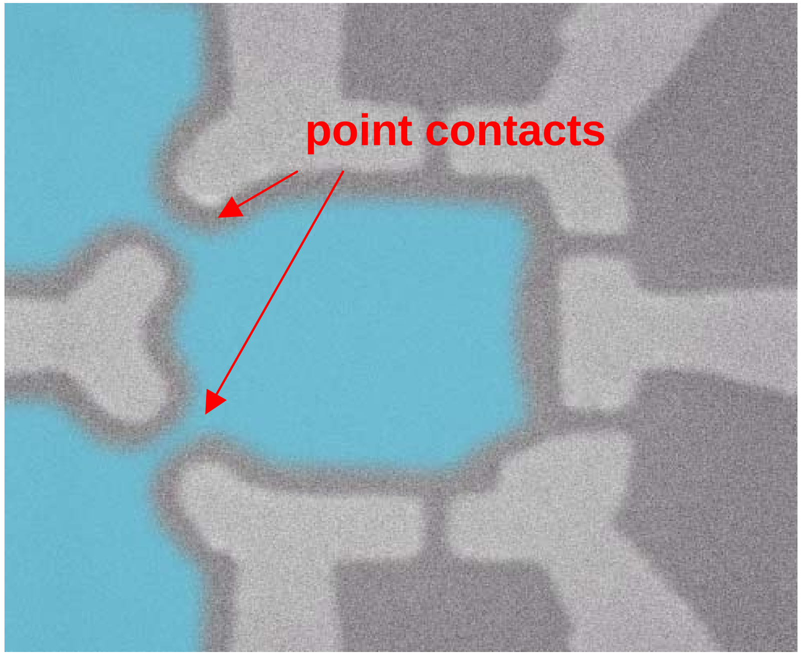
Quantum Dots II

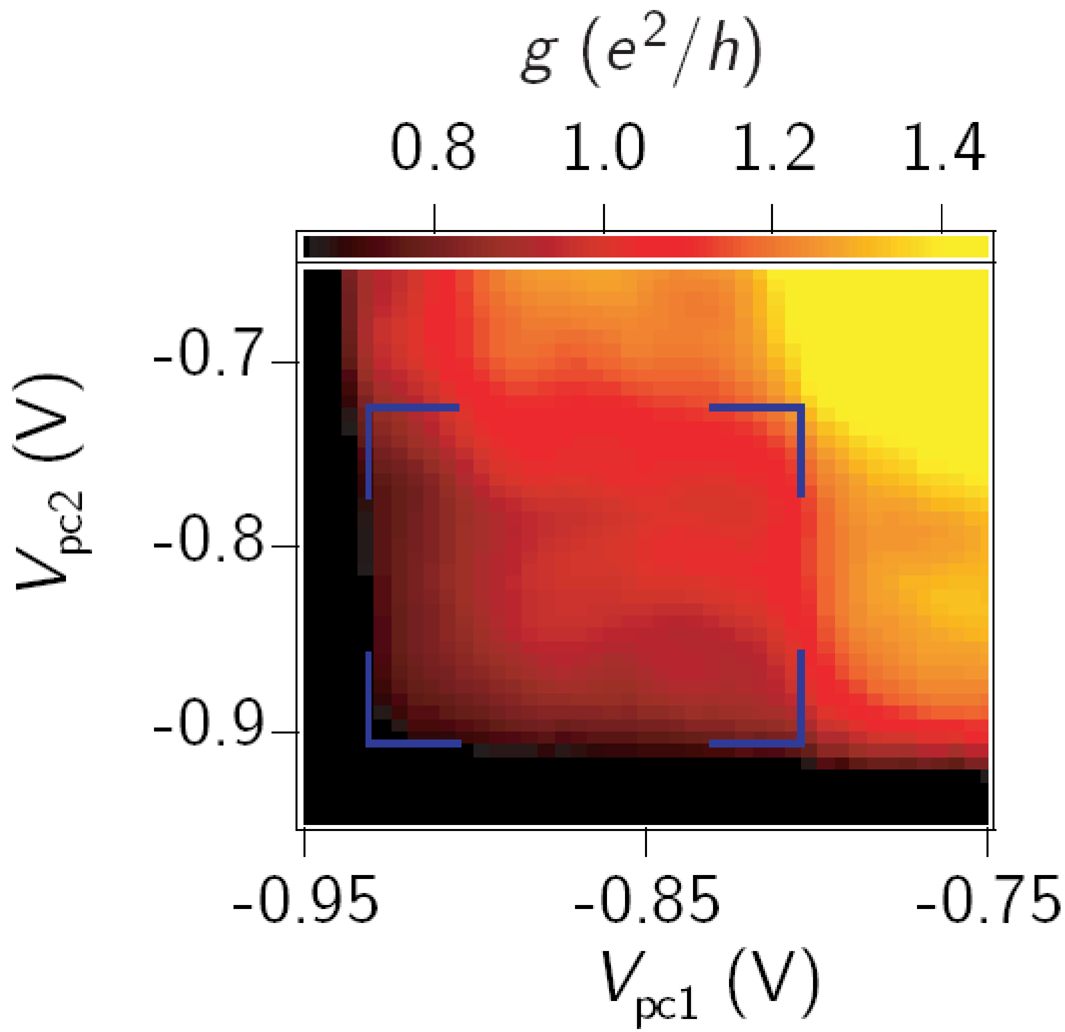
1. Open Dot Experiments
2. Kondo effect
3. Few Electron Dots
4. Double Quantum Dots

Huibers, Ph.D. Thesis (1999)
Huibers et al., PRL83, 5090 (1999)

Open Dot Regime

Open Dot



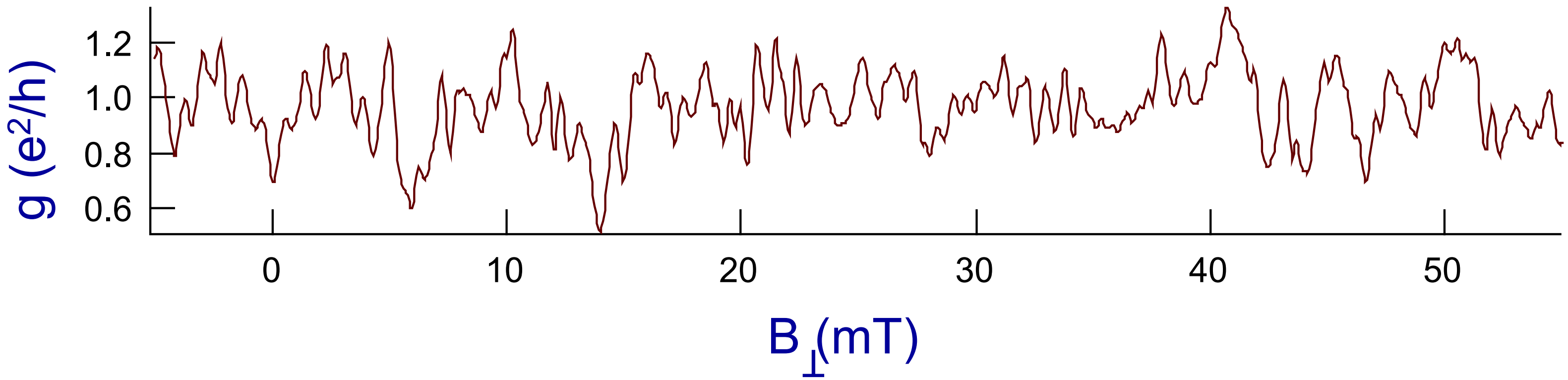


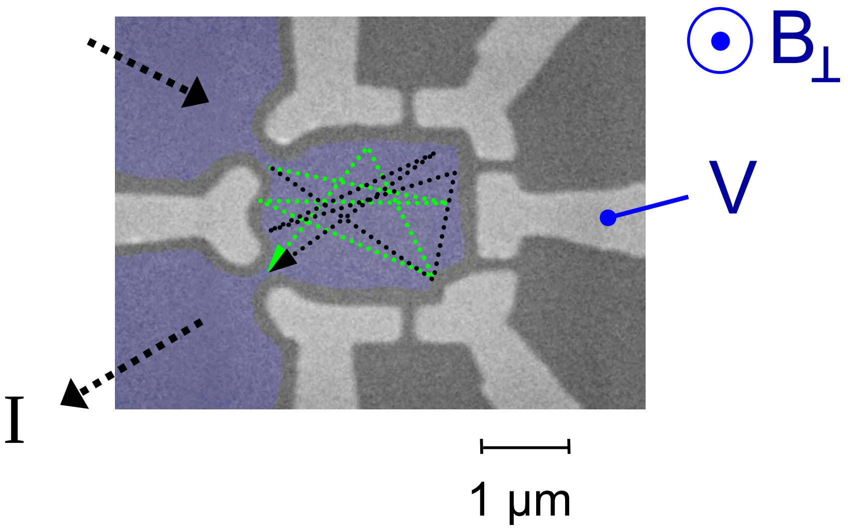
- V_{gate} set to allow ≥ $2e^2/h$ conductance through each point contact
- Dot is well-connected to reservoirs
- Transport measurements exhibit CF and Weak Localization

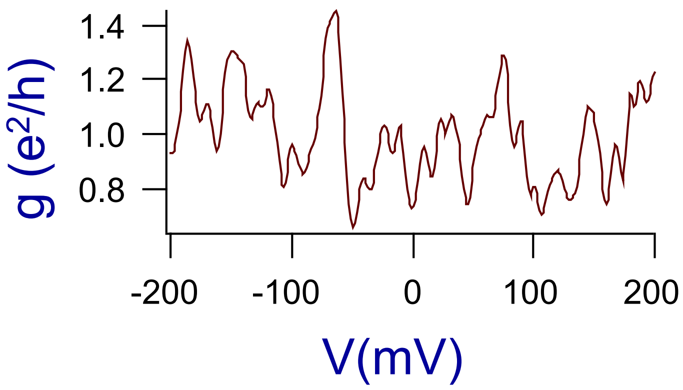
many open dot slides: A. Huibers and J. Folk

Open Dot Regime: Conductance Fluctuations

$N_L=N_R=1$



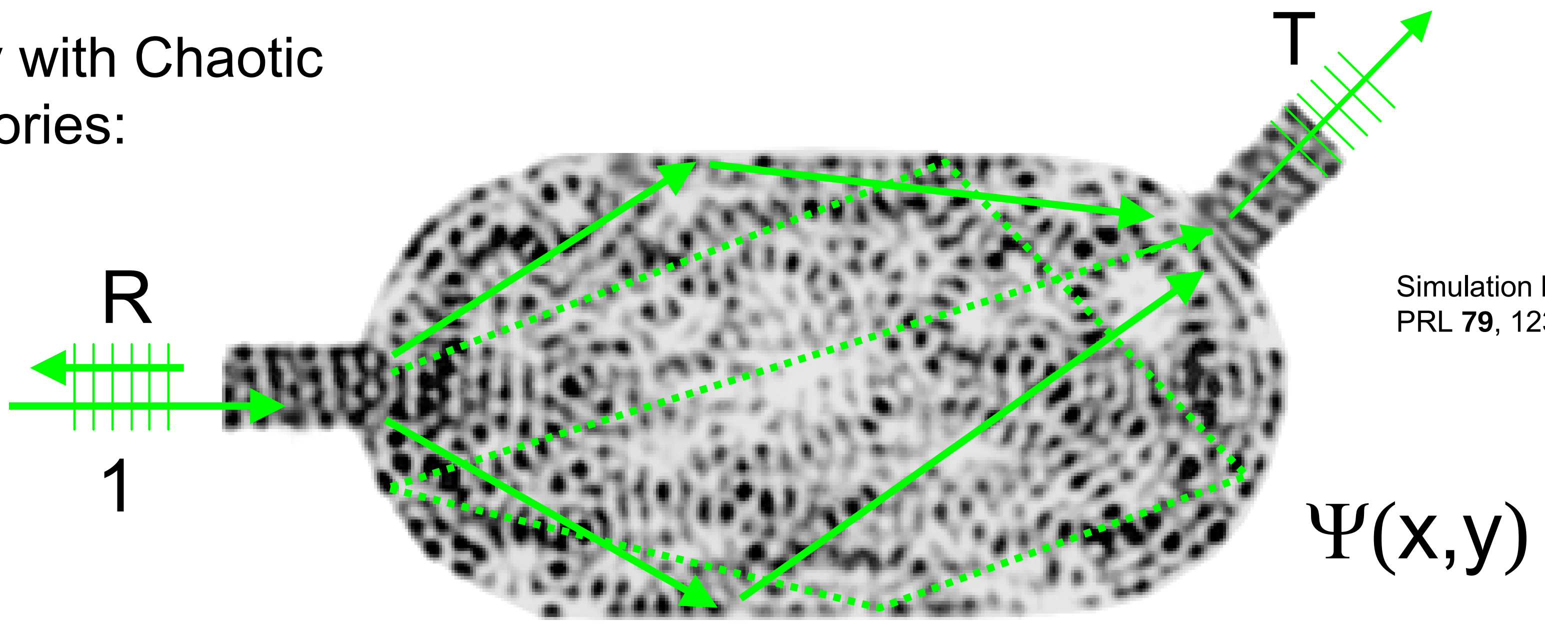




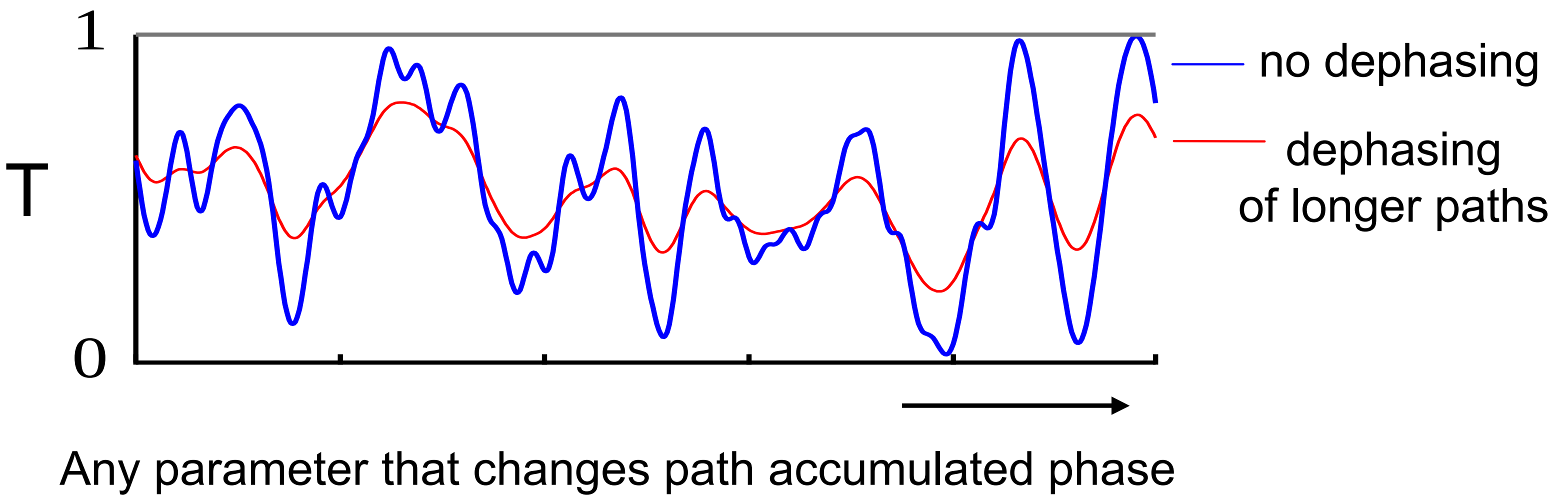
<u>Repeatable</u> random
intereference fluctuations
as function of dot parameters

Two-Dimensional Quantum Dot

2D Cavity with Chaotic trajectories:



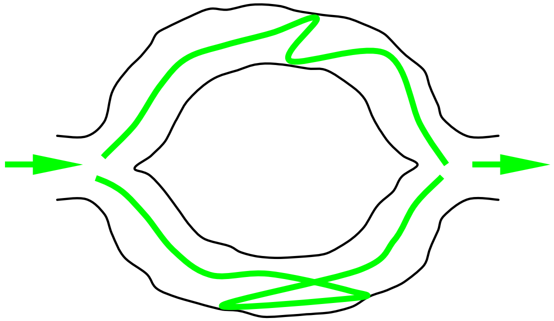
Simulation by R. Akis, PRL **79**, 123 (1997)



Goal: use quantum dot as a probe of quantum phase coherence

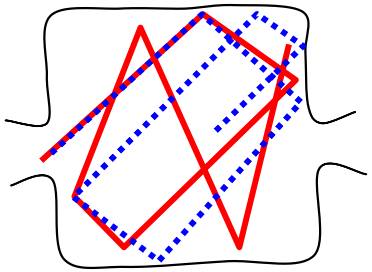
Interferometers

Two-arm:



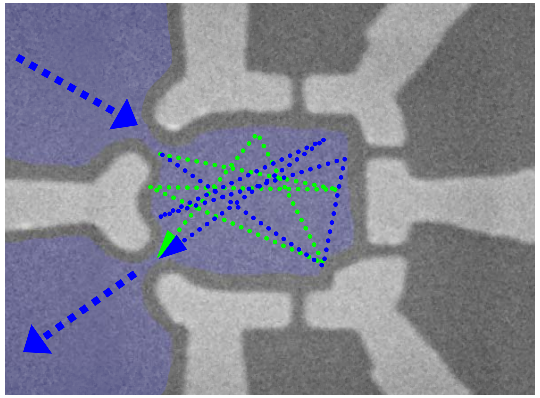
sometimes:
reflections or
small signal

Regular/
Integrable:



Problem:
<u>partially</u> chaotic

Chaotic:

