on the properties of the magnetic impurity.
Thermal Conductivity

- The strength of scattering can depend very strongly on $T$ due to **Kondo effect**: an enhanced inelastic scattering of a cloud of conduction electrons around magnetic moment that are localized on impurity atoms:

$$\rho = \rho_0 + \rho_k \ln(T)$$