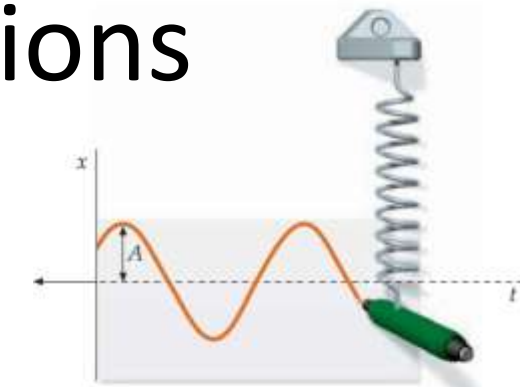
Introduction to Physics I

For Biologists, Geoscientists, & Pharmaceutical Scientists

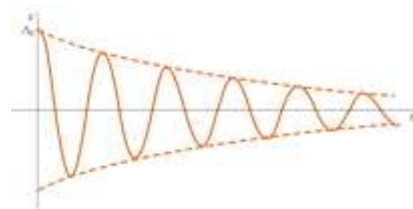
L15, 09.11.2017

Mechanical vibrations



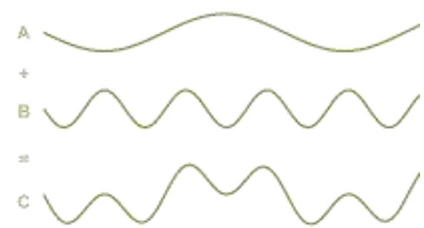
Harmonic oscillators, pendulum

Undamped oscillator –
damped oscillator

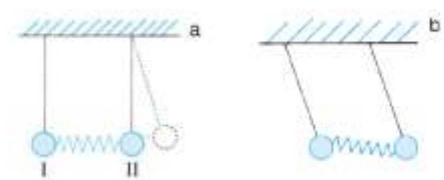


Driven oscillations

Superposition, decomposition, beating



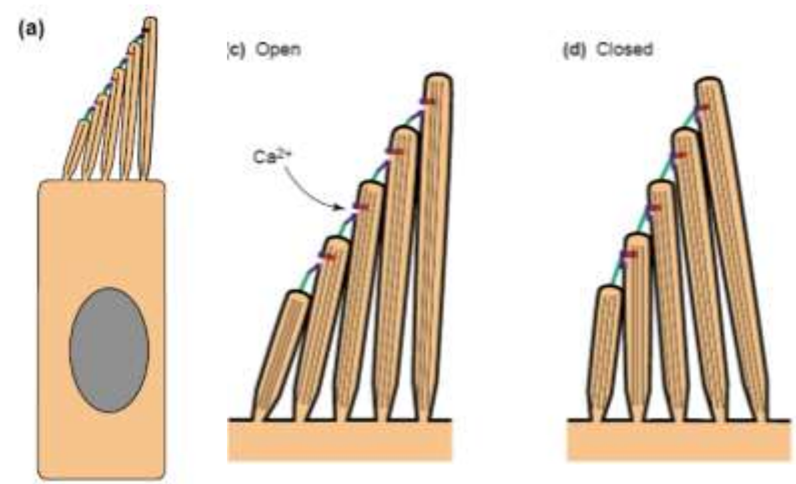
Coupled pendulum



Trautwein (German) Ch. 6 - 6.4
Tipler (English) Ch.14

mechanical vibrations

oscillatory motion



auditory hair bundles
Jülicher COCB (2005)

also: genetic oscillators e.g. circadian clocks

oscillatory motion

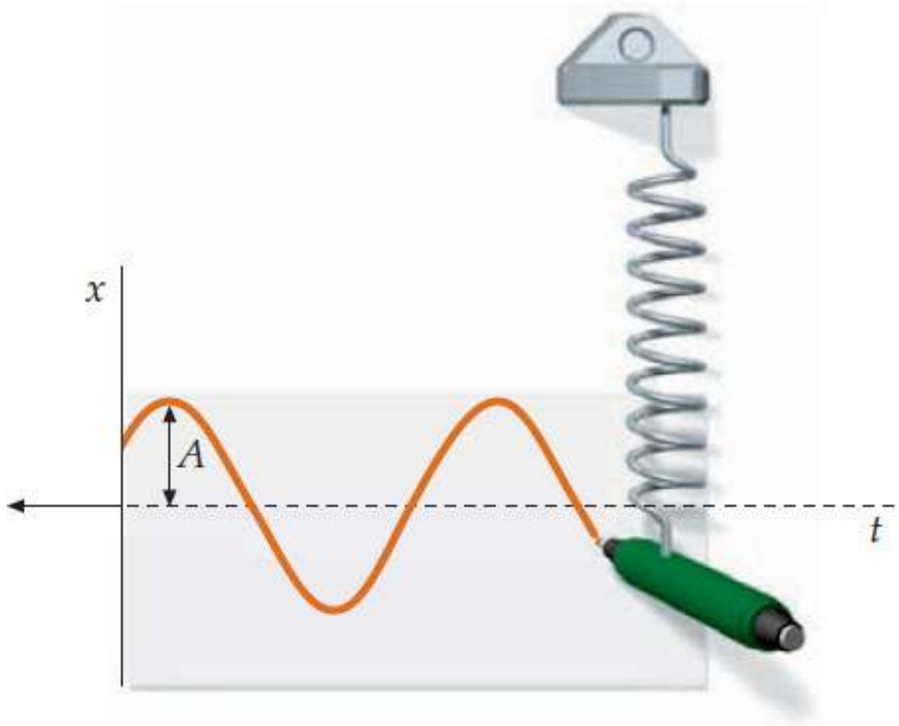


FIGURE 14-2 A marking pen is attached to a mass on a spring, and the paper is pulled to the left. As the paper moves with constant speed, the pen traces out the displacement x as a function of time t. (Here, we have chosen x to be positive when the spring is compressed.)