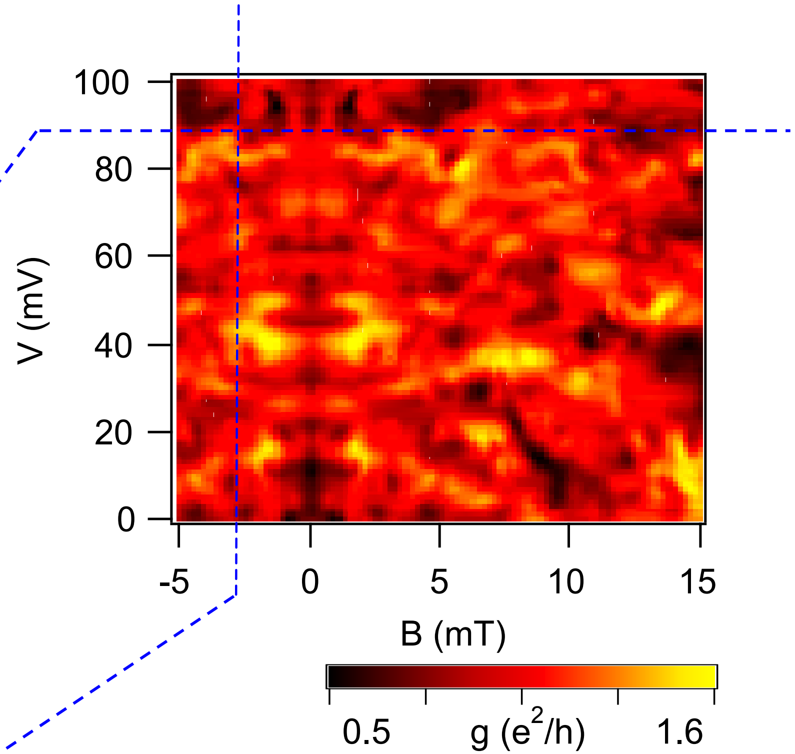


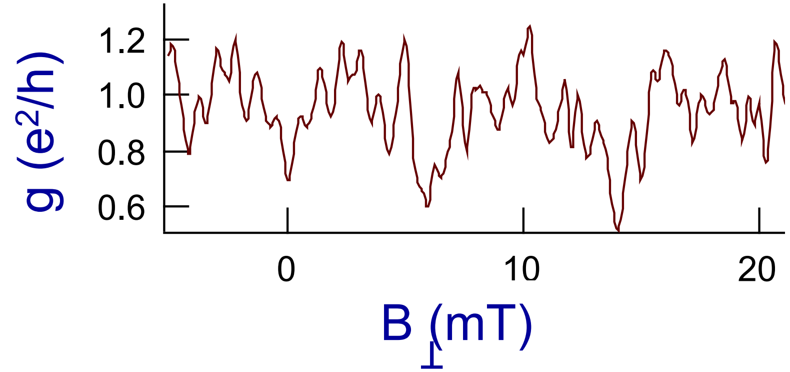
1. Mostly chaotic/ergodic

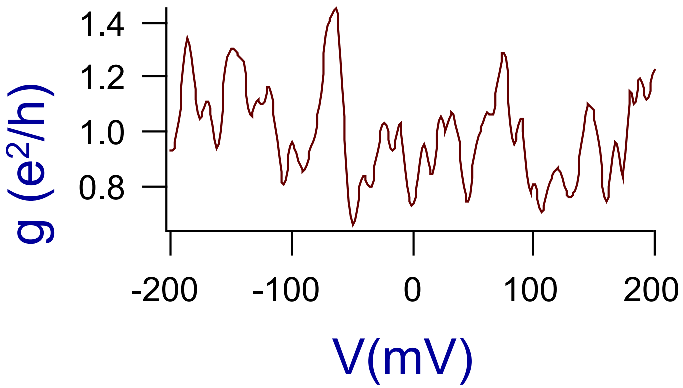
2. Interesting physics
& complete description

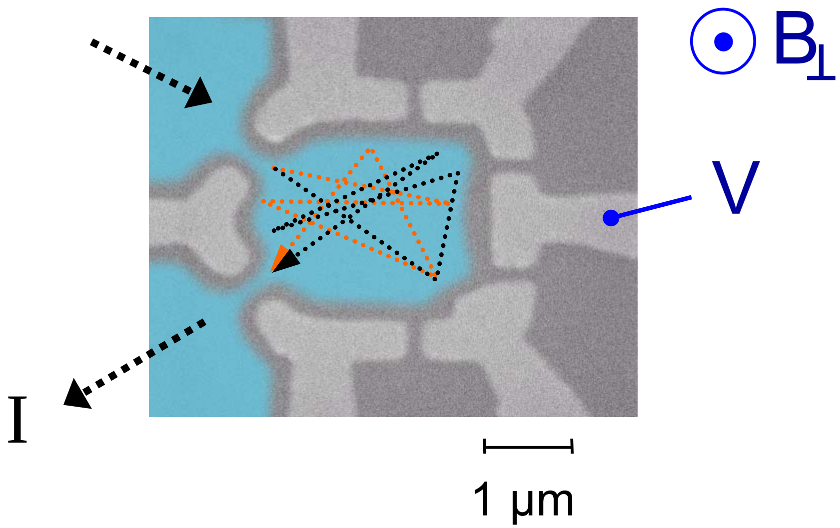
Quantum Interference in Open Dots

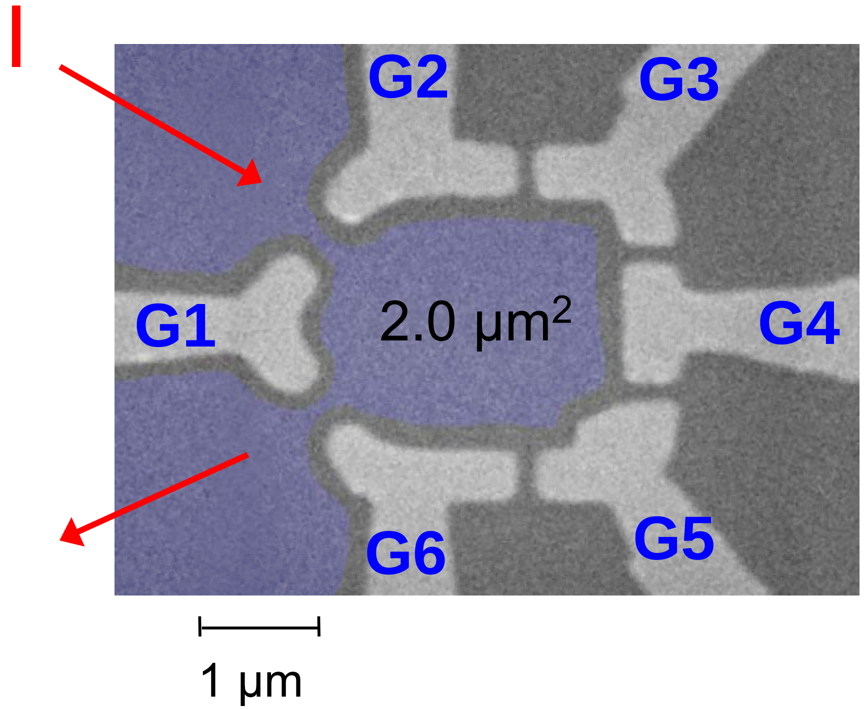
Interference between all possible trajectories gives rise to repeatable random intereference fluctuations as function of dot parameters











2D conductor:

area = 2.0 μm^2

charge density = 2 10^{11} e/cm^2

λ_F = Fermi wavelength = 50 nm

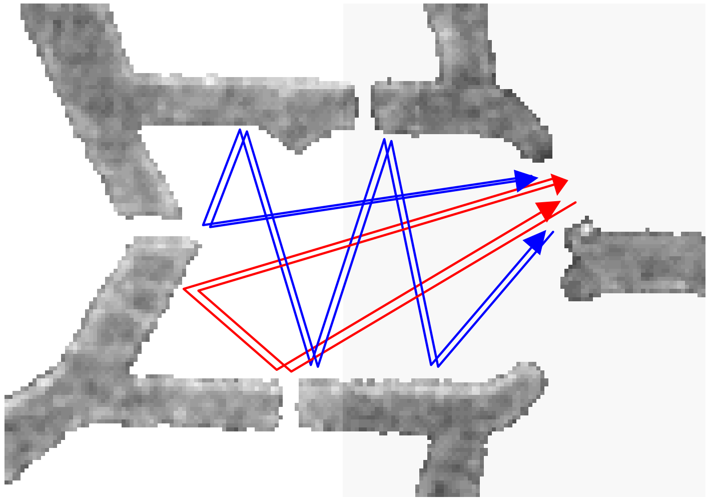
v_F = Fermi velocity = 200 μm/ns

E_F = Fermi energy = 7 meV

Dwell time in dot:	200 ps
Crossing time:	7 ps
	30 bounces

bulk mean free path ℓ_e ~ 2- 10 μm

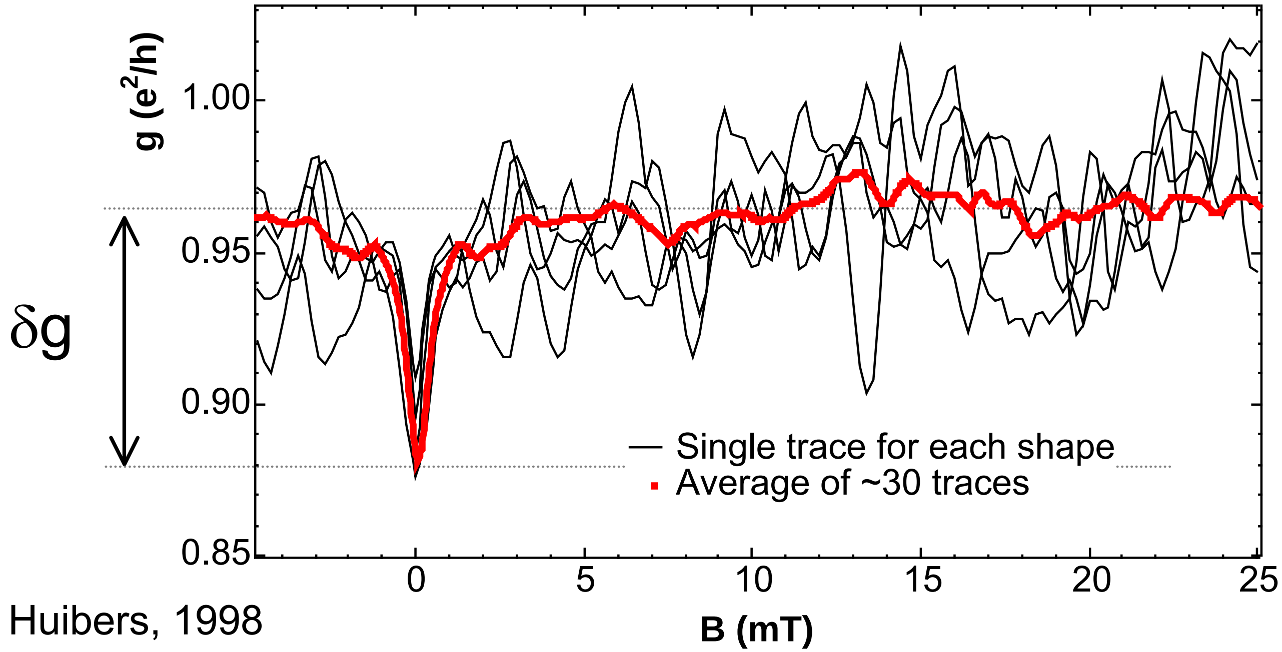
Weak Localization



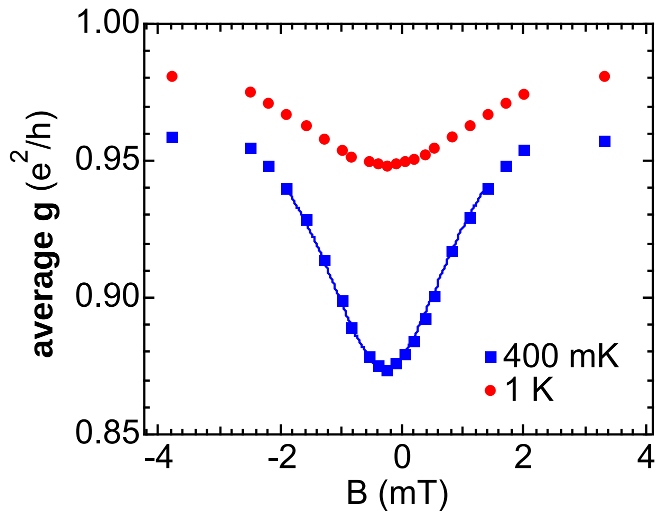
At B=0, phase-coherent backscattering results in "weak localization"

↓

Conductance dip at B=0

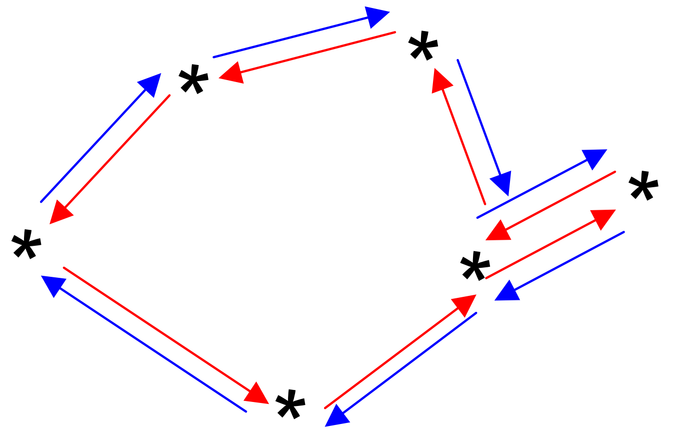


Huibers, 1998



Quantum Correction: Weak Localization

constructive interference of coherently backscattered, time reversed trajectories decreases conductivity



$$\frac{\delta\sigma_{loc}}{\sigma} \propto -\frac{1}{k_F \ell} \ln\left(1+\frac{\tau_\varphi}{\tau}\right) \qquad \text{2D } (L_\varphi << W)$$

$$\frac{\delta\sigma_{loc}}{\sigma} \propto -\frac{L_\varphi}{W}\frac{1}{k_F \ell}\left(1-\left(1+\frac{\tau_\varphi}{\tau}\right)^{-1/2}\right) \qquad \text{1D } (L_\varphi >> W)$$

magnetic field: AB-flux, cut off trajectories of area $A>\phi_0 B$
magnetoconductance

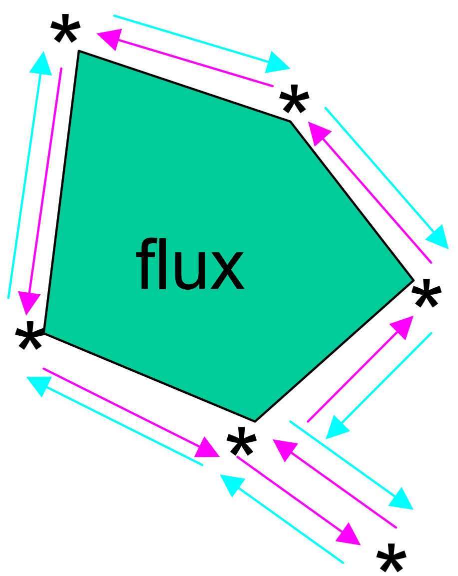
(assuming spinless electrons)

Weak Localization in Magnetic Fields

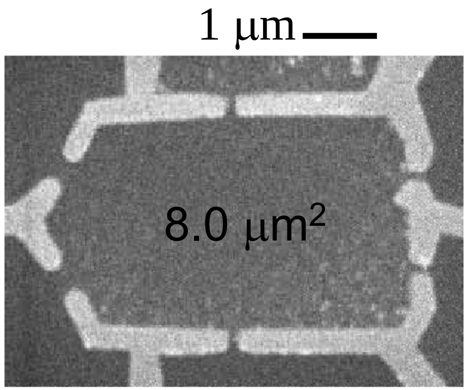
in a given magnetic field B, trajectories enclosing flux acquire additional Aharonov-Bohm phase:

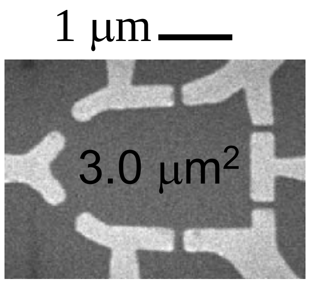
$$\phi = \frac{2e}{\hbar}\int(\nabla \times A)\cdot d\vec{S} = \frac{2eBS}{\hbar}$$

when summing over all trajectories, this ϕ will effectively eliminate trajectories of area $A >> \phi_0/B$. ($\phi_0 = h/e$)



