Outlook

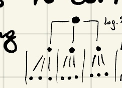
Error Correction
& Surface Code

... a crash course

Backed by the 2016 - chronicles by J. Wootton

Error Corr. & Surface Code, Crash Course

Idea:
- Need redundancies
- Check if they all agree (parity)
- Map that parity to a „sacrificial" ancilla, which you can read-out & therefore decide to perform an error correction or not.

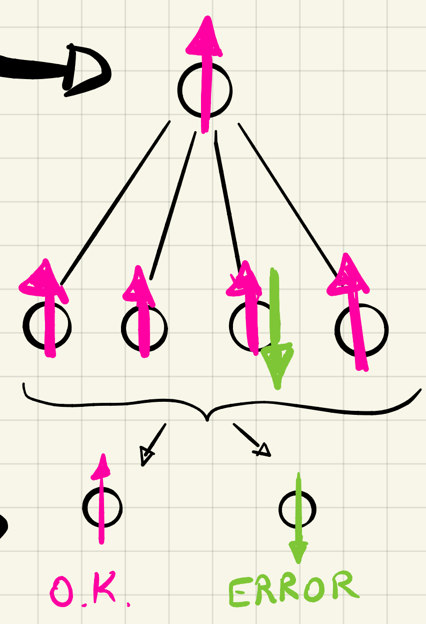
(consider 1^{st} ord. errors:
So we correct only „likely" errors.
For more unlikely errors e.g. 2x flip, we need more redundancies to correct larger subsets. → Stacking


)

logical qubit →

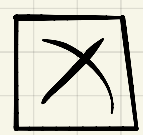
Stabilizer consisting of 4 phys. qubits →