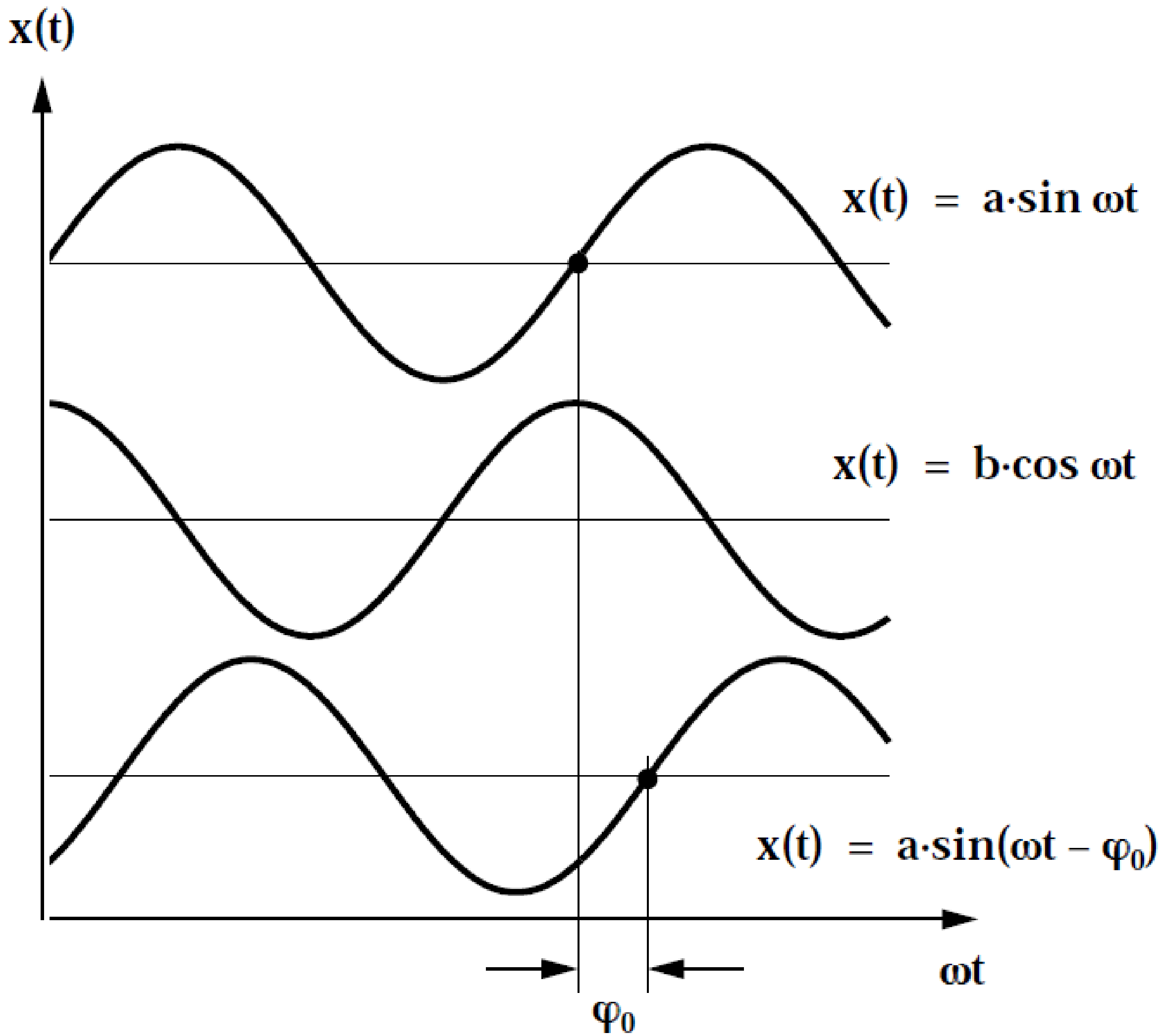
Tipler

oscillatory motion

$x(t) = a \cdot \sin \omega t$

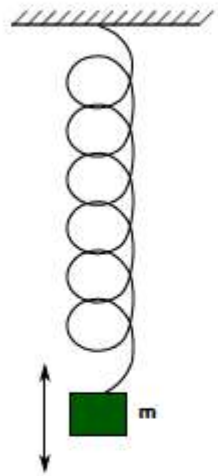
$x(t) = b \cdot \cos \omega t$

$x(t) = a \cdot \sin (\omega t - \varphi_0)$



oscillatory motion

Question

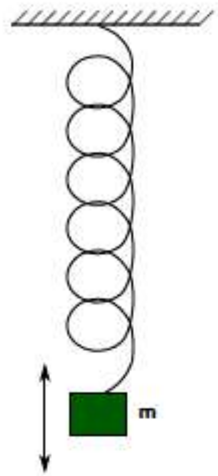


Eine Masse m schwingt wie in der Skizze gezeigt, mit gewisser Frequenz und Amplitude auf und ab. Falls die Amplitude verdoppelt wird, ist die maximale Kraft....

1. vier mal grösser
2. zwei mal grösser
3. unverändert
4. halb so gross
5. ein Viertel der bisherigen Kraft.
6. keines von allem

oscillatory motion

Question



Antwort: 2. Zwei mal grösser

Wird die Amplitude einer Schwingung verdoppelt, wirkt eine doppelt so grosse maximale Kraft.

Eine Masse m schwingt wie in der Skizze gezeigt, mit gewisser Frequenz und Amplitude auf und ab. Falls die Amplitude verdoppelt wird, ist die maximale Kraft....

1. vier mal grösser
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4. halb so gross
5. ein Viertel der bisherigen Kraft.
6. keines von allem