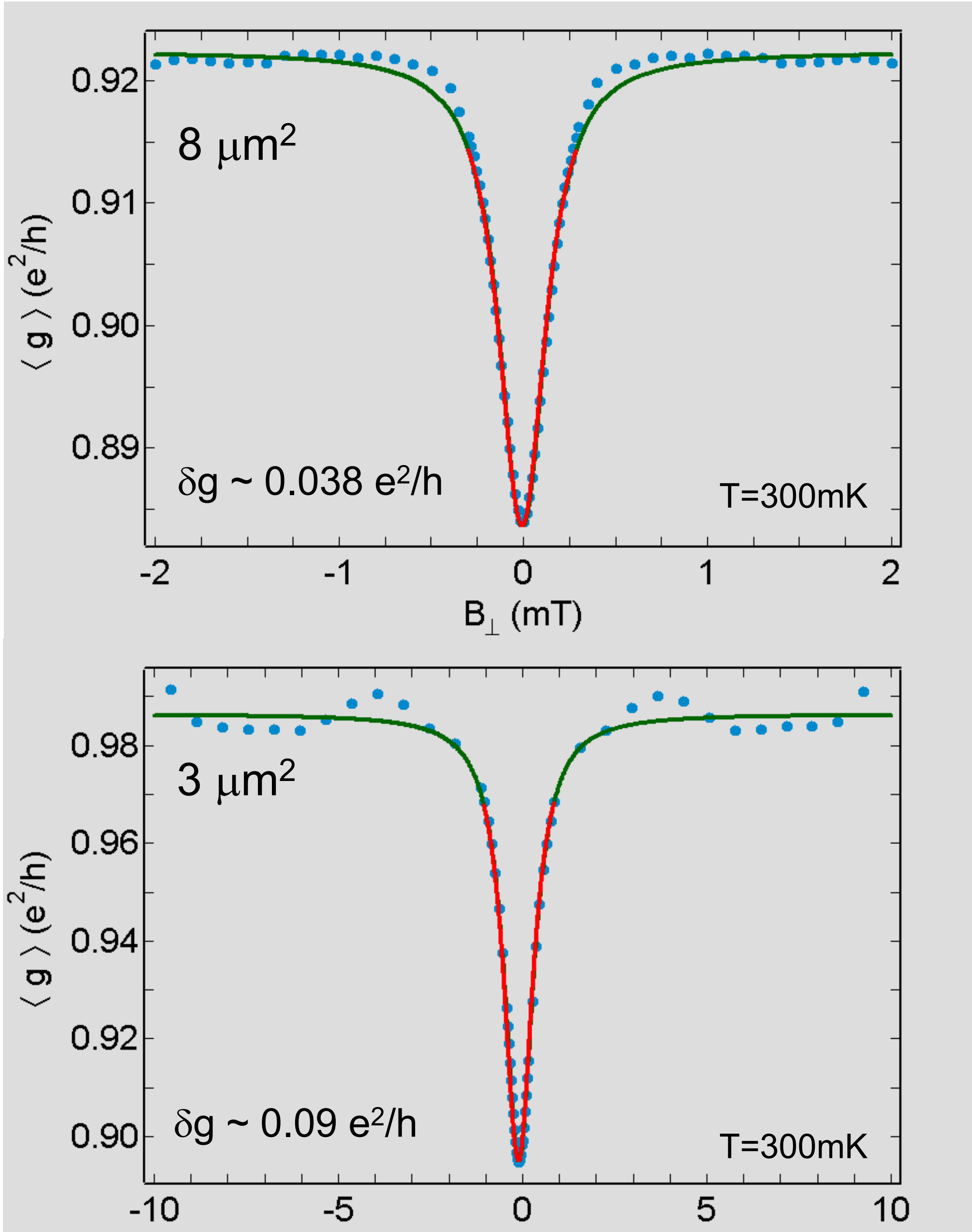
Weak Localization: Measure of Dephasing





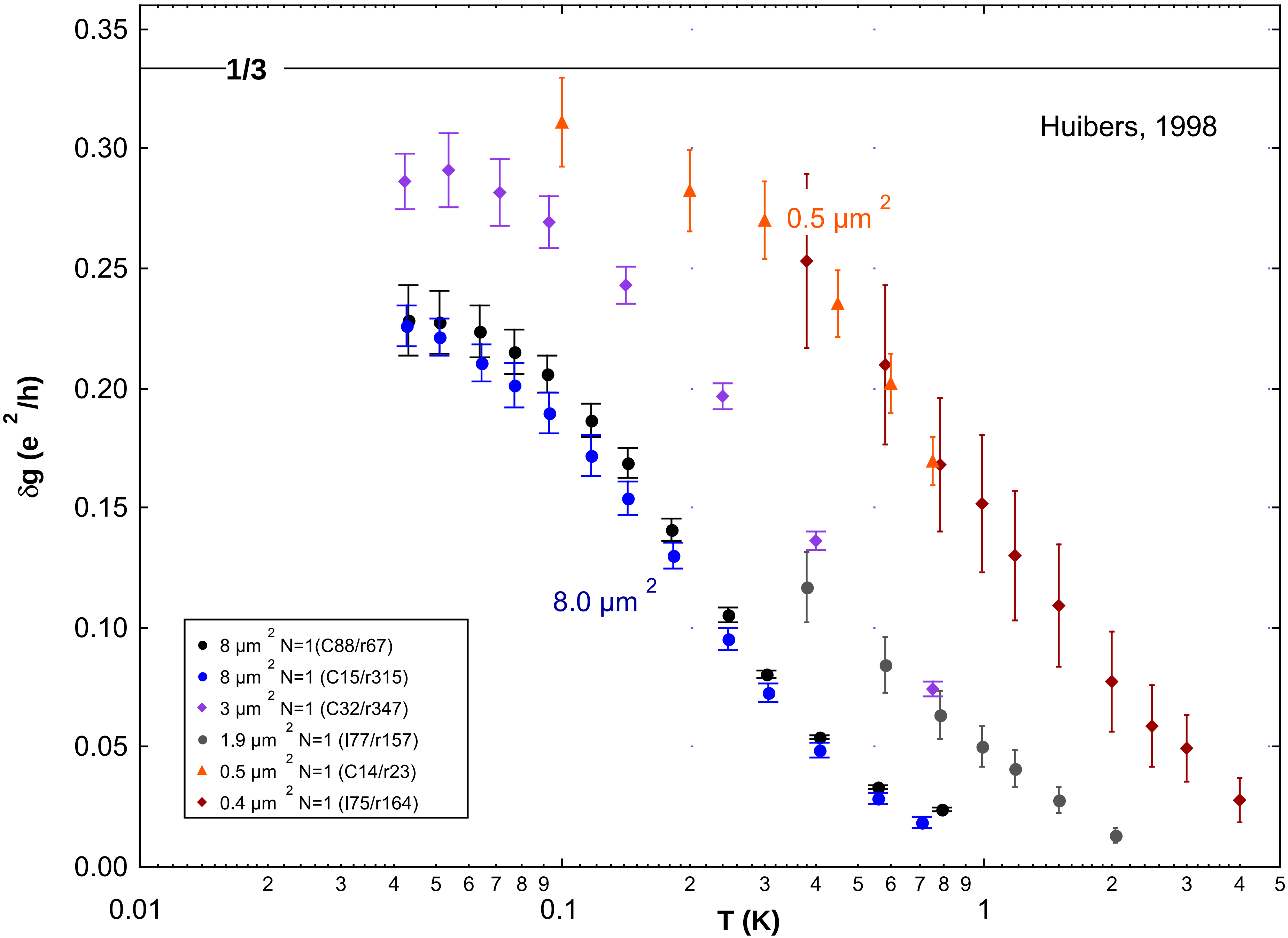
random matrix theory

$$\delta g = \frac{1}{2N + 1 + \gamma_\phi} \qquad \tau_\phi^{-1} = \frac{\Delta}{h}\gamma_\phi$$

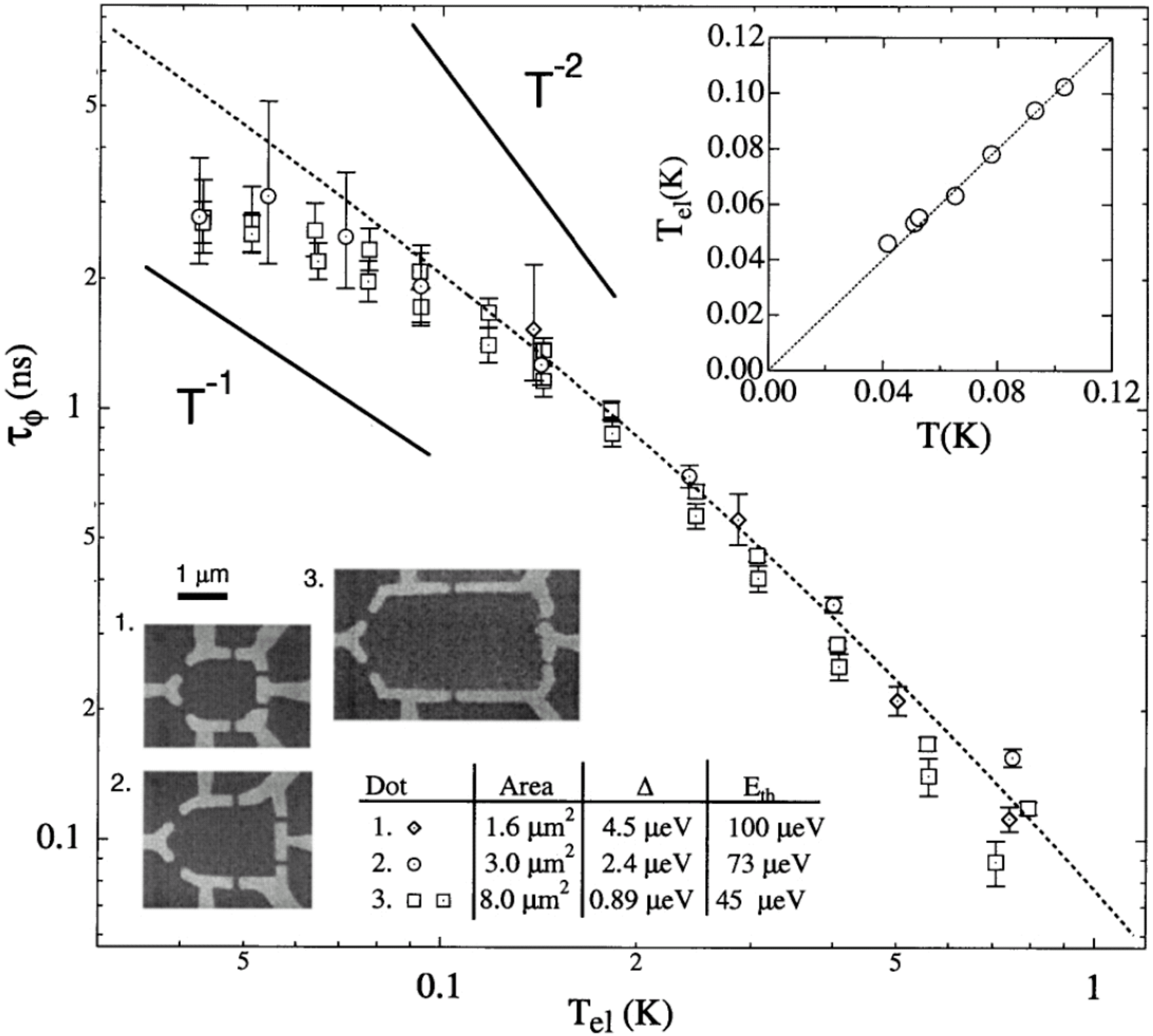


DMZ, 2002

Weak Localization vs T



Low Temperature Saturation?



Huibers et al., PRL83, 5090 (1999)

Spin-Orbit Coupling

electrons move with the Fermi velocity, electric fields in material appear as magnetic fields in the rest frame of the electron

these magnetic fields

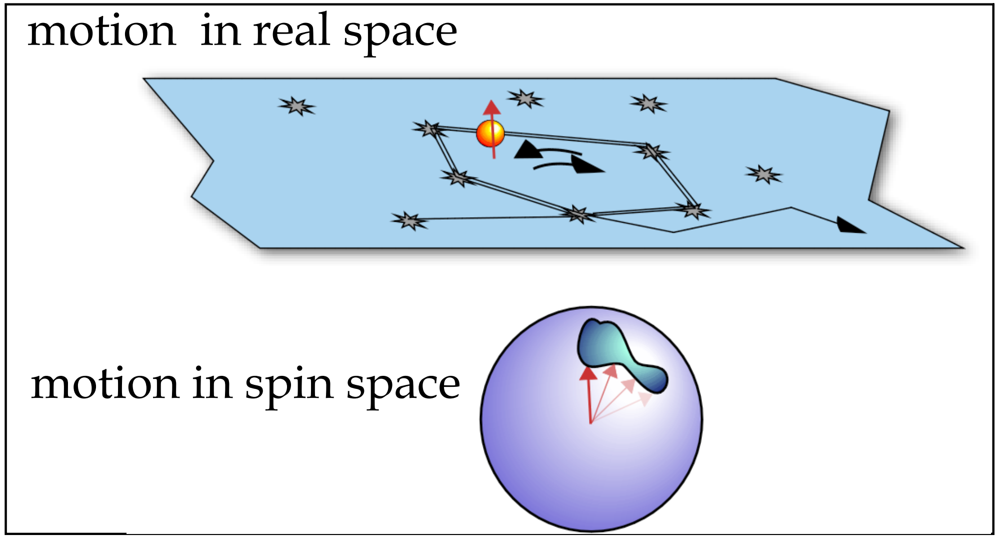
- depend on magnitude of electron velocity (density dependence)
- couple to the electron spin via Zeeman coupling

⟶ spin-precessions

electric fields due to:

- heterointerface (***Rashba***)
- crystalline anisotropy in III-V zincblende crystal (***Dresselhaus***)

spin precession affects phase interference (2π in spin space gives -1 to phase)



Spin-Orbit Coupling

presence of electric fields $\vec{E} = -\frac{1}{e}\vec{\nabla}V$

electrons are moving in these electric fields

rest frame of electrons: effective magnetic field

$$\vec{B}_{so} = -\frac{\vec{v}}{c} \times \vec{E}$$

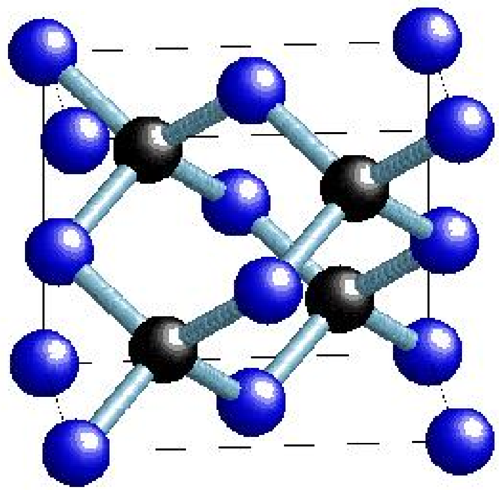
magnetic moment $\vec{\mu} = \frac{e\vec{S}}{mc}$ of electron couples to $\vec{B}_{so}$

$$H_{so} = -\vec{\mu} \cdot \vec{B}_{so}$$

electrons precess around B_{so}
B_{so} depends on the electron momentum

spin rotation symmetry is broken,
time reversal symmetry is NOT broken

Spin-Orbit Coupling due to Crystal Anisotropy



Conventional cell

III-V Semiconductor

Zinkblende crystall structure:
two interpenetrating fcc lattices
with only Ga atoms on one lattice,
only As on the other

absence of inversion symmetry

symmetry considerations:

$$H_{so} = \gamma(\sigma_x k_x (k_y^2 - k_z^2) + \text{cycl.})$$

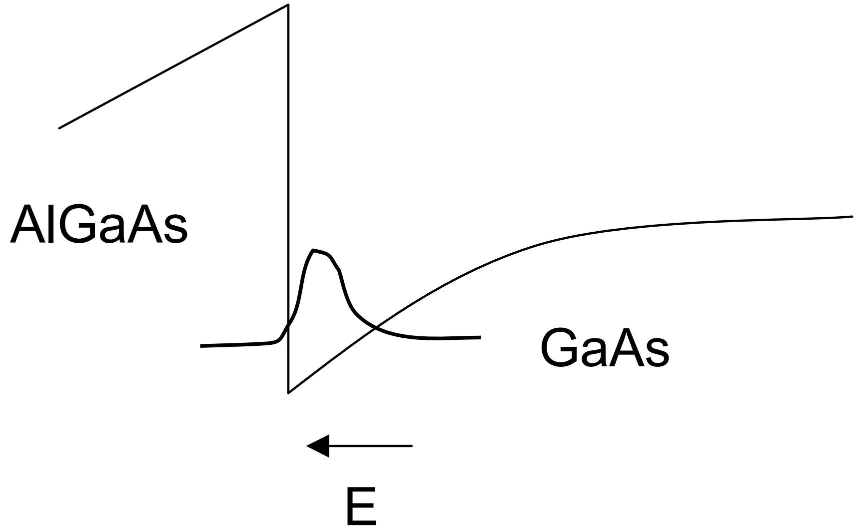
G. Dresselhaus,
Phys. Rev. 100, 580 (1955)

after size quantization (2D): $\langle k_z \rangle = 0 \qquad \alpha = \gamma \langle k_z^2 \rangle$

$$H_D^{(1)} = \alpha(\sigma_x k_x - \sigma_y k_y)$$ linear Dresselhaus term

$$H_D^{(3)} = \gamma(\sigma_y k_y k_x^2 - \sigma_x k_x k_y^2)$$ cubic Dresselhaus term

Spin-Orbit Coupling due to Heterointerface



electric field at heterointerface perpendicular to 2D plane

$$\vec{B}_{so} \propto (k_y E, -k_x E, 0) \perp \vec{k}$$

$$H_R = \beta(\sigma_x k_y - \sigma_y k_x)$$

Rashba term (linear)

coupling strength parameters β and γ can be determined from Band structure, for example in $k \cdot p$ approximation

Weak Antilocalization

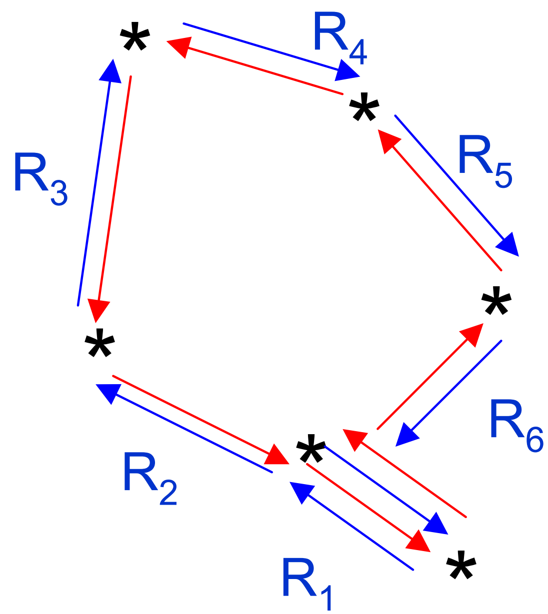
initial state: $|i\rangle$

final (forward): $|f_f\rangle = R_N \dots R_2 R_1 |i\rangle = R|i\rangle$

final (backward): $|f_b\rangle = R_1^{-1} R_2^{-1} \dots R_N^{-1} |i\rangle = R^{-1}|i\rangle$ (TRS)

R_i: spin rotations $R = R_N \dots R_2 R_1$ $R^\dagger R = 1$ $R^{-1} = R^\dagger$

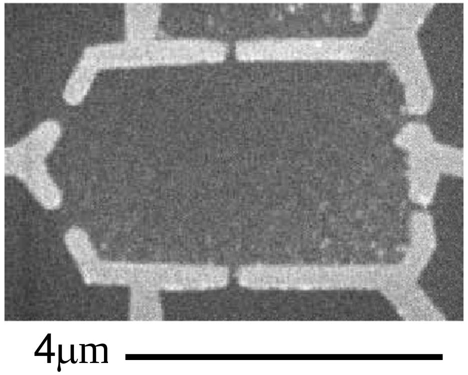
interference term $\langle f_b | f_f \rangle = \langle i | R^2 | i \rangle$



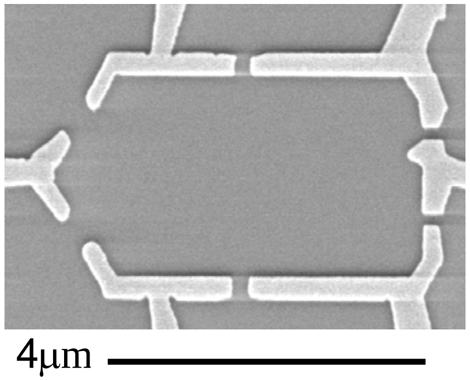
assuming strong spin-orbit coupling, summing over all trajectories is equivalent to averaging R^2 over sphere

$\overline{\langle f_f | f_b \rangle} = -\frac{1}{2}$ **destructive interference**
opposite sign for Magnetoconductance