one phys. qubit →

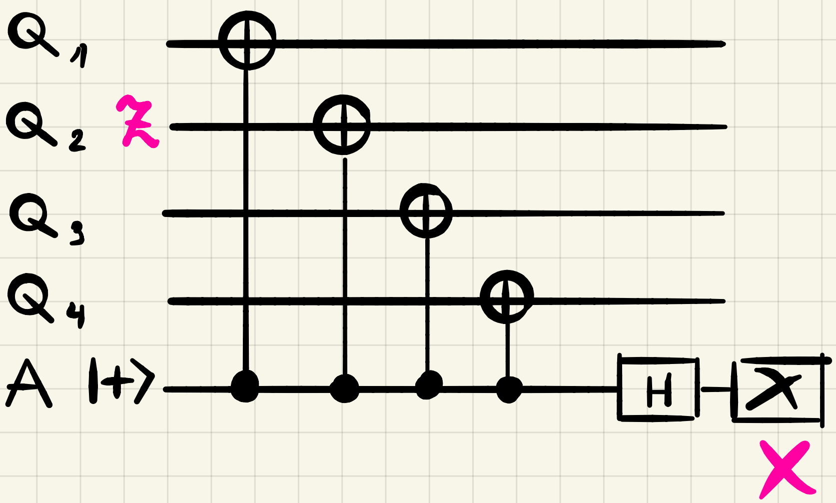


parity ±1

project ancilla

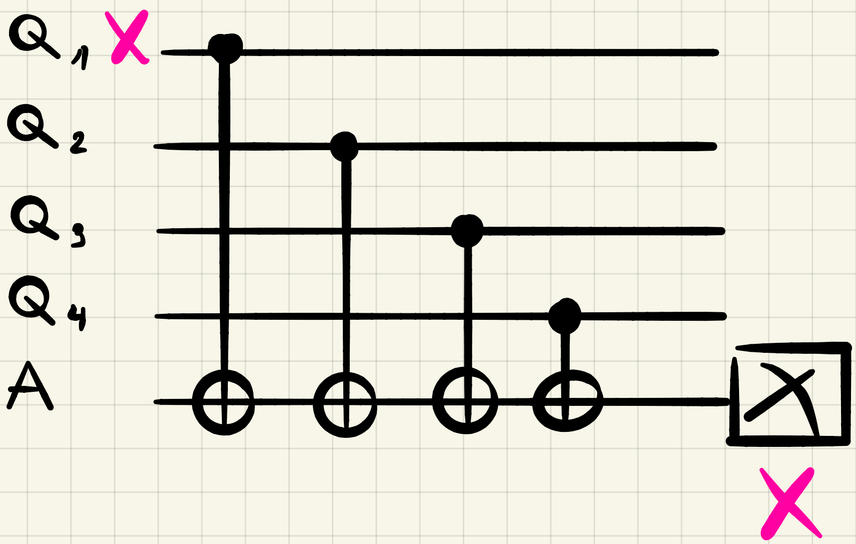


Stabiliser Code /w parity check



ancilla flips if Z error

Z-parity check, stabilizer S_Z



ancilla flips if X error

X-parity check, Stabilizer S_X

Define Stabilizer Code

$XZ = -ZX$

Def: Set of code states

e.g. $S_x = \sigma_x \sigma_x \sigma_x \sigma_x$

$$\boxed{|\psi\rangle \in C : S|\psi\rangle = |\psi\rangle \ \forall \text{ operators } S \in \mathcal{S}}$$

We say S „stabilizes" $|\psi\rangle$ because $|\psi\rangle$ is the $+1$ Eigenstate of S.

