Quantum Transport FS 2014

Problem Set 3 Due: 06.05.14

1. Quantum Dot Energies

(a) Starting from the total energy of a quantum dot (constant interaction model), derive the addition energy and the Coulomb blockade (CB) charging energy following the lecture notes.

(b) Consider the stability diagram of a quantum dot CB diamond. What do the slopes show? Express the slopes of these lines in terms of capacitances.

(c) Derive the average quantum level spacing of a circular quantum dot of area πR^2 starting from the 2D density of states. Further, find an expression for the charging energy, assuming an infinitely thin flat disc of radius R. Compare qualitatively the size dependence of the charging energy and the quantum level spacing.

2. Sequential Tunneling through a Single-Level Quantum Dot

Consider a quantum dot coupled to two reservoirs with Fermi-Dirac distributions at temperature T and tunneling rates Γ_S and Γ_D through the source and drain barriers, respectively. Assume the temperature broadened regime $h\Gamma_{S,D} \ll k_B T$. A source-drain bias $eV_{SD} = \mu_S - \mu_D$ is applied, where μ_S and μ_D are the chemical potentials of the source and drain reservoir. Assume that the dot has only a single quantum level at energy ϵ above μ_D. *Hint:* Assume $\mu_D = 0$ throughout this exercise for simplicity.

(a) Draw a sketch of the situation with reservoirs, dot, energy level and tunnel barriers.

(b) Derive an expression for the sequential tunneling current I through the dot as a function of T, V_{SD}, Γ_S, Γ_D and for arbitrary level energy ϵ.

(c) How can this dot be used as a thermometer?

(d) From the current I, find the differential conductance g as a function of the same parameters as for the current. What is the line shape as a function of gate voltage?

3. Conductance Quantization in a Magnetic Field

Consider a QPC in the presence of an external magnetic field B, applied transverse to the plane of the 2DEG. As discussed in the lecture, the conductance shows plateaus quantized in units of $\frac{2e^2}{h}$ for $B = 0$ when the gate voltage is swept.

(a) What would you expect to happen for $B > 0$? Would the steps survive? If yes, would they be wider or narrower compared to the zero field situation? What about the step height? Degeneracy of the modes?

(b) See Figure 1: What would you expect to measure for two QPCs in series (2DEG region 1-3) that are sufficiently close together such that electrons do not get scattered in between? What about $B > 0$?[1]

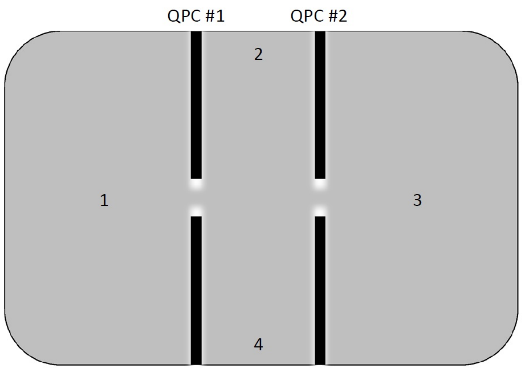

Figure 1: For exercise 3.(b): Two QPCs in series on top of a 2DEG (grey shaded). Numbers 1 to 4 depict different regions of the 2DEG.

4. Weak Antilocalization in the Strong Spin-Orbit Coupling Limit *(optional)*

Consider weak antilocalization in a GaAs 2DEG (cf. lecture notes / slides on weak antilocalization). Derive the disorder averaged quantum interference contribution of the electron spin sector. Assume strong spin-orbit coupling, i.e. the spin direction is fully randomized upon return to the origin, neglecting decoherence effects. *Hint:* A convenient representation of an arbitrary spin rotation for a spin-$^1/_2$ particle is given in terms of Euler angles by the following rotation matrix:

$$\hat{R} = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) e^{\frac{i(\phi+\psi)}{2}} & i\sin\left(\frac{\theta}{2}\right) e^{\frac{-i(\phi-\psi)}{2}} \\ i\sin\left(\frac{\theta}{2}\right) e^{\frac{i(\phi-\psi)}{2}} & \cos\left(\frac{\theta}{2}\right) e^{\frac{-i(\phi+\psi)}{2}} \end{pmatrix}$$

[1] Optional: *After* you have made your own thoughts to these questions, you can have a look at Staring *et al.*, PRB **41**, 8461 (1990).