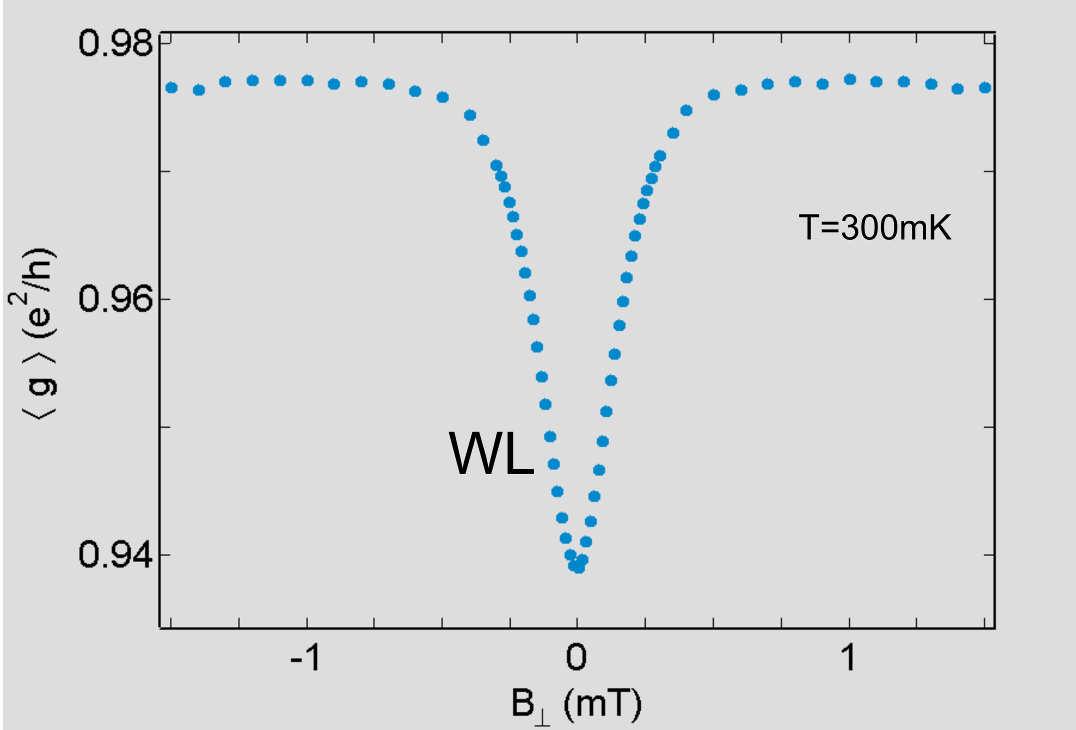
low density
weaker SO coupling
weak localization (WL)

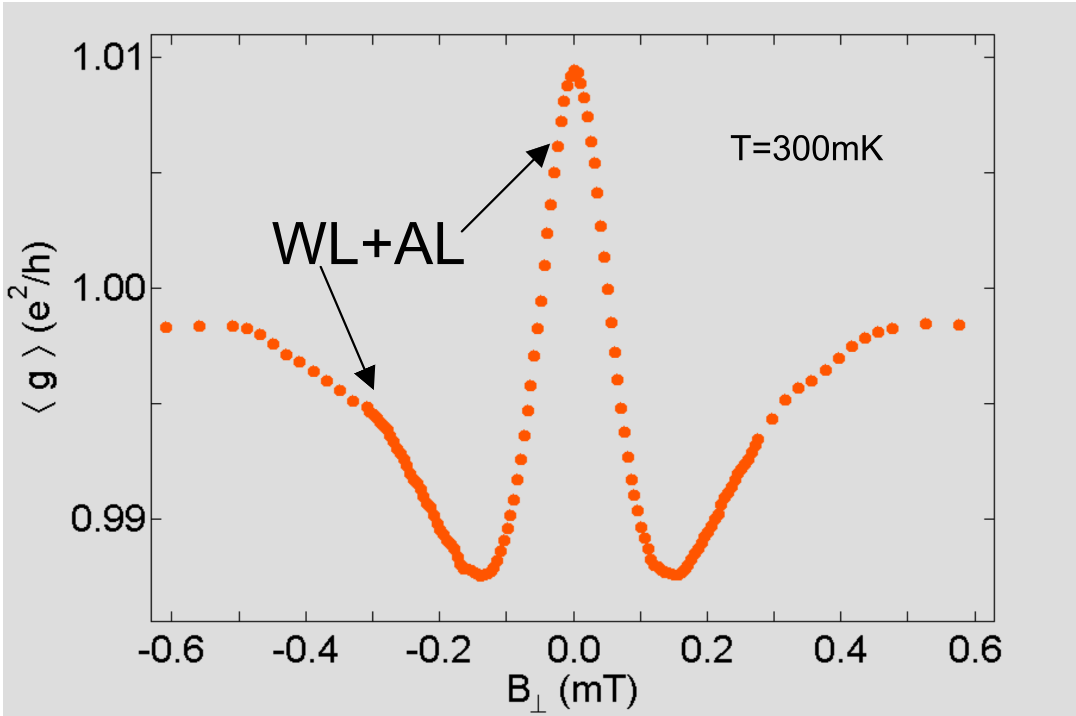


dots are on **different** wafers



high density
stronger SO coupling
antilocalization (AL)





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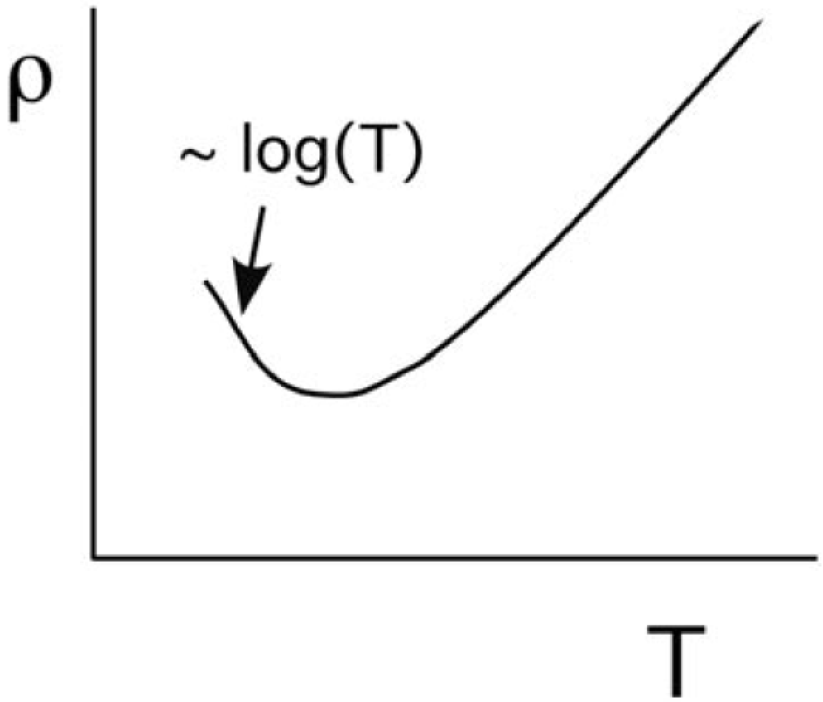
Goldhaber-Gordon et al., Nature **391**, 156 (1998)
Cronenwett et al., Science **281**, 540 (1998)
S. Cronenwett, Ph. D. Thesis (2001)

Kondo Effect in Metals

1930s experiments:



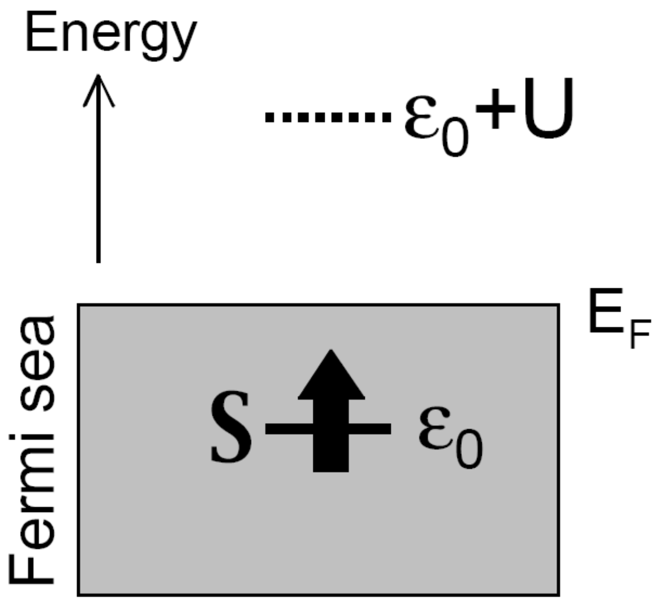
1960s: (exp) related to magnetic impurities

theoretical explanation by Jun Kondo
spin-fip scattering on mag. impurities

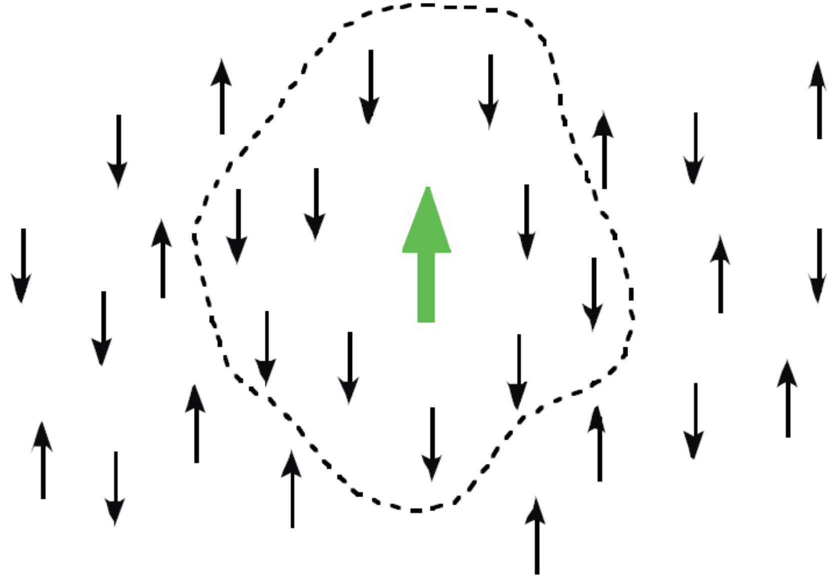
$$\rho \sim \rho_0 + aT^5 - b\log(T)$$

lattice phonons

Kondo Effect in Metals: Model



new energy scale: Kondo temperature T_K
formation of spin-singlet screening cloud



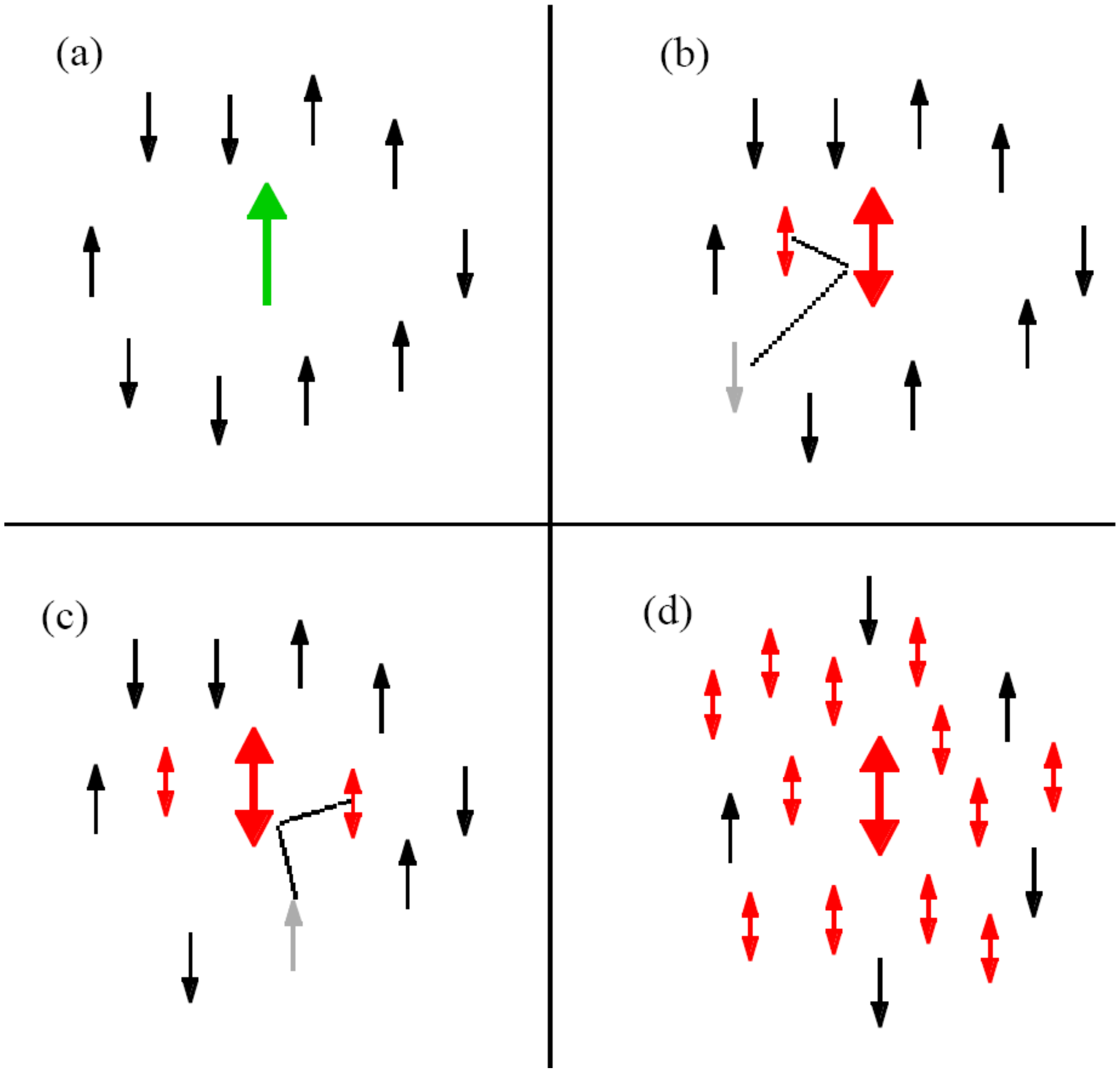
cloud: more effective scatterer
increase in resistance

Anderson Hamiltonian

$$H_A = \sum_{\sigma;k<k_f} \varepsilon_{k\sigma} c^{\dagger}_{k\sigma} c_{k\sigma} + \sum_{\sigma} \varepsilon_{\sigma} d^{\dagger}_{\sigma} d_{\sigma} + \frac{1}{2} U n_{\sigma} n_{\sigma'} + \sum_{\sigma;k<k_f} t_{k\sigma} c^{\dagger}_{k\sigma} d_{\sigma} + H.c..$$

free electrons — localized electrons — on site charging — coupling between localized and free ele.

Kondo Effect in Metals: spin flip scattering

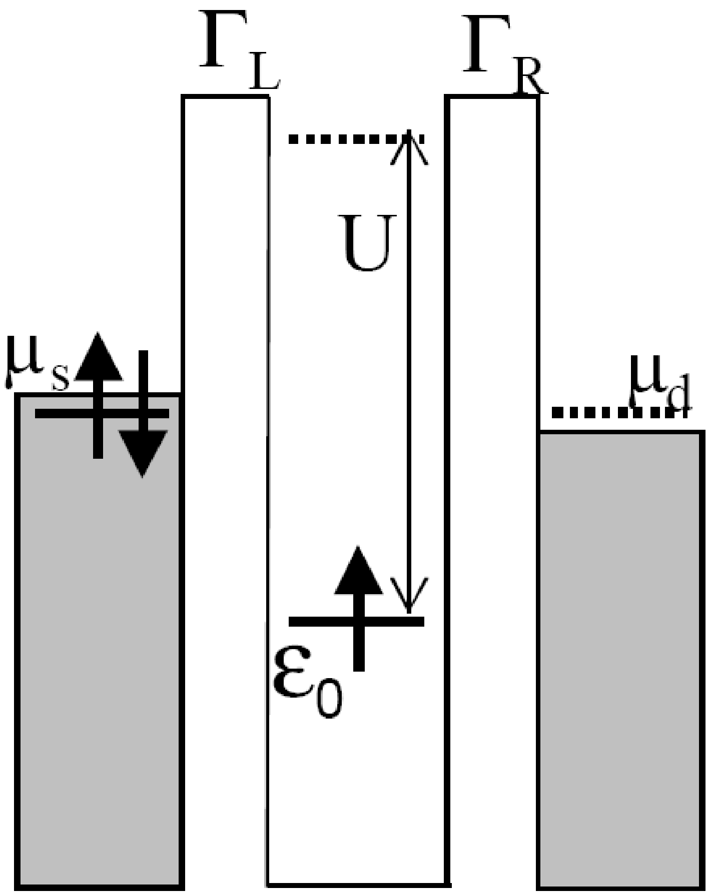


each scattering event engangles impurity with conduction electron

singlet cloud formation

temperature scale T_K

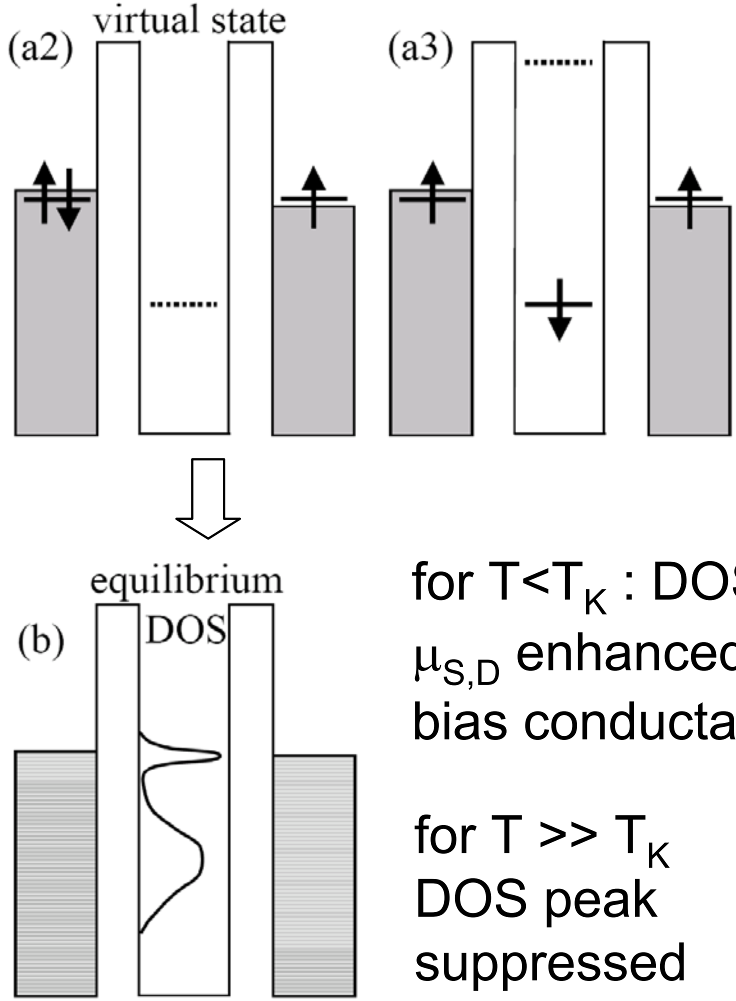
Kondo Effect in Quantum Dots



unpaired spin

$$T_K \propto \sqrt{U\Gamma}\exp(\pi\epsilon_0/2\Gamma)$$
$$\Gamma = \Gamma_L + \Gamma_R$$

spin-flip cotunneling (elastic)