oscillatory motion

Question

m1=m2
k1=k2

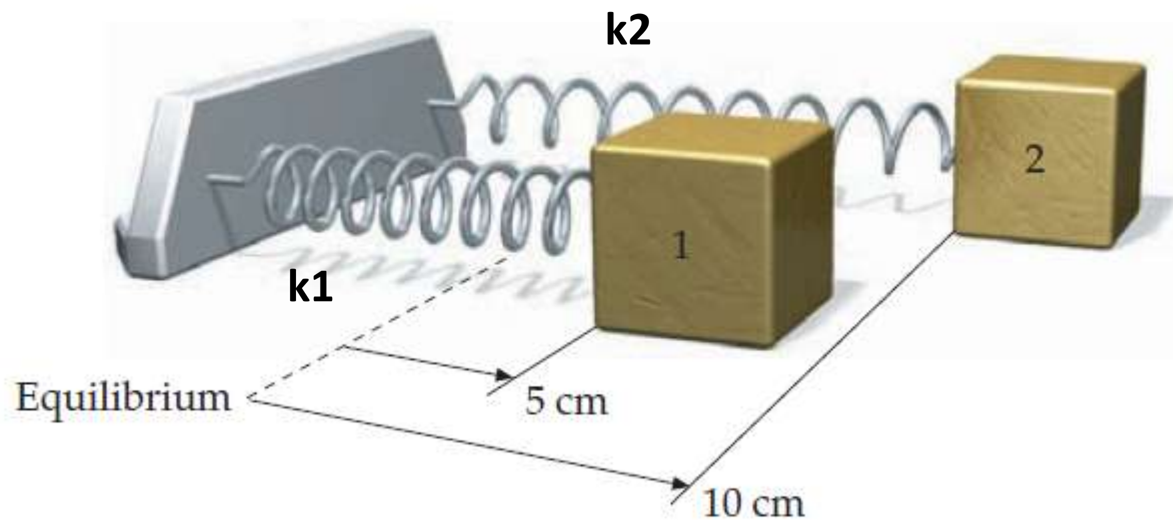


FIGURE 14-3 Two identical mass-spring systems.

If they are released at the same time, which object reaches the equilibrium position first?

a) Object 1
b) Object 2
c) at the same time

Tipler

oscillatory motion

Answer

at the same time

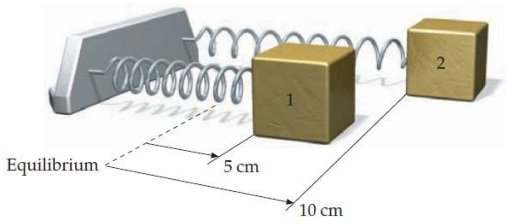


FIGURE 14-3 Two identical mass-spring systems.

The frequency (and thus the period) of simple harmonic motion is independent of the amplitude.

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

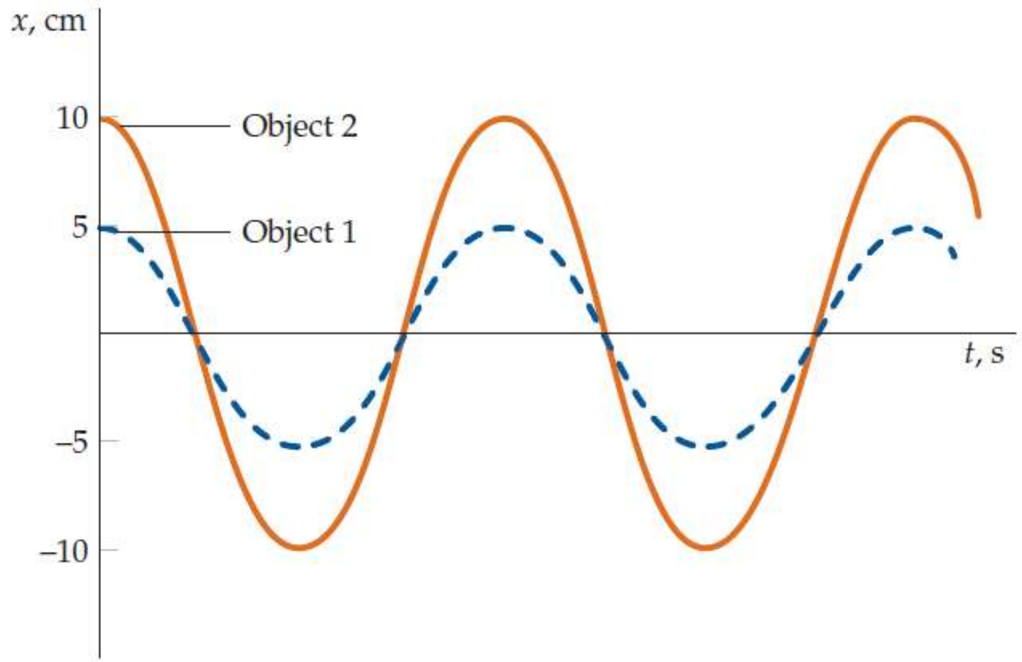


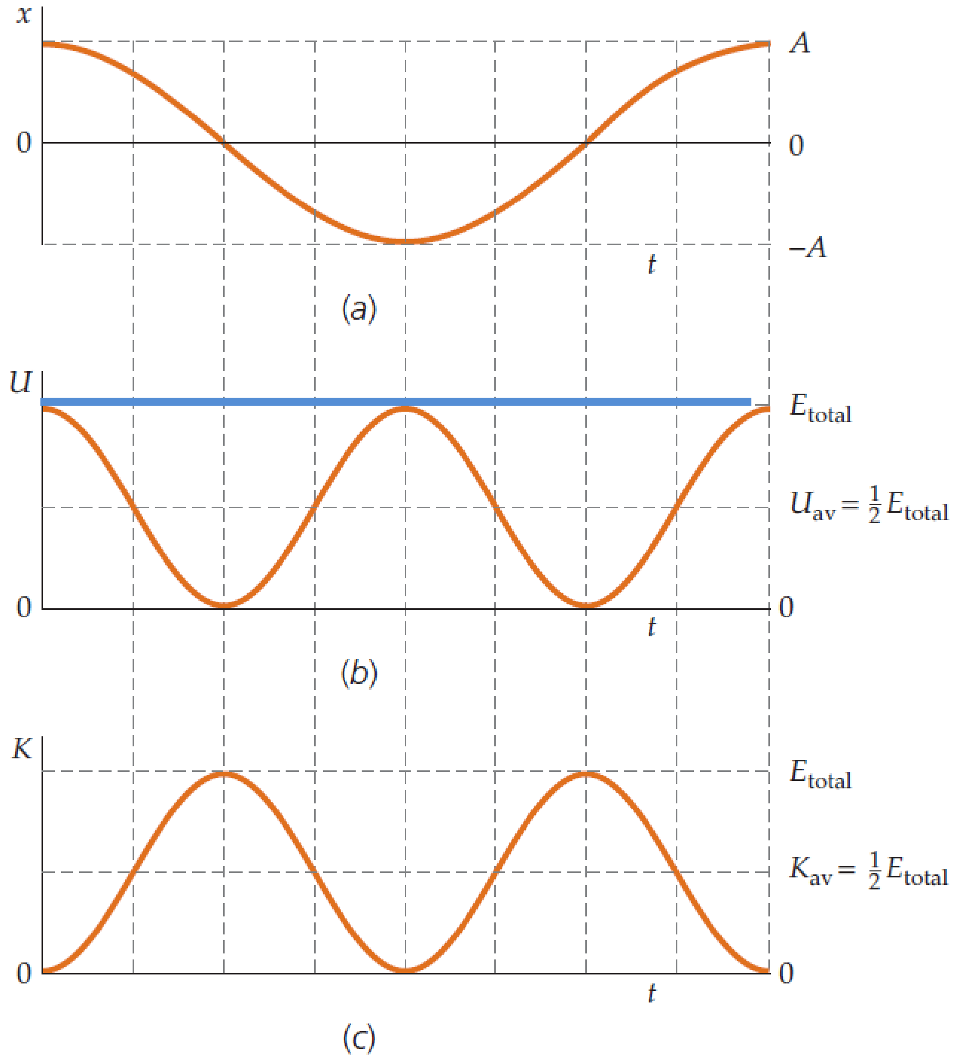
FIGURE 14-4 Plots of x versus t for the systems in Figure 14-3. Both reach their equilibrium positions at the same time.