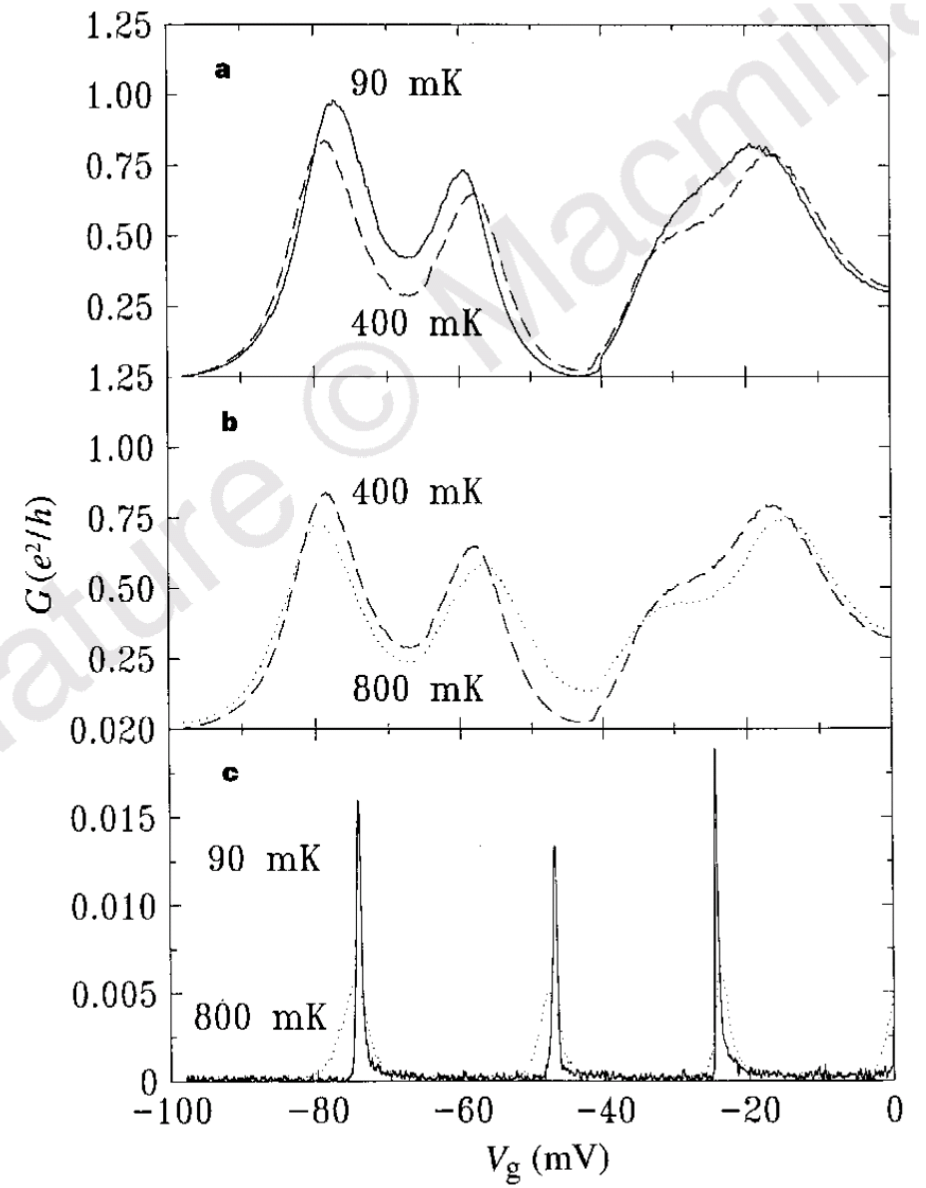


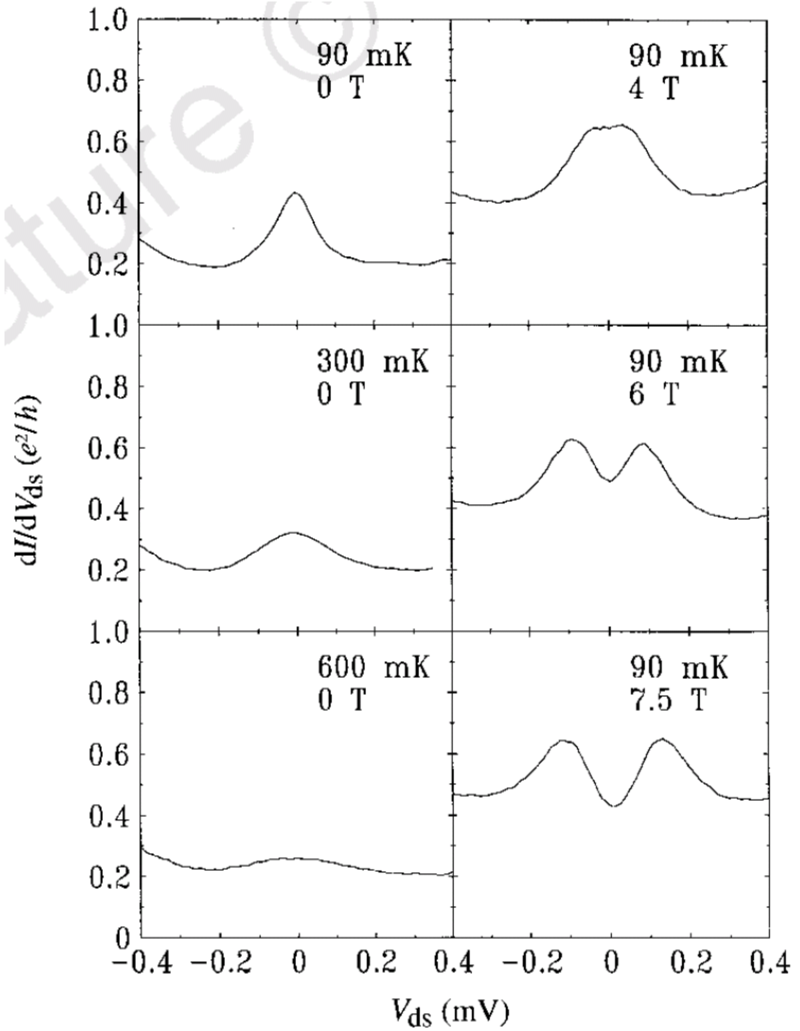
for T<T_K : DOS at $\mu_{S,D}$ enhanced zero bias conductance!!

for T >> T_K DOS peak suppressed

dots: parameters tunable
SINGLE impurity

Kondo Effect in Quantum Dots: Experiment





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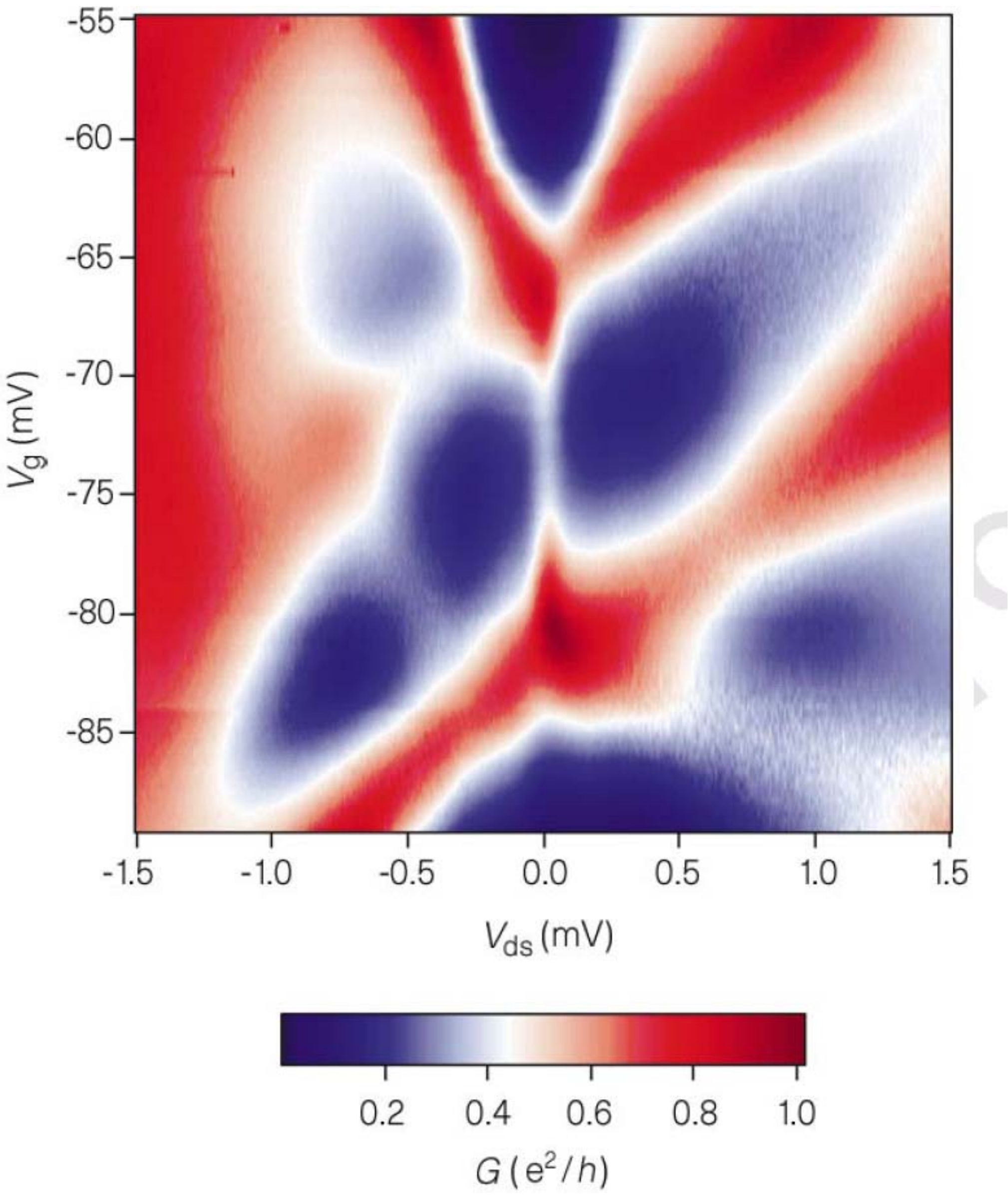
Kondo Effect in Quantum Dots: Experiments



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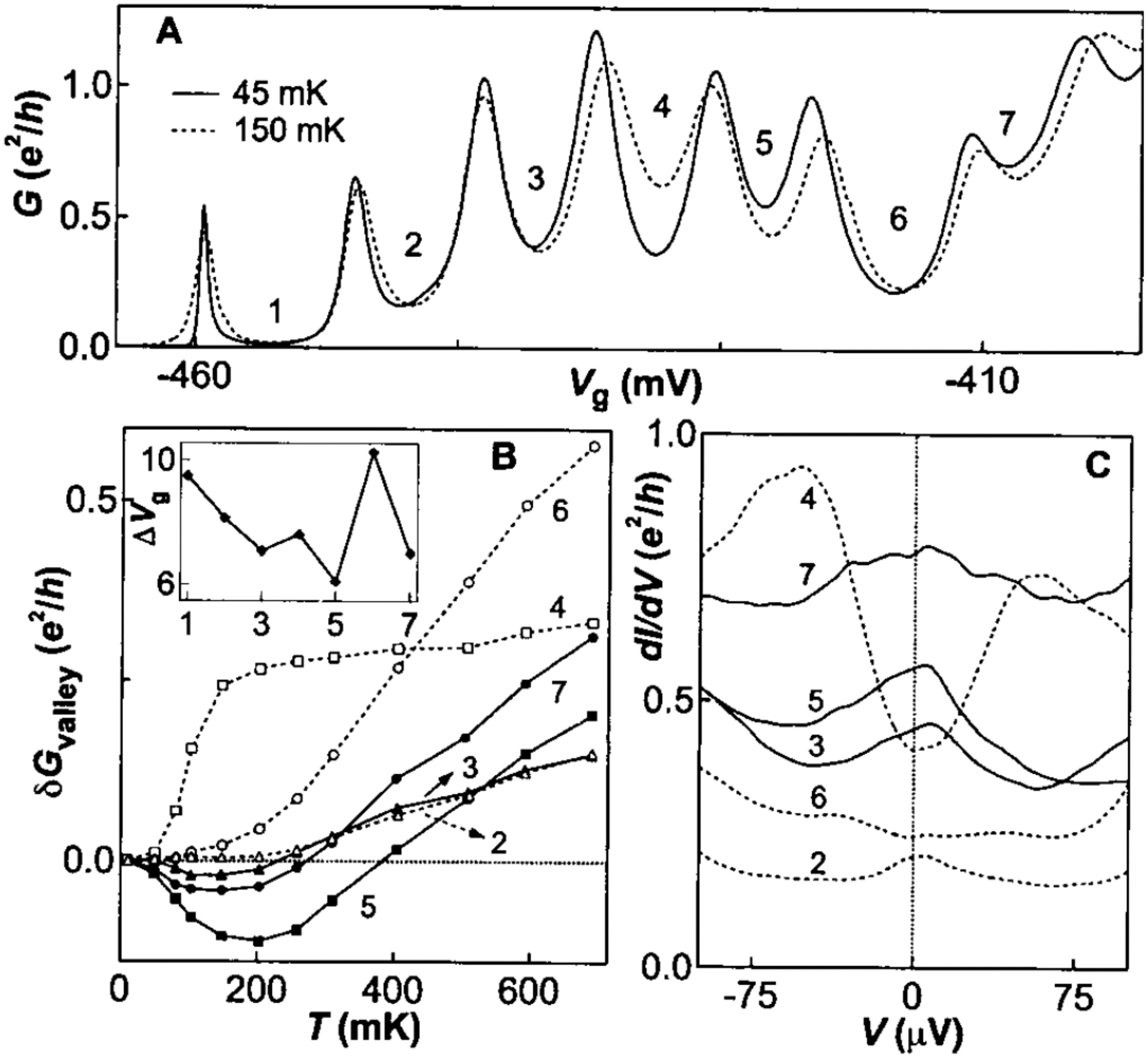
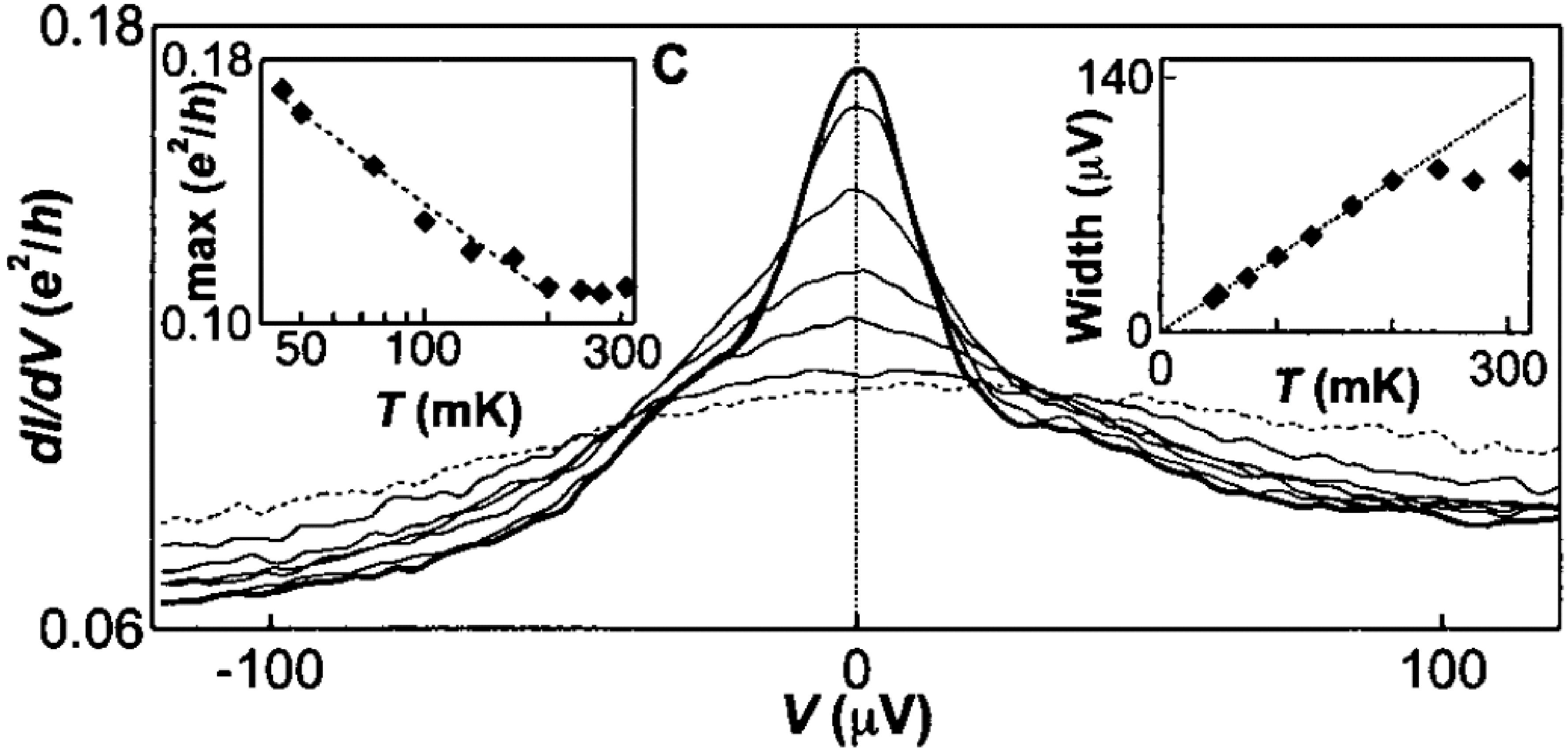
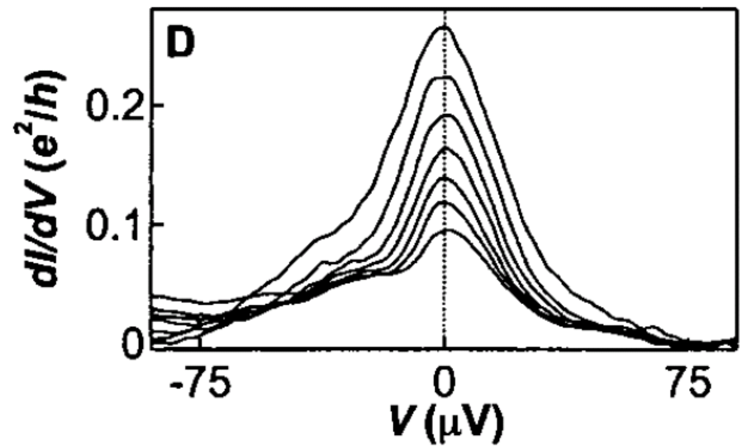


Cronenwett et al., Science **281**, 540 (1998)

Kondo Effect in Quantum Dots: Experiments





gate voltages
into odd valley

Cronenwett et al., Science **281**, 540 (1998)

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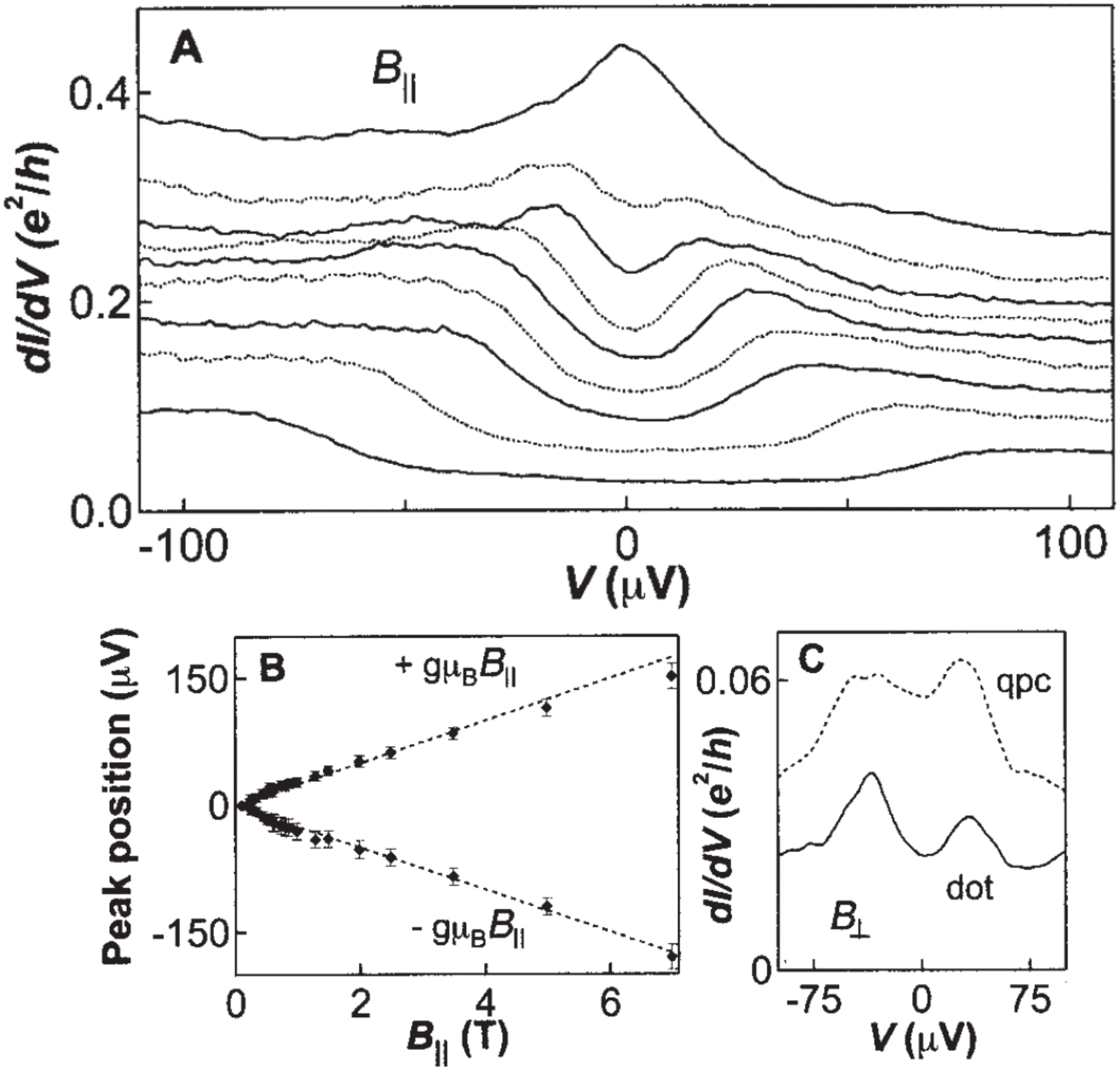

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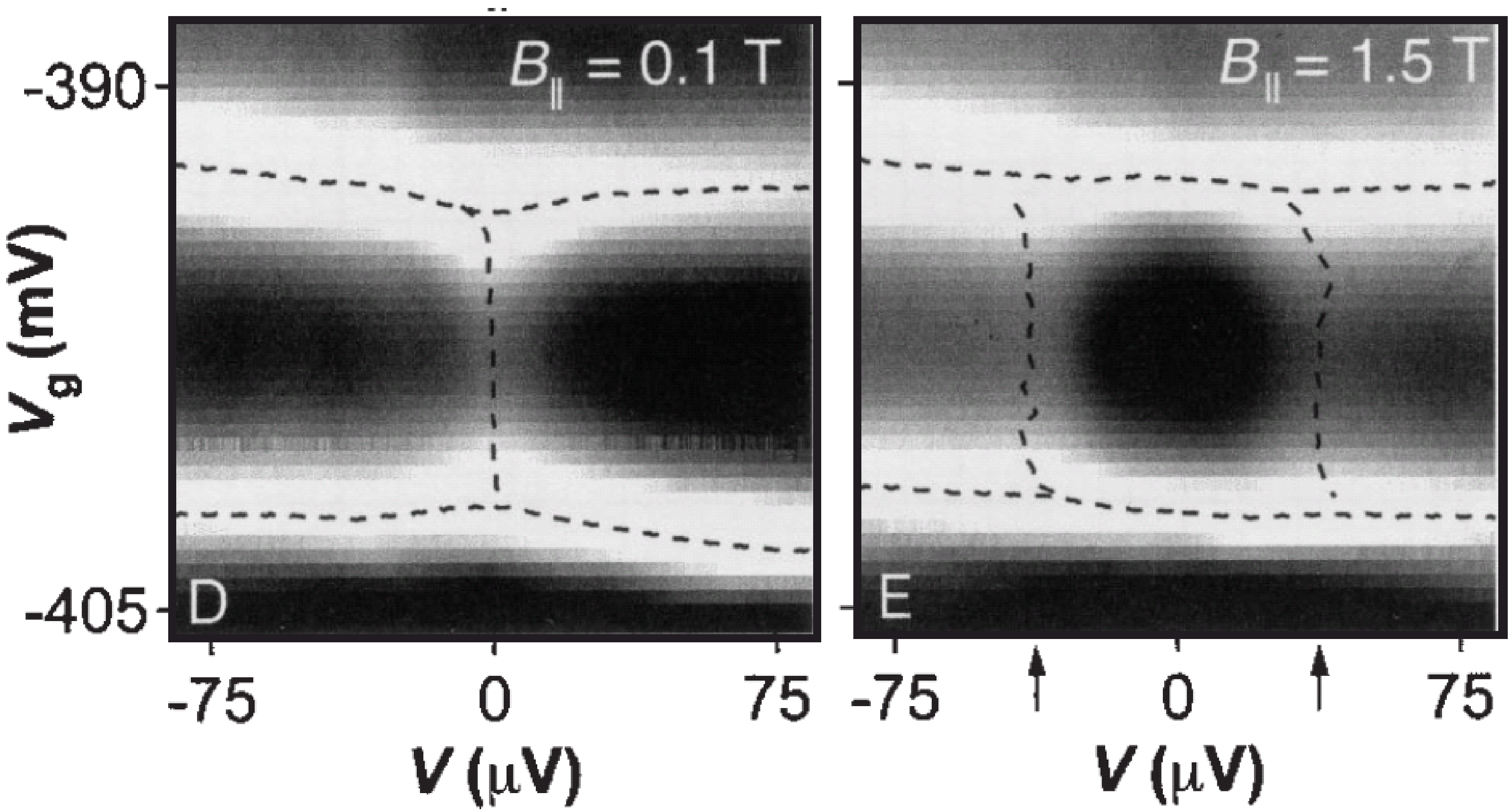

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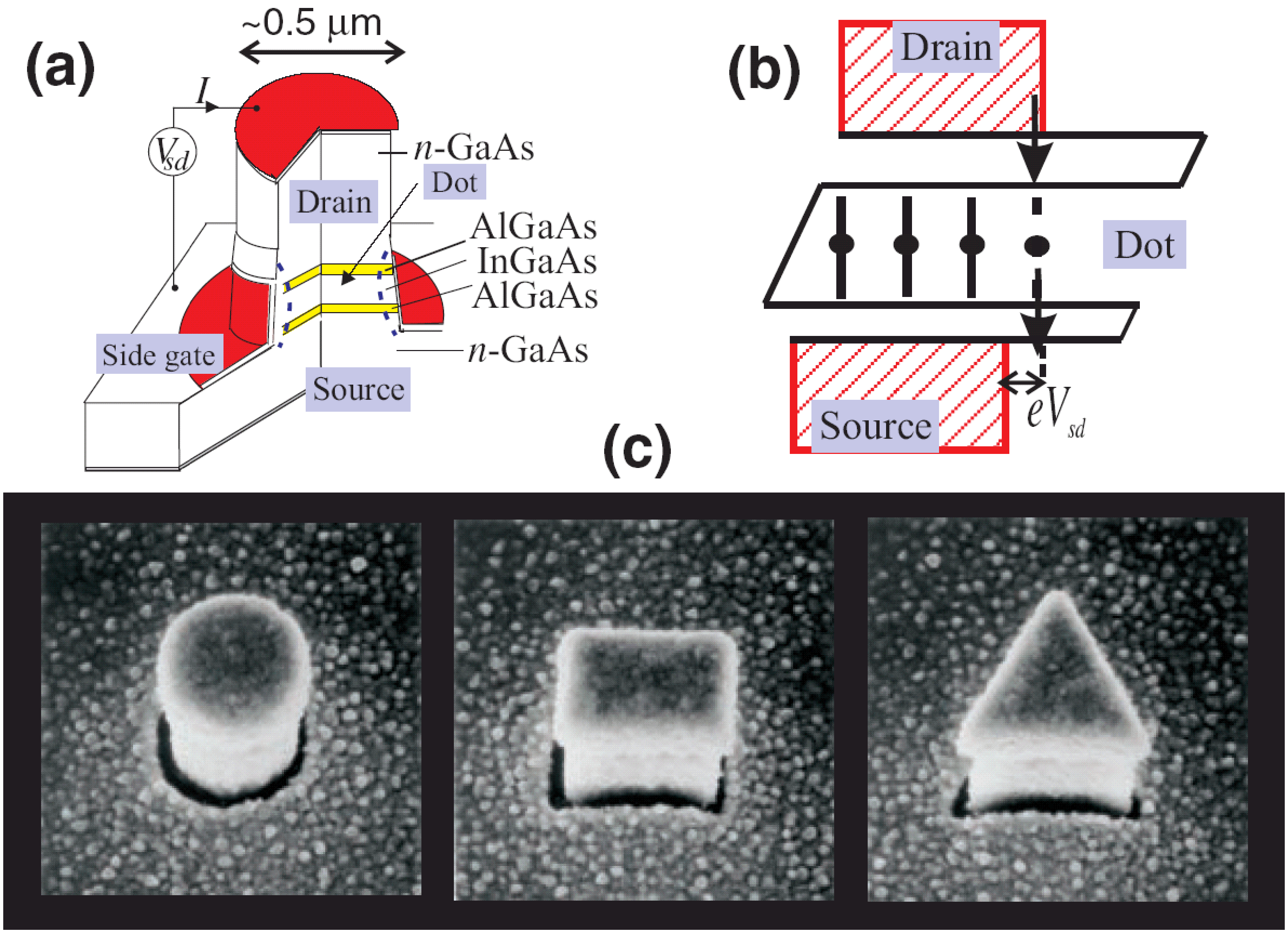
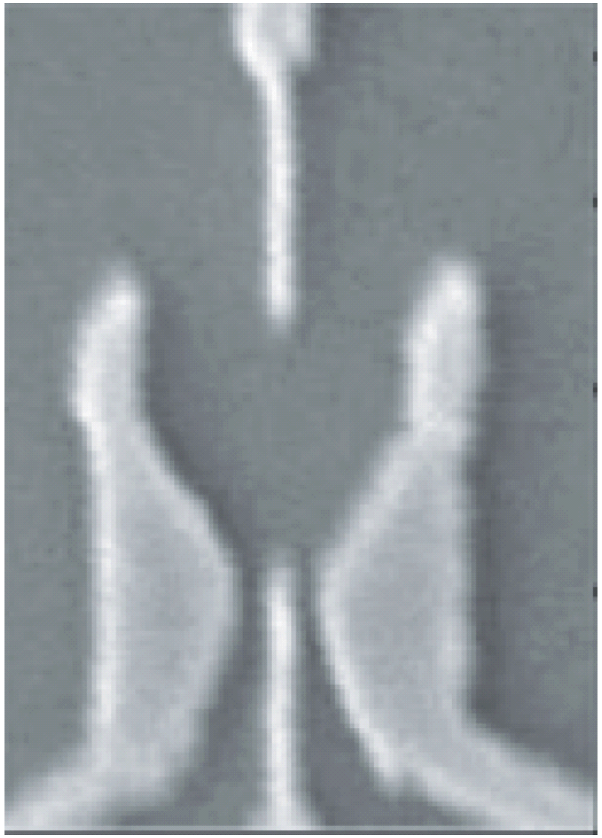
Kouwenhoven, Austing and Tarucha, RPP 64, 701 (2002)
Tarucha et al., PRL77, 3613 (1996)
Kouwenhoven et al., Science 278, 1788 (1997)

Few Electron Quantum Dots: Vertical

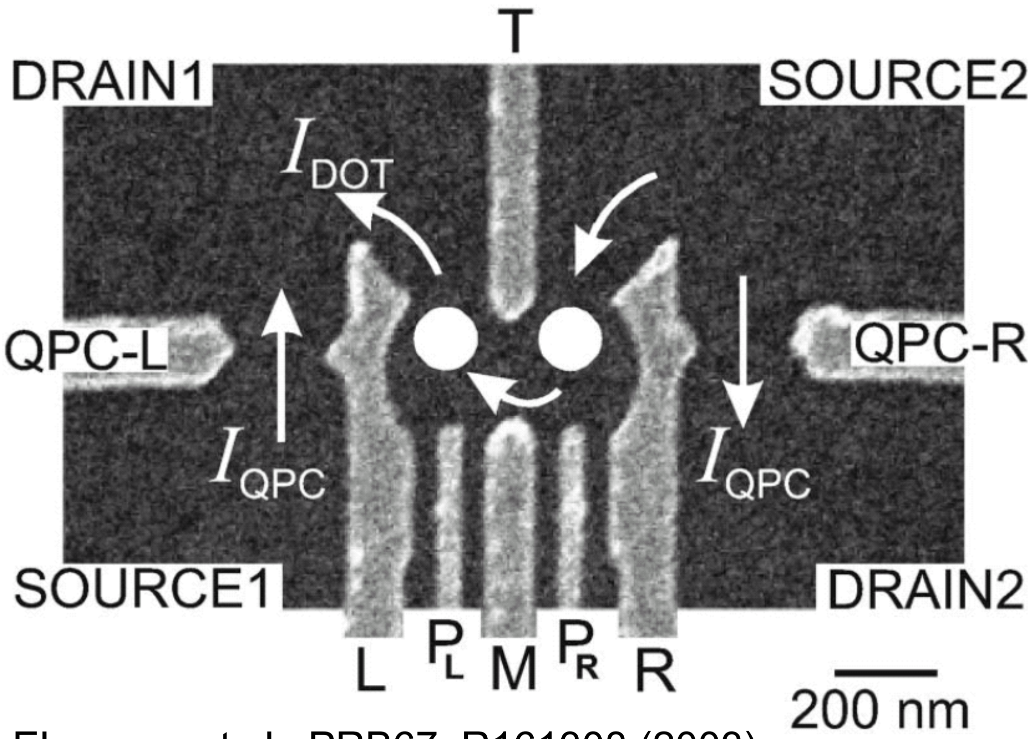


Kouwenhoven, Austing and Tarucha, RPP 64, 701 (2001)

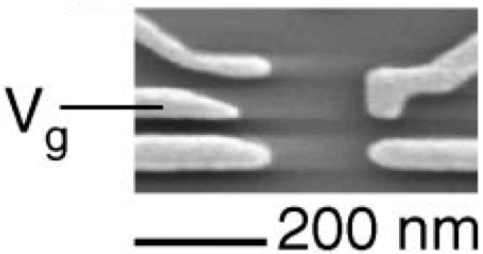
Few Electron Quantum Dots: Lateral



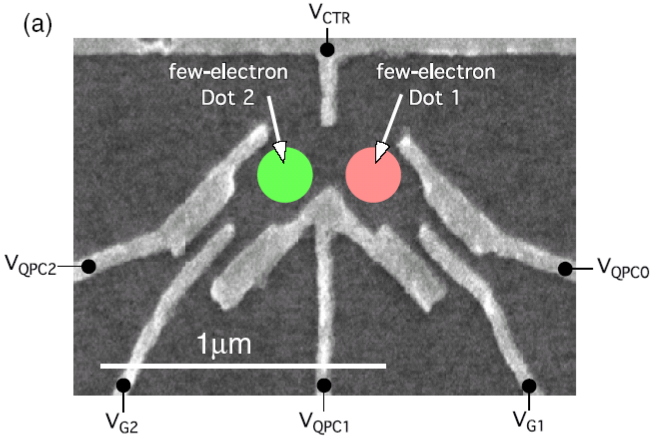
Ciorga et al., PRB61, R16315 (2000)