Tipler

harmonic oscillation, energy

$U = E_{pot}$

$K = E_{kin}$



$$E_{total} = U + K = E_{pot} + E_{kin}$$

FIGURE 14-7 Plots of x, U, and K versus t.

Tipler

harmonic oscillation, energy

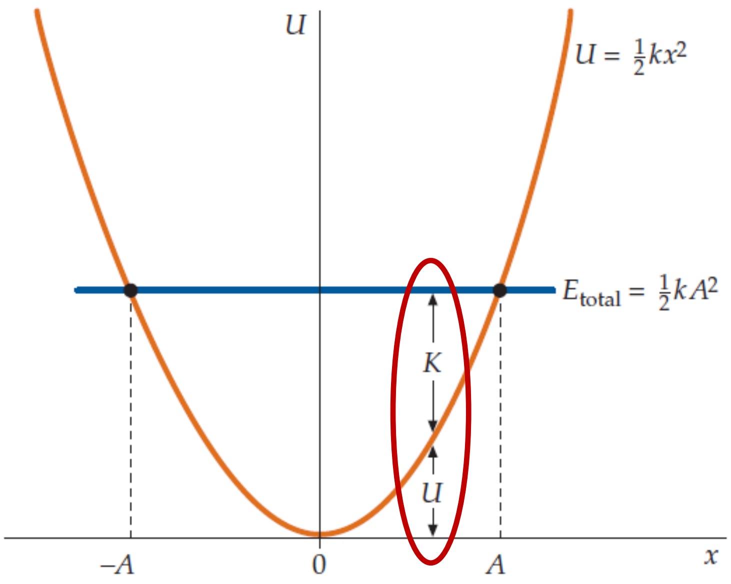


FIGURE 14-8 The potential-energy function $U = \frac{1}{2}kx^2$ for an object of mechanical mass m on a (massless) spring of force constant k. The horizontal blue line represents the total mechanical energy E_{total} for an amplitude of A. The kinetic energy K is represented by the vertical distance $K = E_{total} - U$. $E_{total} \geq U$, so the motion is restricted to $-A \leq x \leq +A$.

harmonic approximation

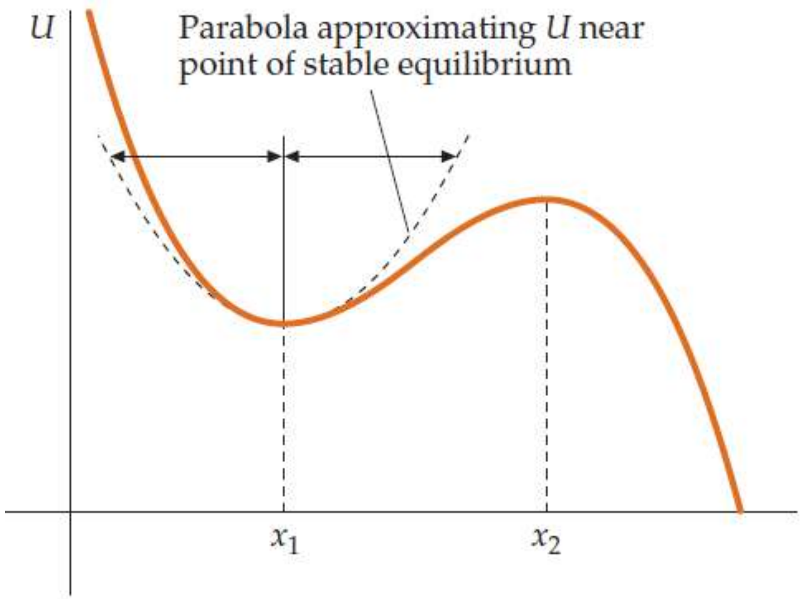


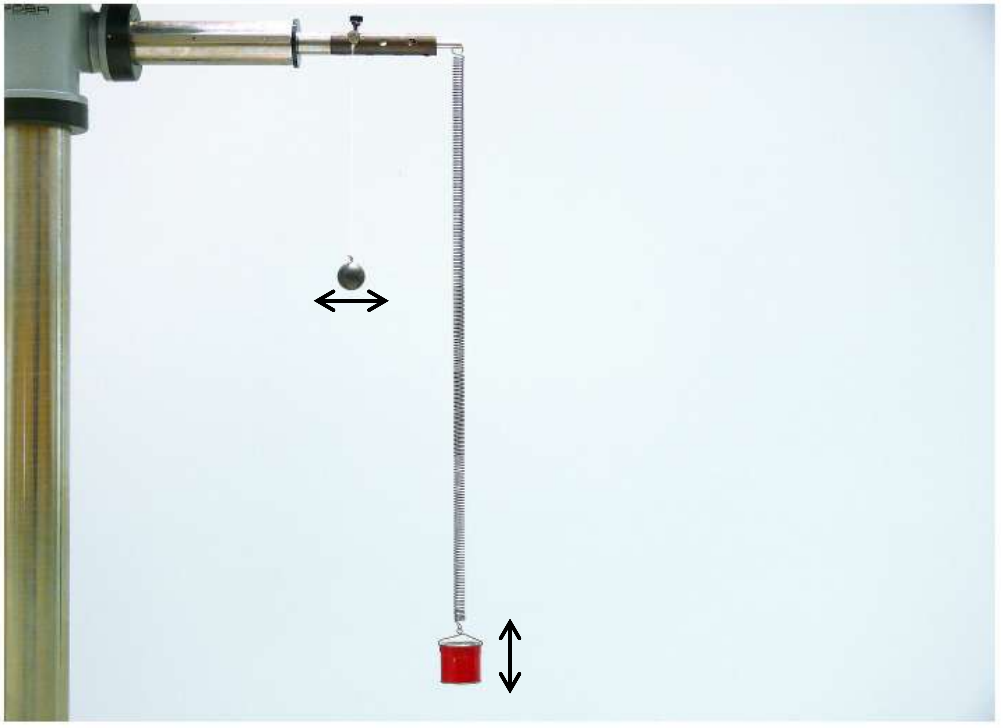
FIGURE 14-9 Plot of U versus x for a force that has a position of stable equilibrium (x_1) and a position of unstable equilibrium (x_2).