Elzerman et al., PRB67, R161308 (2003)
Petta et al., PRL93, 186802 (2004)



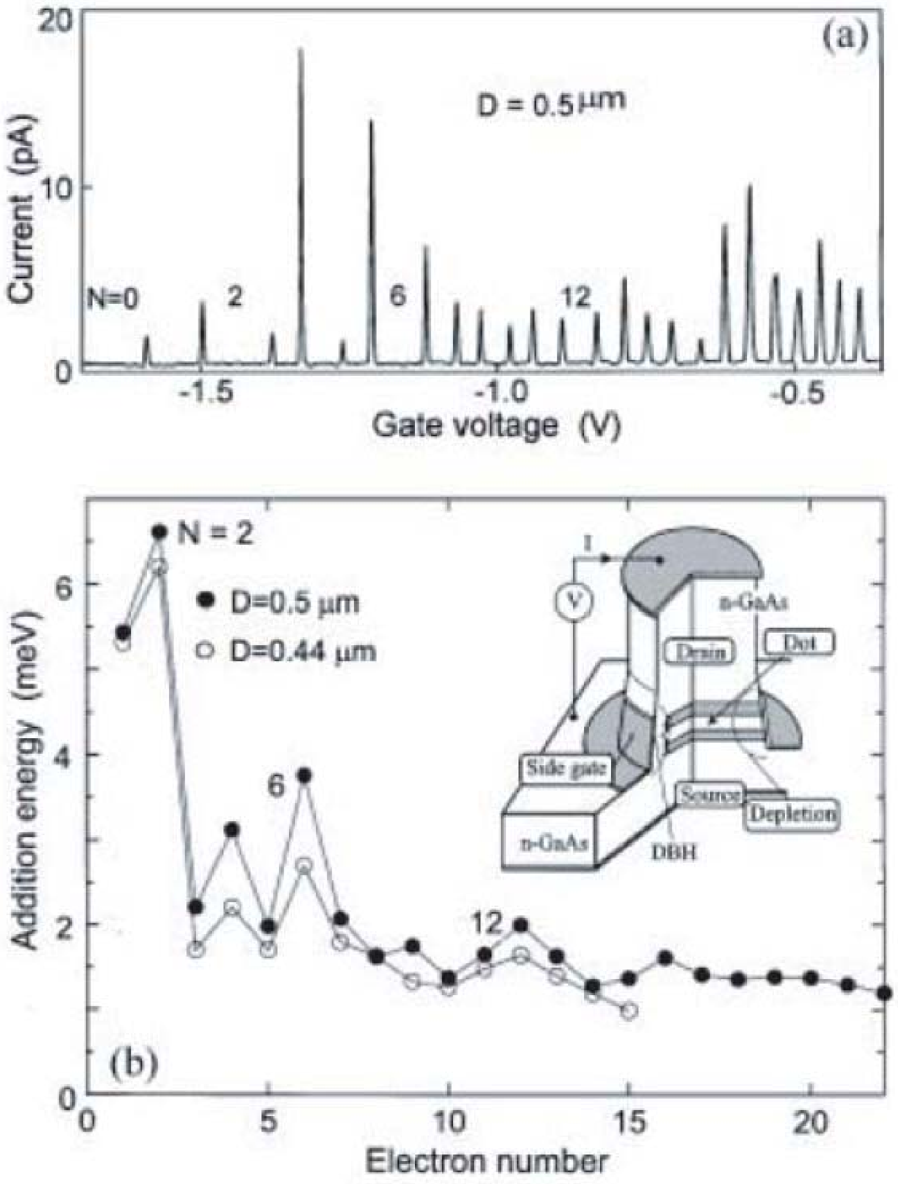
Zumbuhl et al., PRL93, 256801 (2004)



Chan et al., Nanotech. 15, 609 (2004)

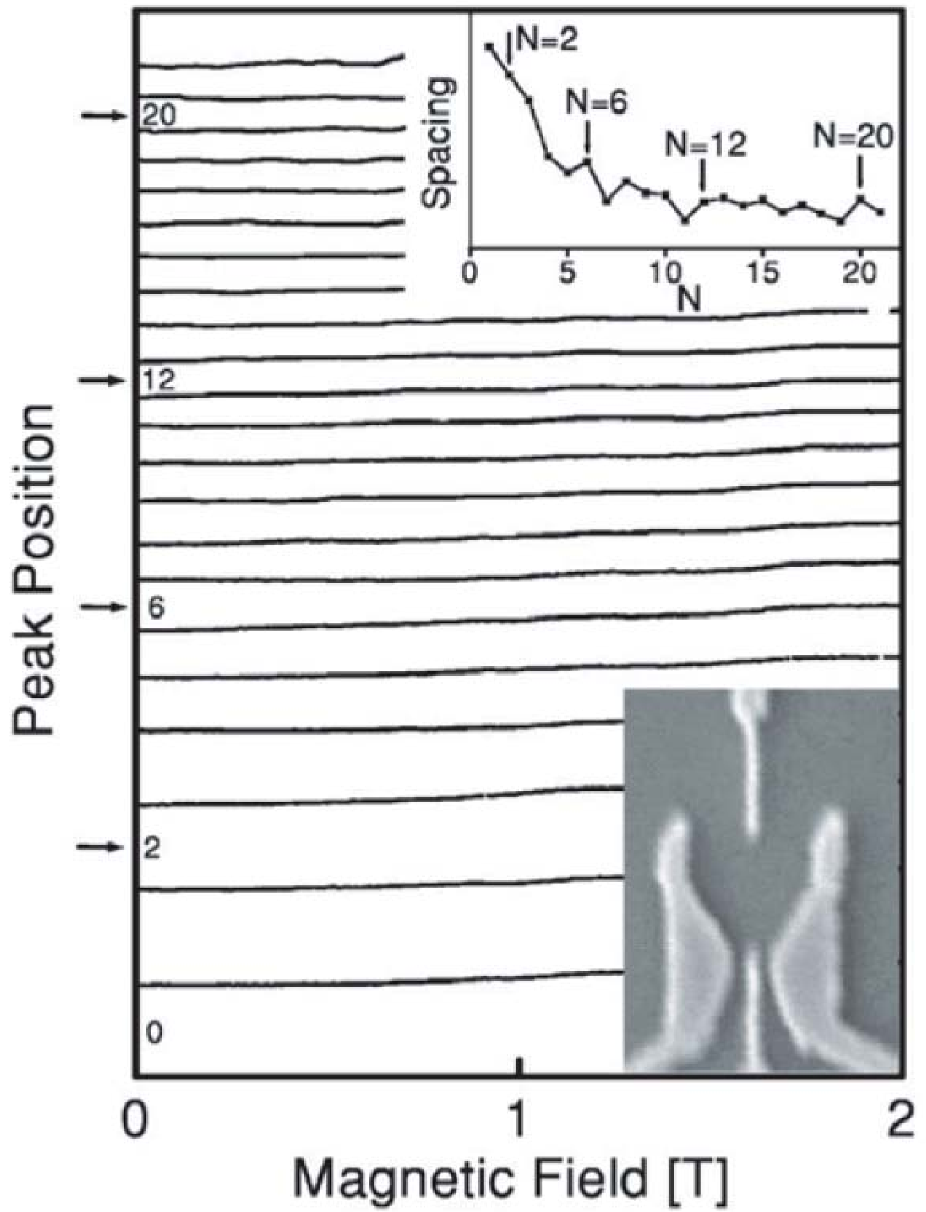
Rotation Symmetry and Angular Momentum

circular symmetry: 2D shell filling



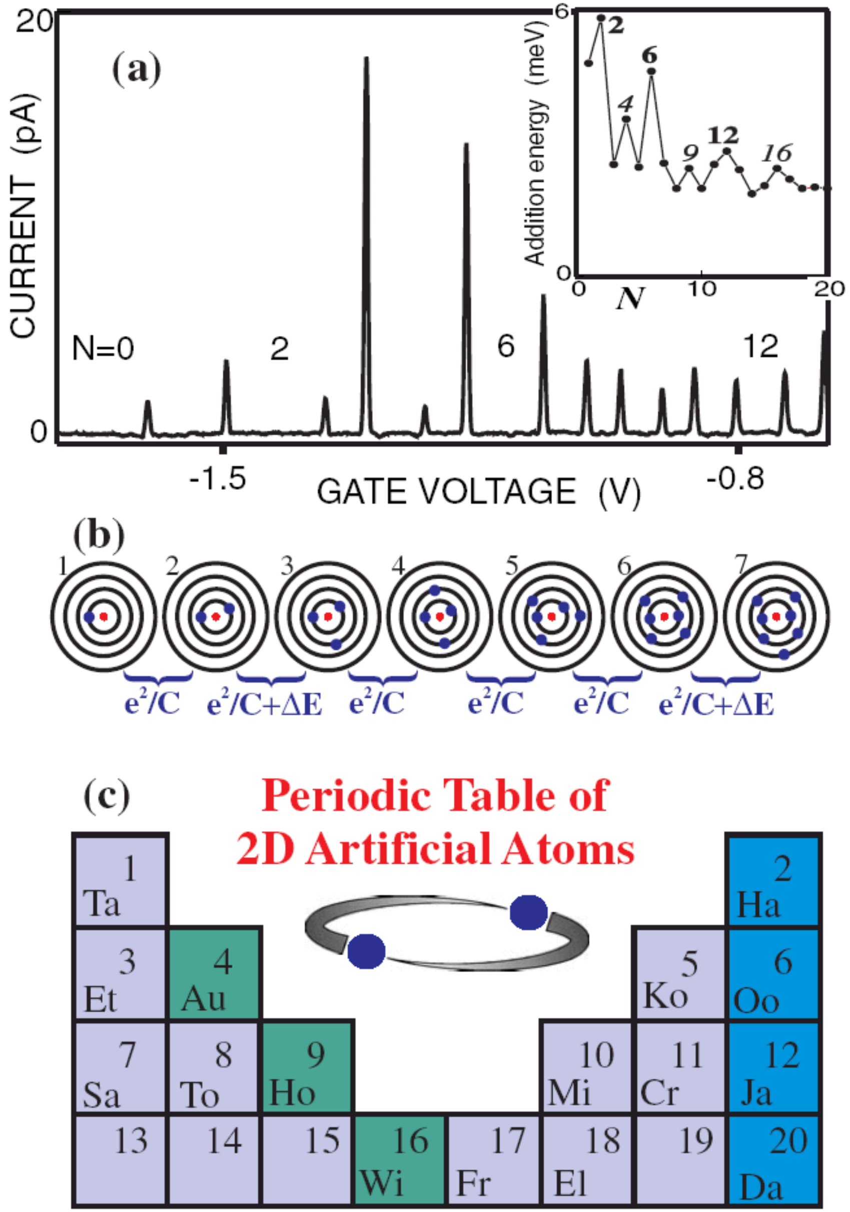
Tarucha et al., PRL77, 3613 (1996)

circular symmetry broken



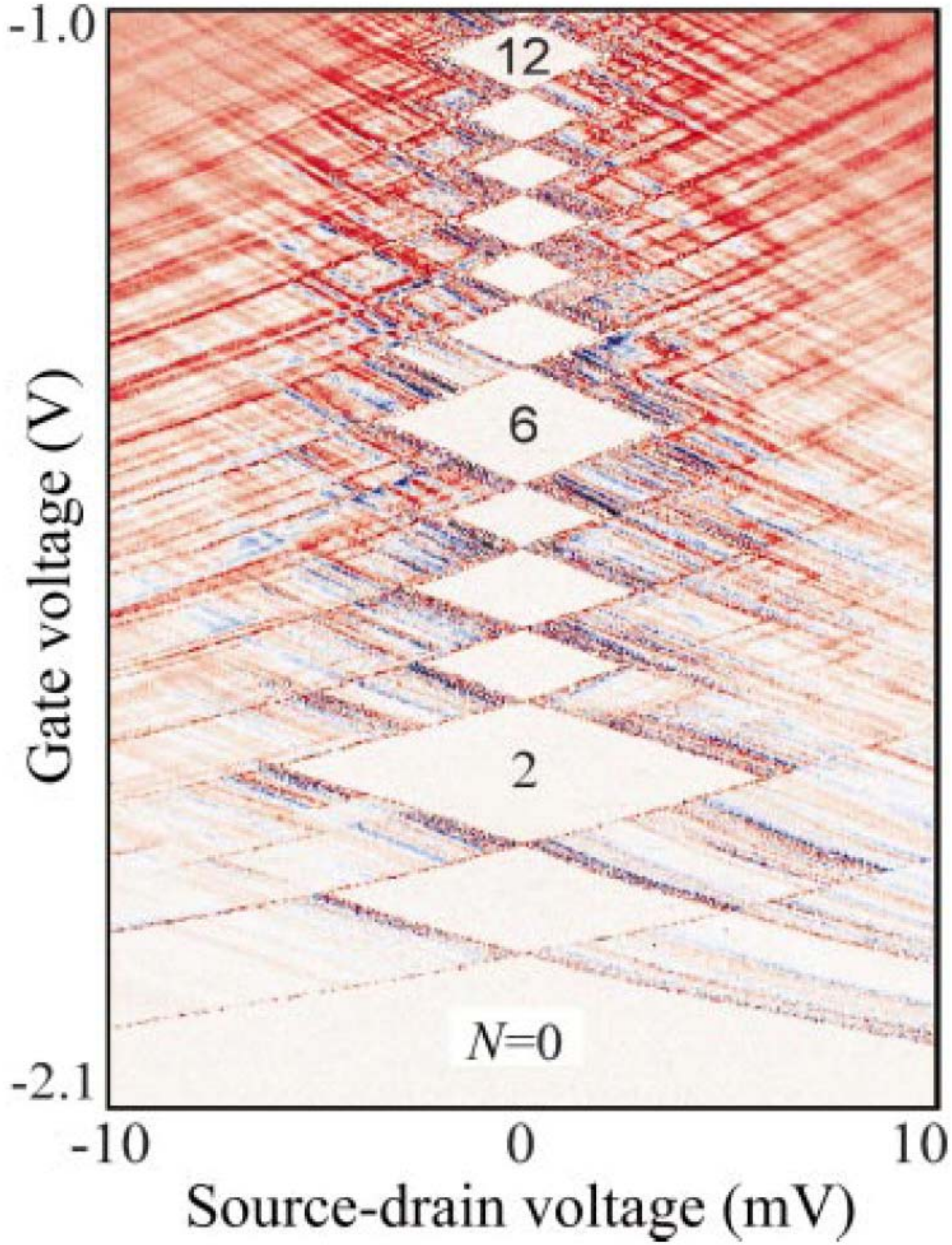
Ciorga et al., PRB61, R16315 (2000)

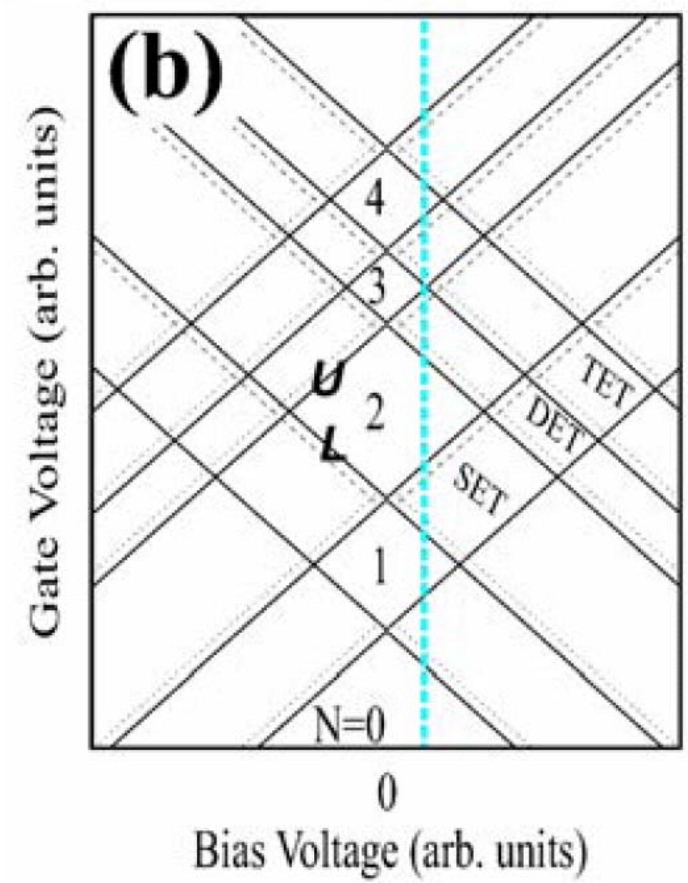
2D Periodic Table of Elements



Kouwenhoven, Austing and Tarucha, RPP 64, 701 (2001)

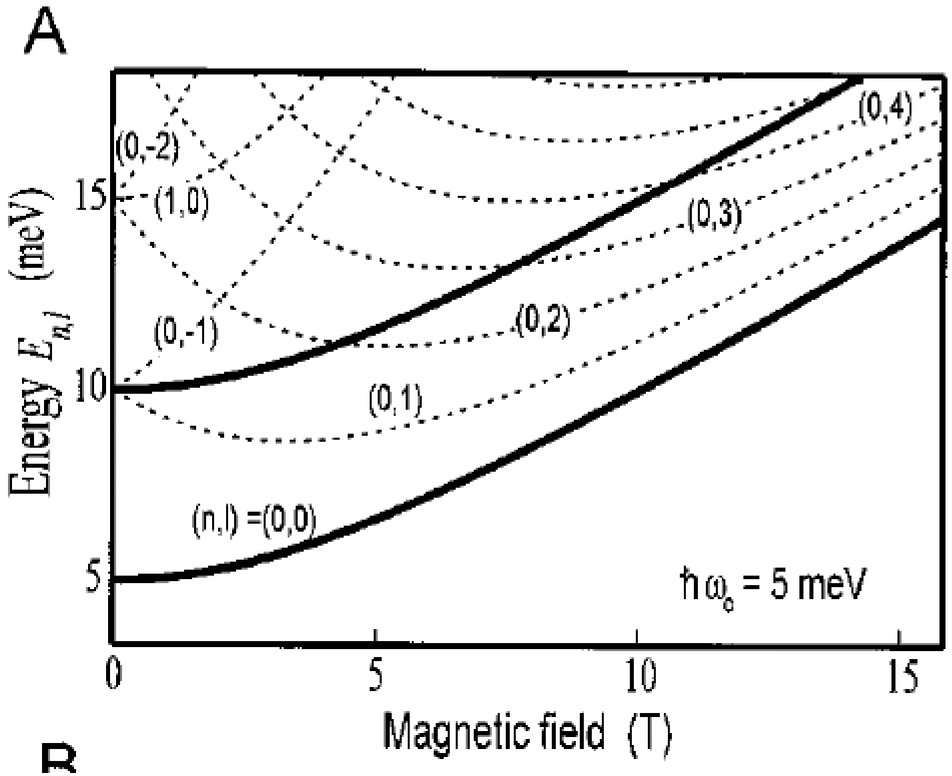
Excitation Spectra of Circular, Few Electron Dots

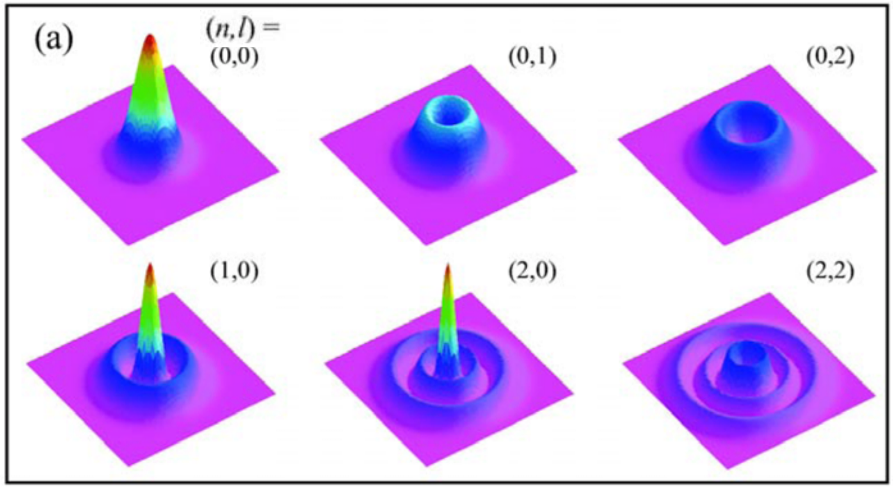




Kouwenhoven et al., Science 278, 1788 (1997)

Fock-Darwin States: Single Particle Levels





Fock-Darwin Energies

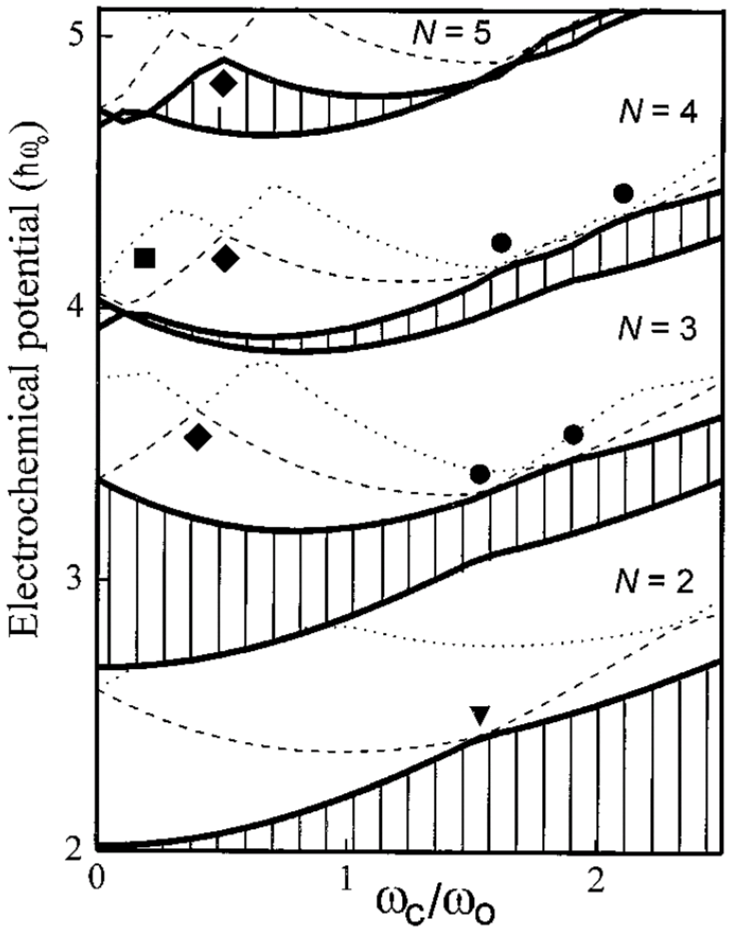
$$E_{n,\ell} = (2n + |\ell| + 1)\hbar\sqrt{\left(\frac{1}{4}\omega_c^2 + \omega_o^2\right)} - \frac{1}{2}\ell\hbar\omega_c$$

$n = 0, 1, 2, \ldots$ radial

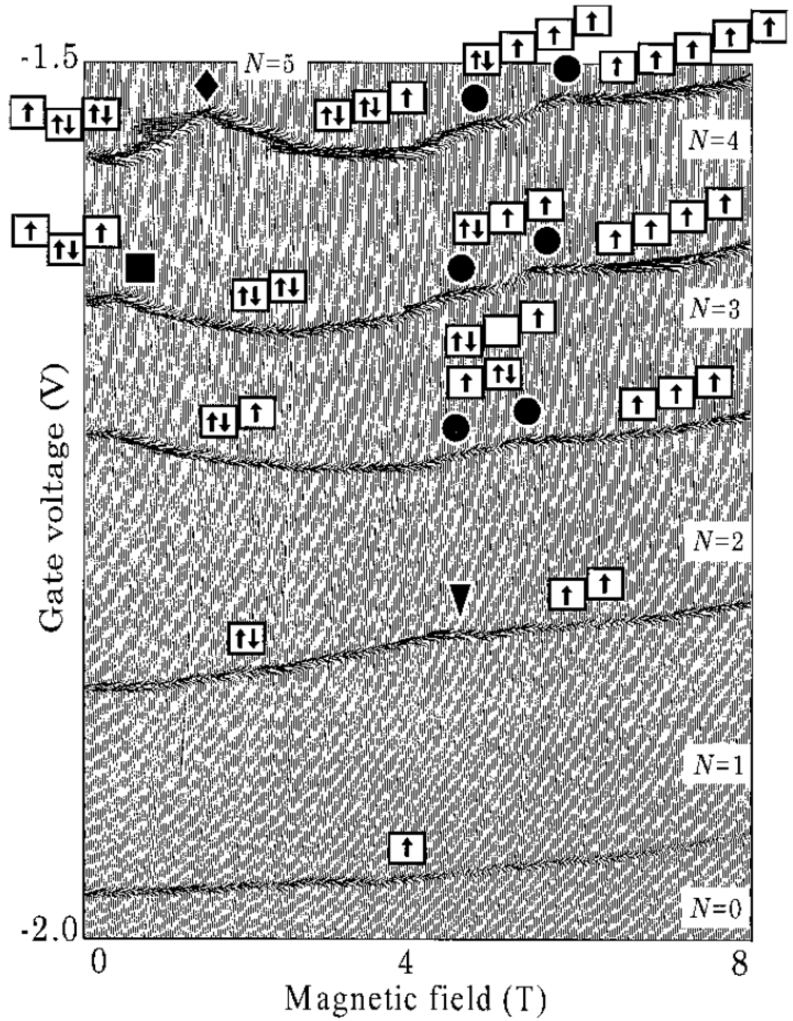
$l = 0, \pm 1, \pm 2, \ldots$ angular momentum

Kouwenhoven, Austing and Tarucha, RPP 64, 701 (2001)

Magnetic Field Transitions



exact calculation

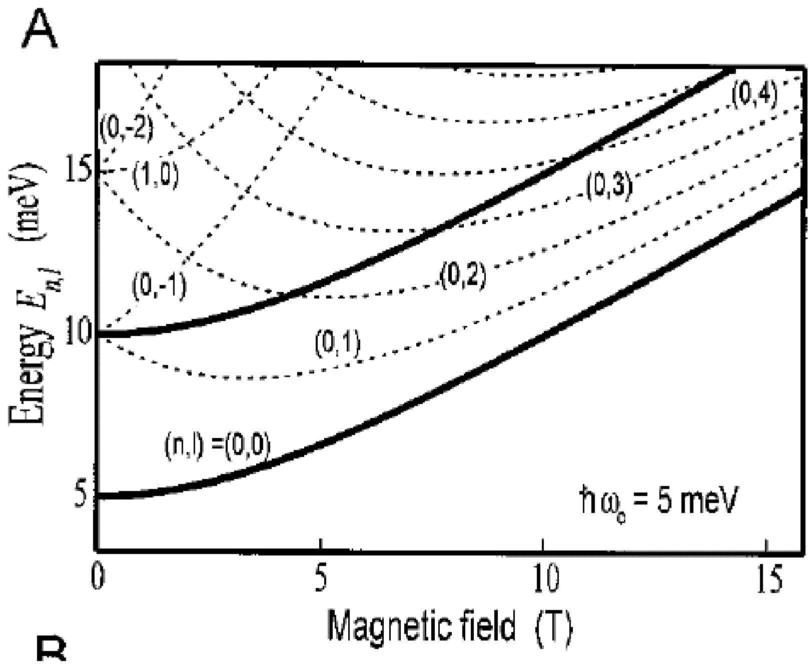


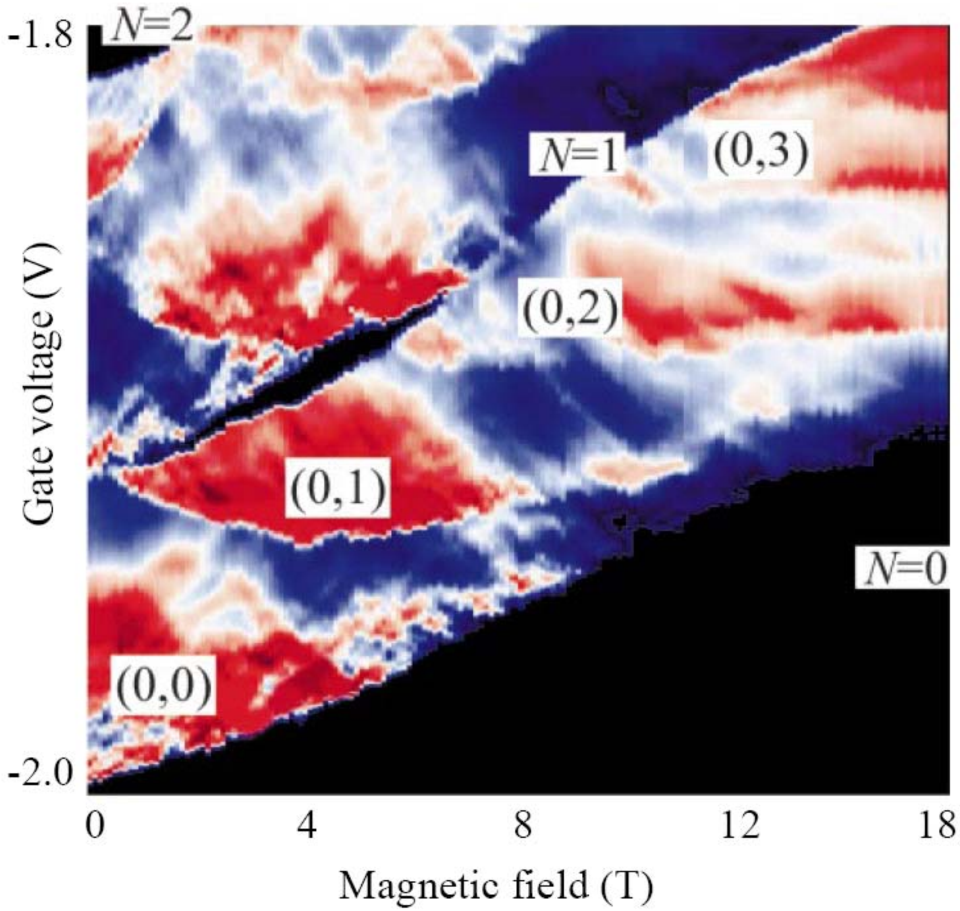
experiment

“atomic physics” like experiments not accessible in real atoms!!

Kouwenhoven et al., Science 278, 1788 (1997)

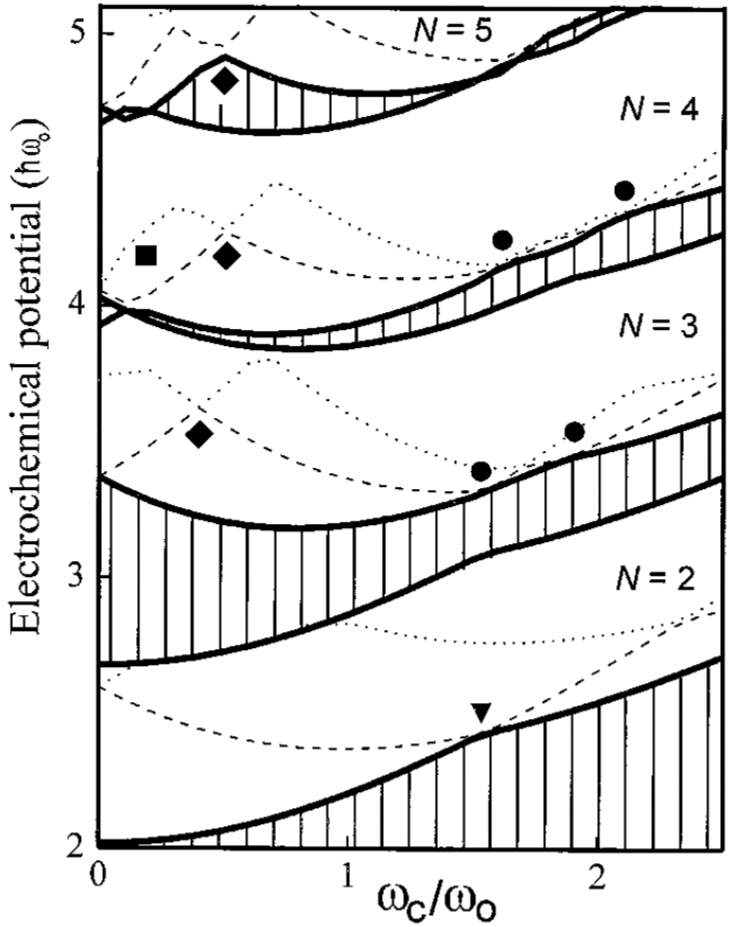
Zero to One Electron Transition

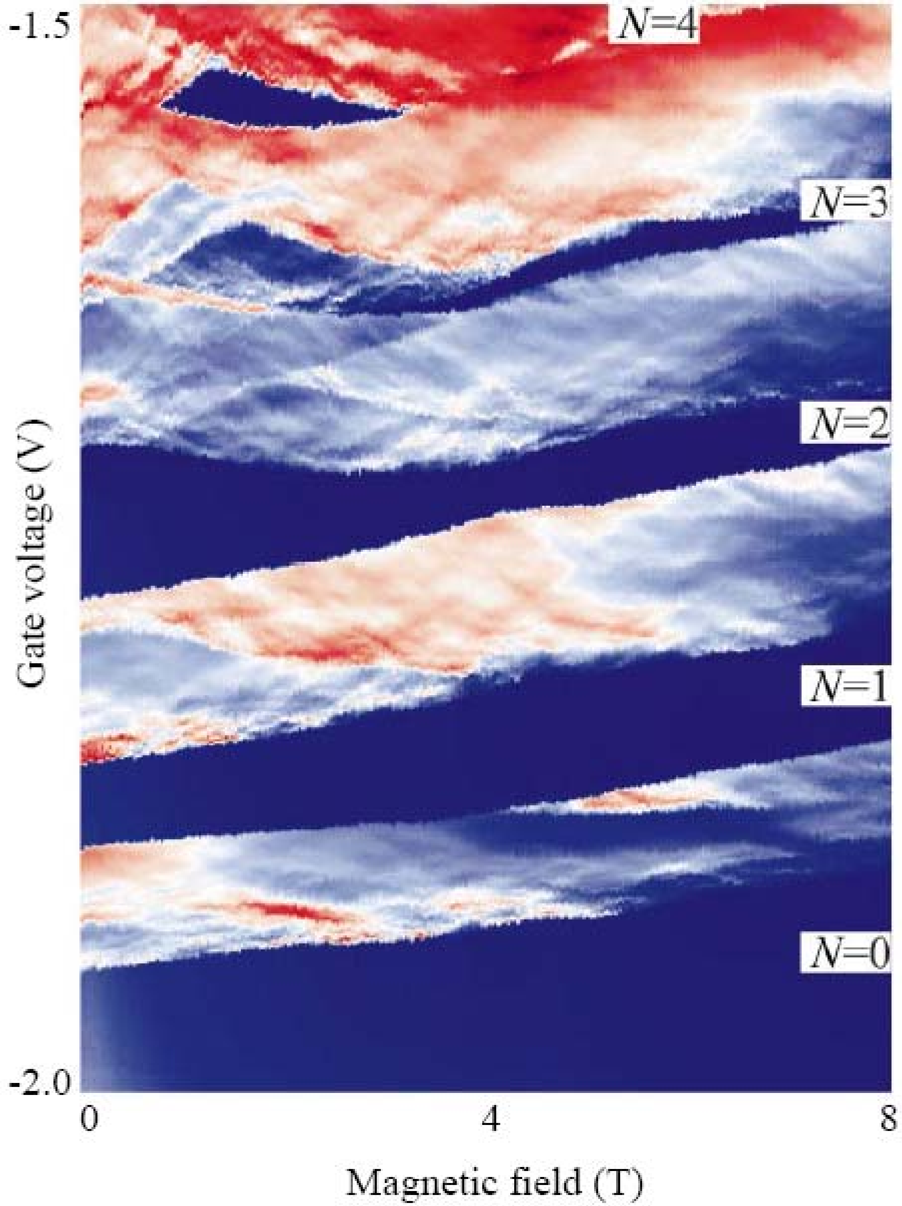




Kouwenhoven et al., Science 278, 1788 (1997)

Higher Transitions





Kouwenhoven et al., Science 278, 1788 (1997)