Tipler

harmonic oscillation

Man hat ein Fadenpendel und ein Federpendel. Die Feder hat eine **Federkonstante k**. An beiden Pendeln hängt die gleiche **Masse m**.

Wie lange muss das Fadenpendel sein, damit die Pendel die gleiche Schwingungsperioden haben?



1. Die Pendel müssen die gleiche Länge haben.
2. $l \propto k/m$
3. $l \propto m/k$
4. kann man mit diesen Informationen nicht sagen.

harmonic oscillation

Man hat ein Fadenpendel und ein Federpendel. Die Feder hat eine **Federkonstante k**. An beiden Pendeln hängt die gleiche **Masse m**.

Wie lange muss das Fadenpendel sein, damit die Pendel die gleiche Schwingungsperioden haben?

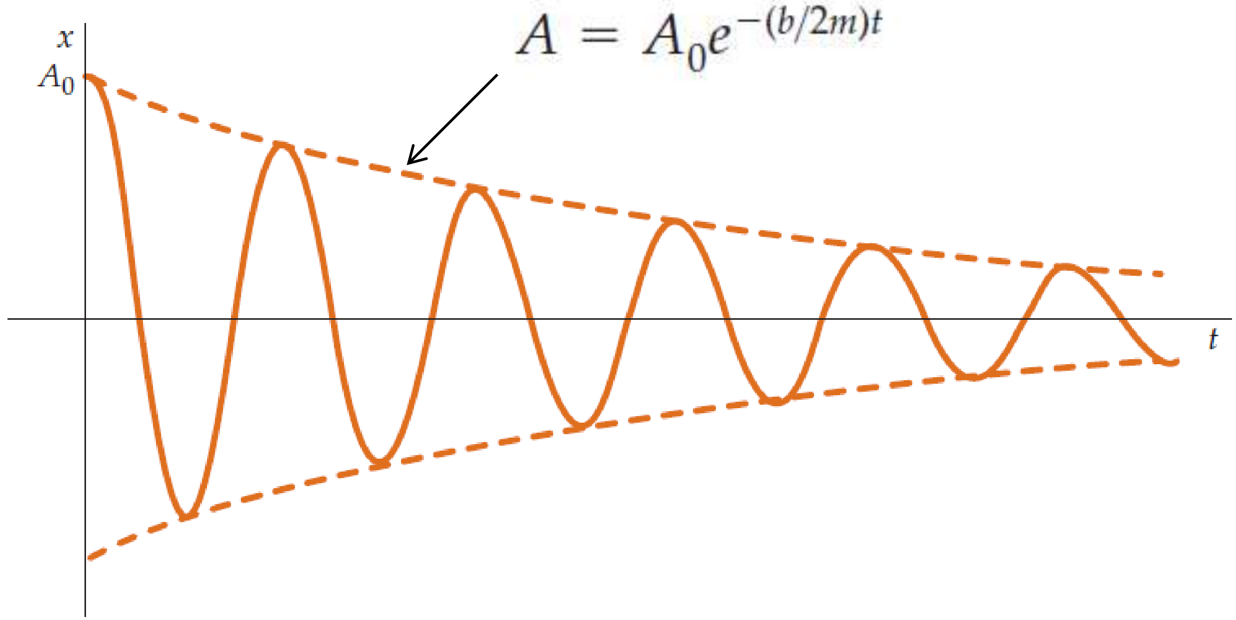
Antwort: 3. $l = gm/k$

Die Schwingungsperioden für Faden und Federpendel lauten:

$$T = 2\pi\sqrt{\frac{l}{g}} \quad , \quad T = 2\pi\sqrt{\frac{m}{k}}$$

Durch Gleichsetzen folgt die obige Beziehung. Zu beachten ist, dass die Schwingungsperiode des Fadenpendels nicht von der Masse abhängt. Die Massenabhängigkeit entsteht aufgrund des Federpendels.

damped oscillator

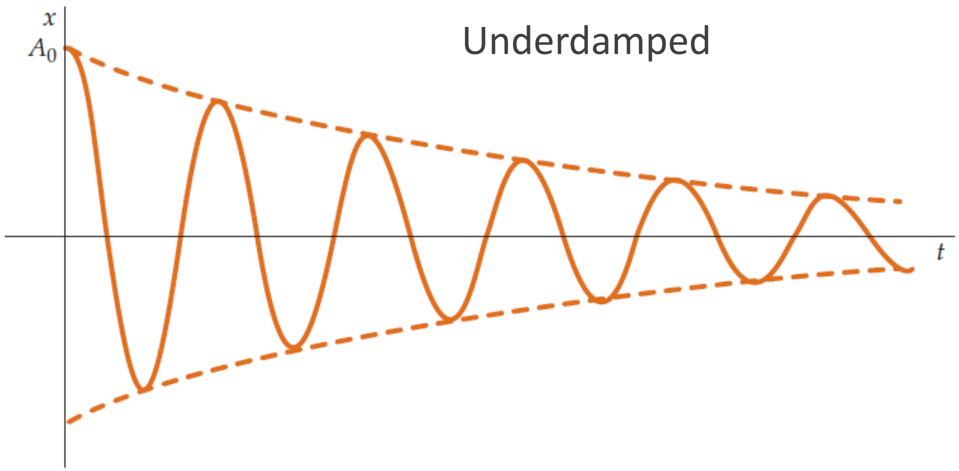


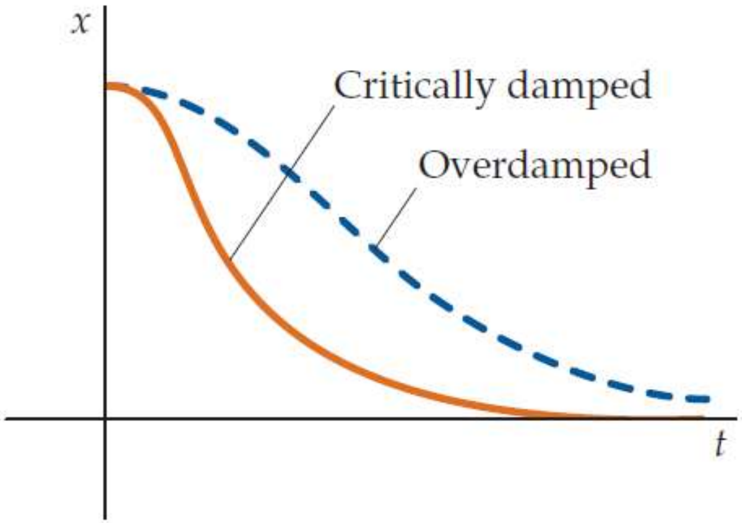
$$x = A_0 e^{-(b/2m)t} \cos(\omega' t + \delta)$$

$$\omega' = \omega_0 \sqrt{1 - \left(\frac{b}{2m\omega_0}\right)^2}$$

Tipler

damped oscillator





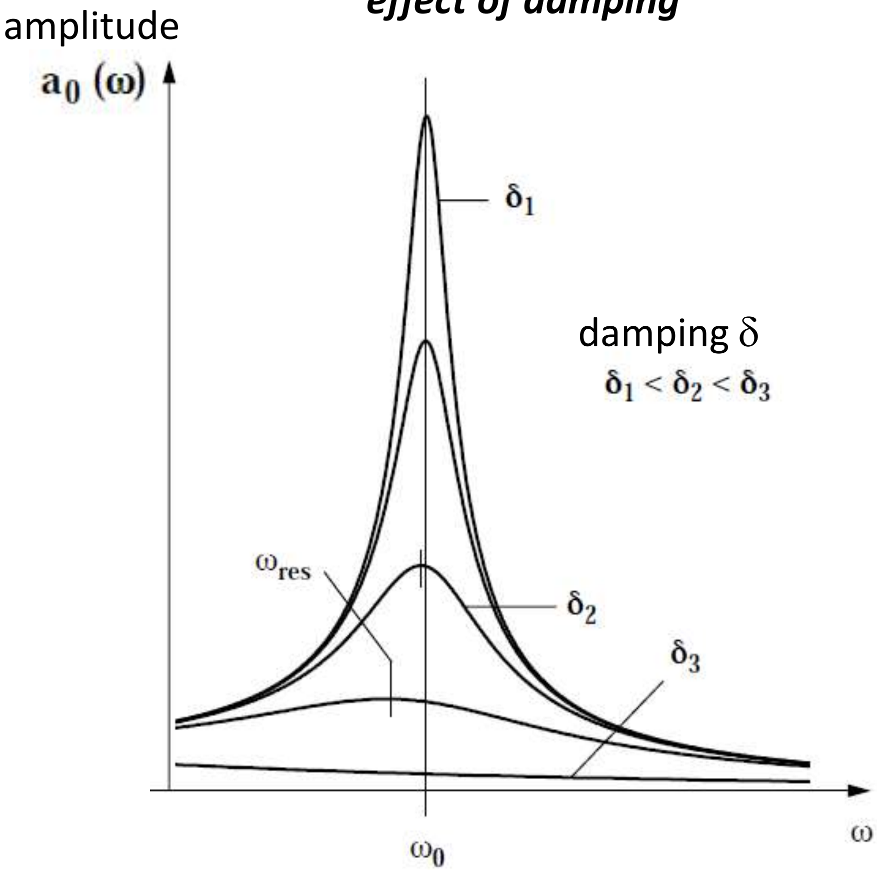
Tipler

driven oscillator

Tipler

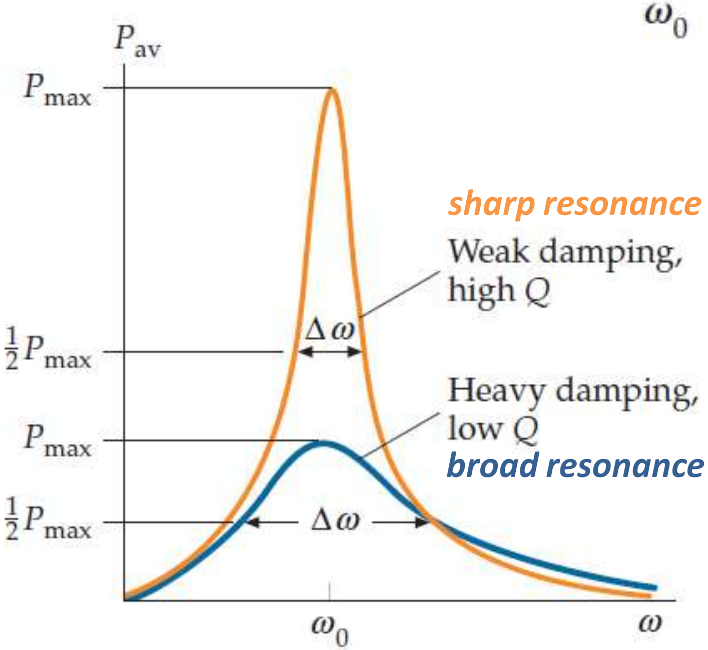
driven oscillator: resonance

effect of damping



average power delivered to an oscillator as a function of the driving frequency for two different values of damping

Q: quality factor $$\frac{\Delta\omega}{\omega_0} = \frac{1}{Q}$$



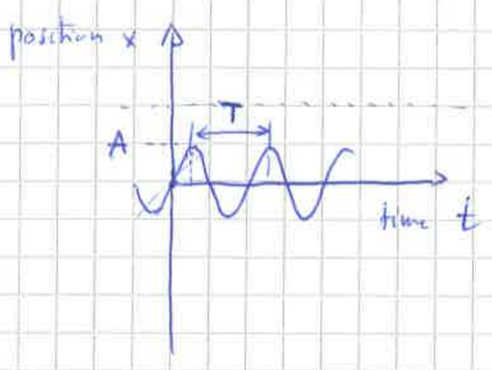
Tipler

Ch. Mechanical vibrations

Trautwein 6.1 – 6.4
Tipler Ch 14

(slide) topic of chapter : oscillatory motion :
a) syst. at equilibrium
b) disturbance
c) periodic motion (time dependent spatial displacement) around equilibrium position.

(slide) representation of oscillatory ~~motion~~ motion : (pendulum/pen)



simple harmonic motion, e.g. (no external force)

$x(t) = A \cdot \sin(\omega t)$, $x(t=0) = 0$

A : amplitude
ω: angular frequency
T : period

$f = 1/T$: frequence

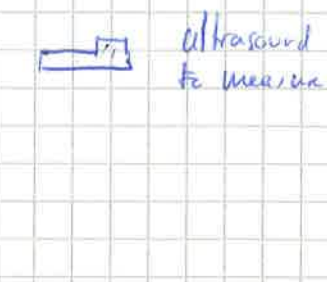
$\omega = 2\pi \cdot f$. $[\omega] = \frac{1}{s}$ (units / dimension)

(units for ω : $\frac{\text{radians}}{s}$)

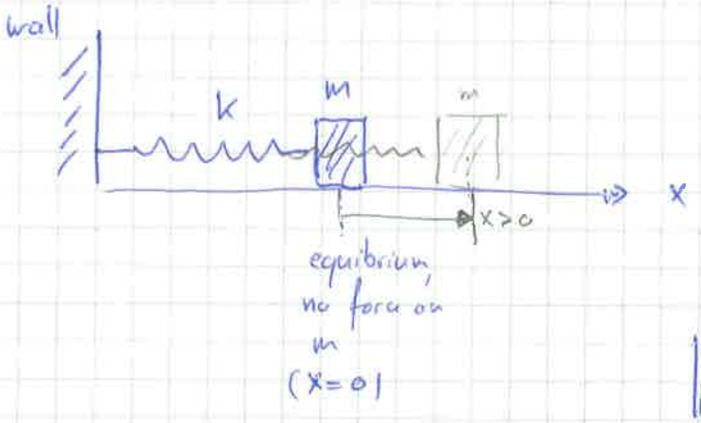
$A(t) \neq const$

- damped oscillation (decay)
  → e.g. friction force
- forced oscillation : external force to maintain / enhance motion

exp : pendulum (Federpendel)
(108-1)



harmonic oscillator



if $x > 0$, $F_x = -k \cdot x$, restoring force (Hooke's law)

Newton: $F_x = m \cdot a_x$

$-kx = m \cdot a_x$, $a_x = \frac{d^2x}{dt^2}$

$$\frac{d^2x}{dt^2} + \frac{k}{m} \cdot x = 0 \qquad (1)$$

a) Note: acceleration $a_x \propto (-x)$ (x: displacement); acceleration opposite to displacement, harmonic / oscillatory motion.

b) general solution for (1)

$$x(t) = A \cdot \cos(\omega t + \varphi_0)$$

amplitude, phase

A: amplitude; max. displacement from equilibrium
ω: ang. frequency
φ_0: phase constant, ~~[illegible]~~

differentiate:

$$v(t) = \dot{x}(t) = \frac{dx(t)}{dt} = -A \cdot \omega \cdot \sin(\omega t + \varphi_0) \quad \propto \omega \qquad \cos'(x) = -\sin'(x)$$

$$a(t) = \dot{v}(t) = \ddot{x}(t) = \frac{d^2x(t)}{dt^2} = -A \cdot \omega^2 \cdot \cos(\omega t + \varphi_0) \quad \propto \omega^2$$

in (1):

$$-A\omega^2 \cdot \cos(\omega t + \varphi_0) + \frac{k}{m} \cdot A \cdot \cos(\omega t + \varphi_0) = 0$$

$$\Rightarrow \omega = \sqrt{\frac{k}{m}}$$

and $x(t) = A \cdot \cos(\omega t + \varphi_0)$, with $\omega = \sqrt{\frac{k}{m}}$
is solution of the differential eq. for a harmonic oscillator

oscillation frequency $f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$;

$f \searrow$ if $m \nearrow$

$f \nearrow$ if $k \nearrow$

(slide) NB: $x(t) = B \cdot \sin(\omega t + \varphi_0)$ also solution

or $x(t) = A \cos(\omega t + \varphi_A) + B \cdot \sin(\omega t + \varphi_B)$ also solution

Questions

slides

1) $A \rightarrow 2.A$; max $F = ?$

$F = -kx$, $\propto x$; if $A_2 = 2.A_1$, $F_2 = 2.F_1$

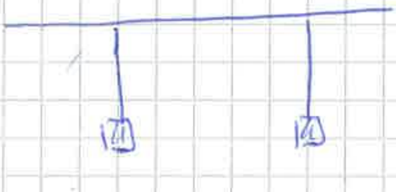
2) 2 masses ($m_1 = m_2$, $k_1 = k_2$) launched at same time
reach equilibrium at same time even if $A_1 > A_2$

$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$, indep of A — no damping, small amplitude ($F = -kx$ valid)

and $v \propto A$, $v_2 = 2.v_1$ if $A_2 = 2.A_1$

exp 2 identical pendulum.

1)

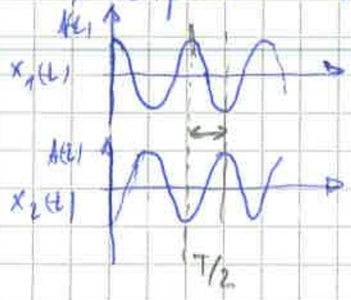


• show effect question 2) above
f indep of A.

2) phase/ dephasing $x(t) = A \cos(\underbrace{\omega t + \varphi_0}_{\text{phase}})$

a) 2 pend. in phase

b) dephased π (180°)



$x_1 = A \cdot \cos(\omega t)$, $\varphi = 0$

$x_2 = A \cdot \cos(\omega t + \pi)$, $\varphi = \pi$

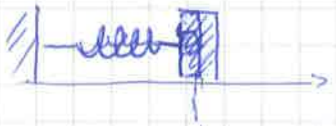
$\cos(0) = 1$

$\cos(\pi) = -1$

kinetic and potential energy for pendulum

$E = E_{pot} + E_{kin}$, total mechanical energy

$= \frac{1}{2} k \cdot x^2(t) + \frac{1}{2} m \cdot v^2(t)$



$x = 0$, rest position

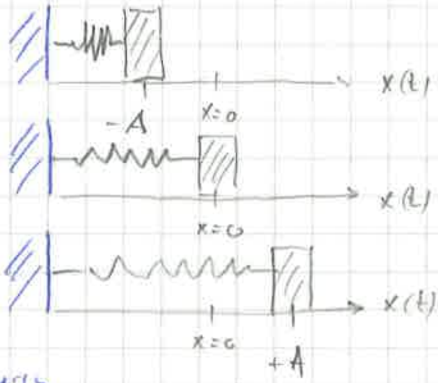
~~$t=0$~~ no oscillation

a) loading, $t = 0$

$E_{pot} = \frac{1}{2} k A^2$, $E_{kin} = 0$

b) $E_{pot} = 0$, $E_{kin} = \frac{1}{2} m v^2$, max speed

c) $E_{pot} = \frac{1}{2} \cdot k A^2$, $kin = 0$



$t = 0$

$t = \frac{T}{4}$

$t = T/2$

T: period

- oscillation between potential & kinetic energy

Ansatz: $x(t) = A \cdot \cos(\omega t)$; $\varphi_0 = 0$

$v(t) = \frac{d\,x(t)}{dt} = -A \cdot \omega (\sin \omega t)$

$v = v_{max}$ $(t = \frac{T}{4})$ for $\sin \omega t = 1$, hence $|v_{max}| = A \cdot \omega$

- Energy conservation: $E = E_{pot} + E_{kin} = const$, $E_a = E_b = E_c$

$\Rightarrow \frac{1}{2} k \cdot A^2 = \frac{1}{2} m v^2$ with $v = v_{max}$

$= \frac{1}{2} m \cdot A^2 \cdot \omega^2$

$\omega^2 = \frac{k}{m}$ and $\omega = \sqrt{\frac{k}{m}}$

(slide) energy

1) $x(t)$

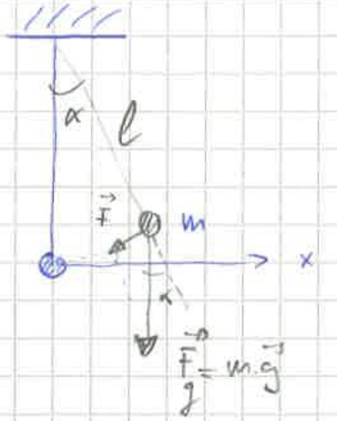
$E_{pot}(t)$; $E_{tot} = const$

$E_{kin}(t)$

2) a) $E_{pot}(t) = \frac{1}{2} k x^2$, parabolic function

b) harmonic approx. for general potential -

Application to the pendulum (mathematical pendulum, point mass)



force on m, along trajectory

$F = F_g \cdot \sin\alpha = mg \sin\alpha$

if α small, $\sin\alpha \approx \alpha$

and $F = mg\alpha$

Eq. of motion: $m \ddot{x} + mg\alpha = 0$ $\qquad \ddot{x} = \frac{d^2x}{dt^2}$

small α => $\alpha \approx \frac{x}{\ell}$

and $\cancel{m}\ddot{x} + \frac{\cancel{m}g}{\ell} x = 0$

Similarly to before, Ansatz: $x(t) = A \cdot \cos(\omega t)$

hence $\quad \| \; \omega = \sqrt{\frac{g}{\ell}}$, for small angles

Question slide

Pendulum and spring oscillator

$T = 2\pi \sqrt{\frac{\ell}{g}}$, $\omega = \sqrt{\frac{g}{\ell}}$

$T = 2\pi \sqrt{\frac{k}{m}}$, $\omega = \sqrt{\frac{m}{k}}$

damped oscillations:

so far damping ignored: air friction, friction dissipated mechanical energy

underdamped: small damping; slowly decreasing amplitude
example: child on swing

overdamped: large damping: e.g. pendulum in honey...
oscillator fails to complete even 1 cycle of oscillation;
(it ~~approaches~~ moves towards eq. position with a speed approaching zero as the object approaches eq. position)

critically damped: minimum damping for non-oscillatory motion

underdamped motion:

damping force $\vec{F}_d = -b\,\vec{v}$

b = const, linearly damped
$F \propto v$, opposite to motion
e.g. Stokes: $\vec{F} = -6\pi\eta R\,\vec{v}$
viscosity η, R: radius sphere

Eq. of motion (Newton):

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

$-kx$: restoring force (spring)
$-b\frac{dx}{dt}$: drag $\propto \frac{dx}{dx} = v$

and $$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + k\cdot x = 0$$

Solution:
$$x(t) = A_0\, e^{-(b/2m)t}\cdot\cos(\omega' t + \varphi_0)$$
$$= A_0\, e^{-t/\tau}\cdot\cos(\omega' t + \varphi_0), \quad \tau = \frac{2m}{b}$$
decay time, characteristic time constant

and $$\omega' = \omega_0\sqrt{1 - \left(\frac{b}{2m}\frac{1}{\omega_0}\right)^2}$$
$$= \omega_0\sqrt{1 - \left(\frac{1}{\omega_0\tau}\right)^2}, \quad \omega_0 = \sqrt{\frac{k}{m}}$$

(slide): graph $x(t)$
dashed line: $A = A_0 e^{-t/\tau_0}$

- weak damping: b small width $\frac{b}{2m\omega_0} \ll 1$

  and $\omega' \approx \omega_0$

- critical damping: (min damping for non oscillatory motion)

  b ↗ until $b_c = 2m\omega_0$

  $\omega' = \omega_0 \sqrt{1 - \left(\frac{b}{2m} \cdot \frac{1}{\omega_0}\right)^2}$ — $= 0$ if $b = 2m\omega_0$

(slide) underdamped, crit damped, overdamped

exp. (108-6) damped pendulum

exp. (108-6) metal blade without/with damping (rubber tube)

show oscillation on oscilloscope

skip exp. 108-7 #1011 galvanometer Skip