Assume an error $E \in$ Paulis, $|\psi\rangle \longmapsto E|\psi\rangle$

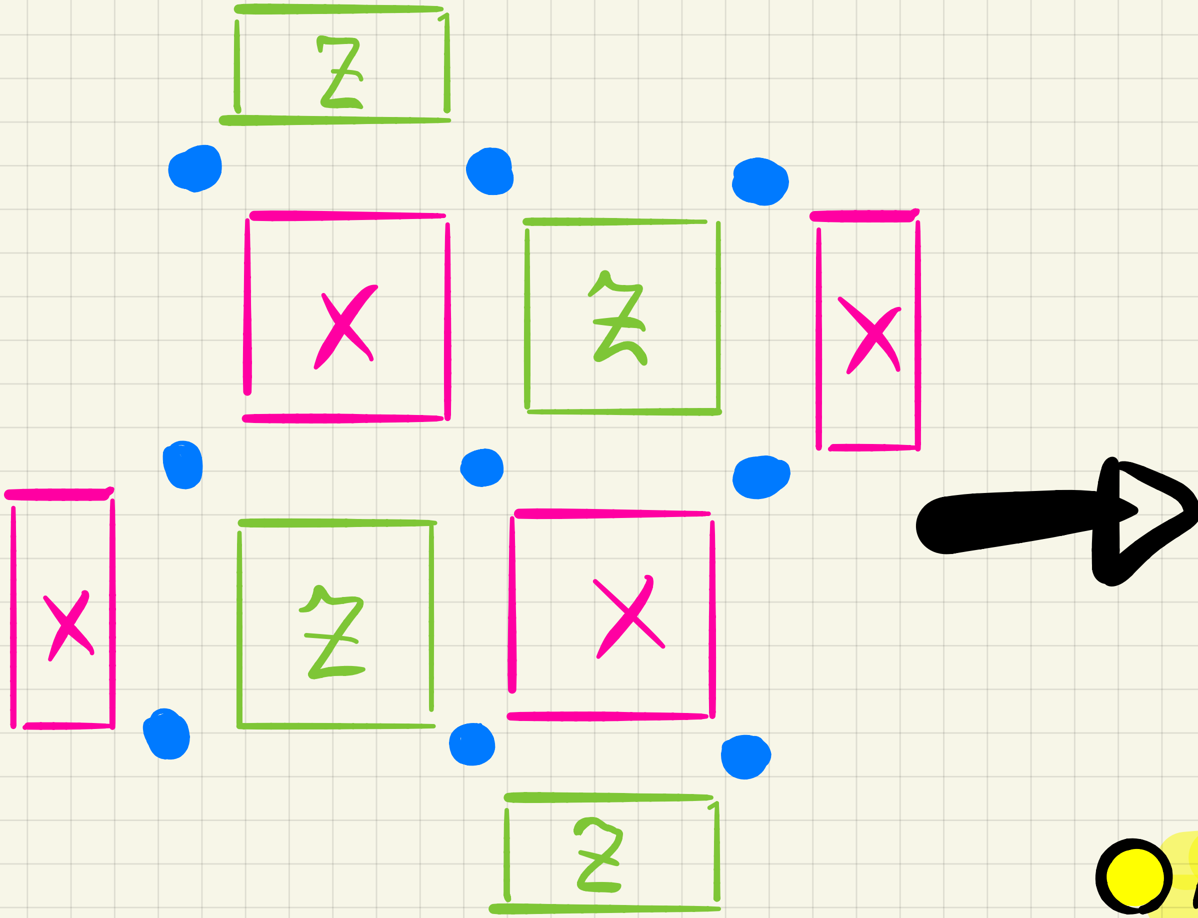
$ES = -SE$

$E^2 = \mathbb{1}$

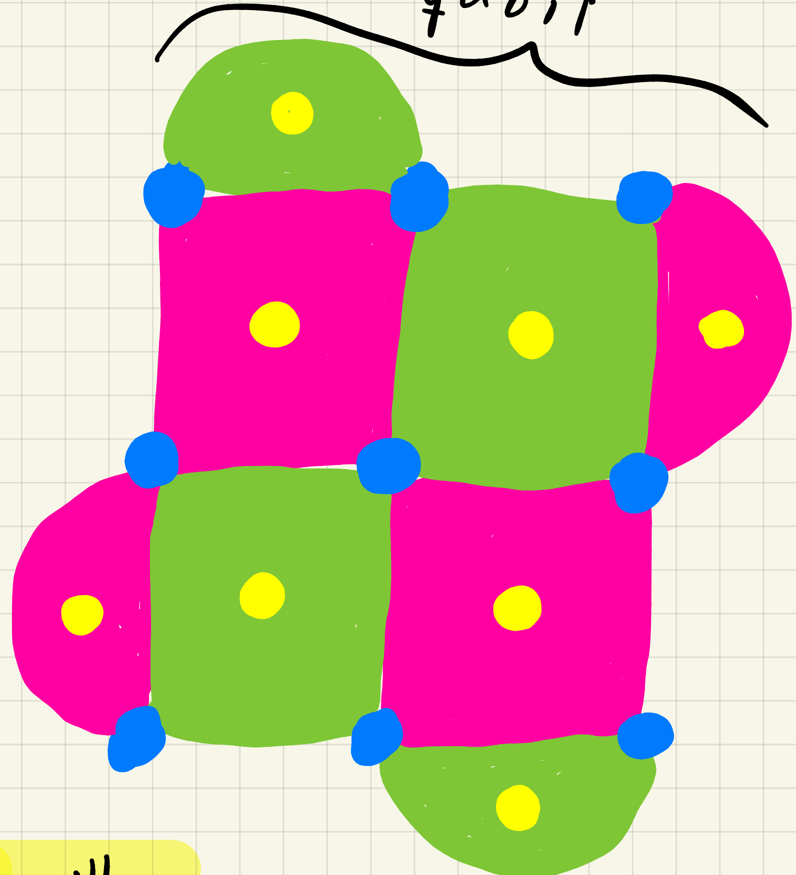
$$\left.\begin{aligned} \therefore \langle\psi|ESE|\psi\rangle &= -1 \\ \langle\psi|S|\psi\rangle &= +1 \end{aligned}\right\} \text{E has phys measurable effect.}$$

• qubits

Def: Stabilizers X, Z

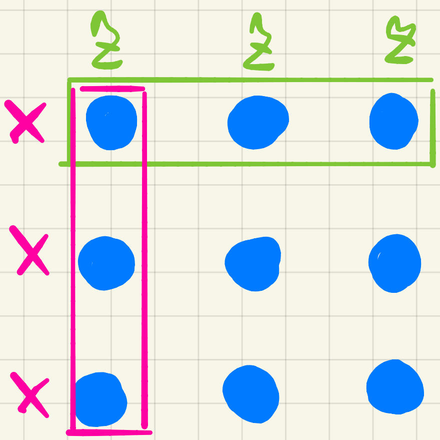


One logical qubit



Ancilla

$$(X\otimes X \cdot Z\otimes Z = Z\otimes Z \cdot X\otimes X)$$



logical Z, L_Z

logical X, L_X

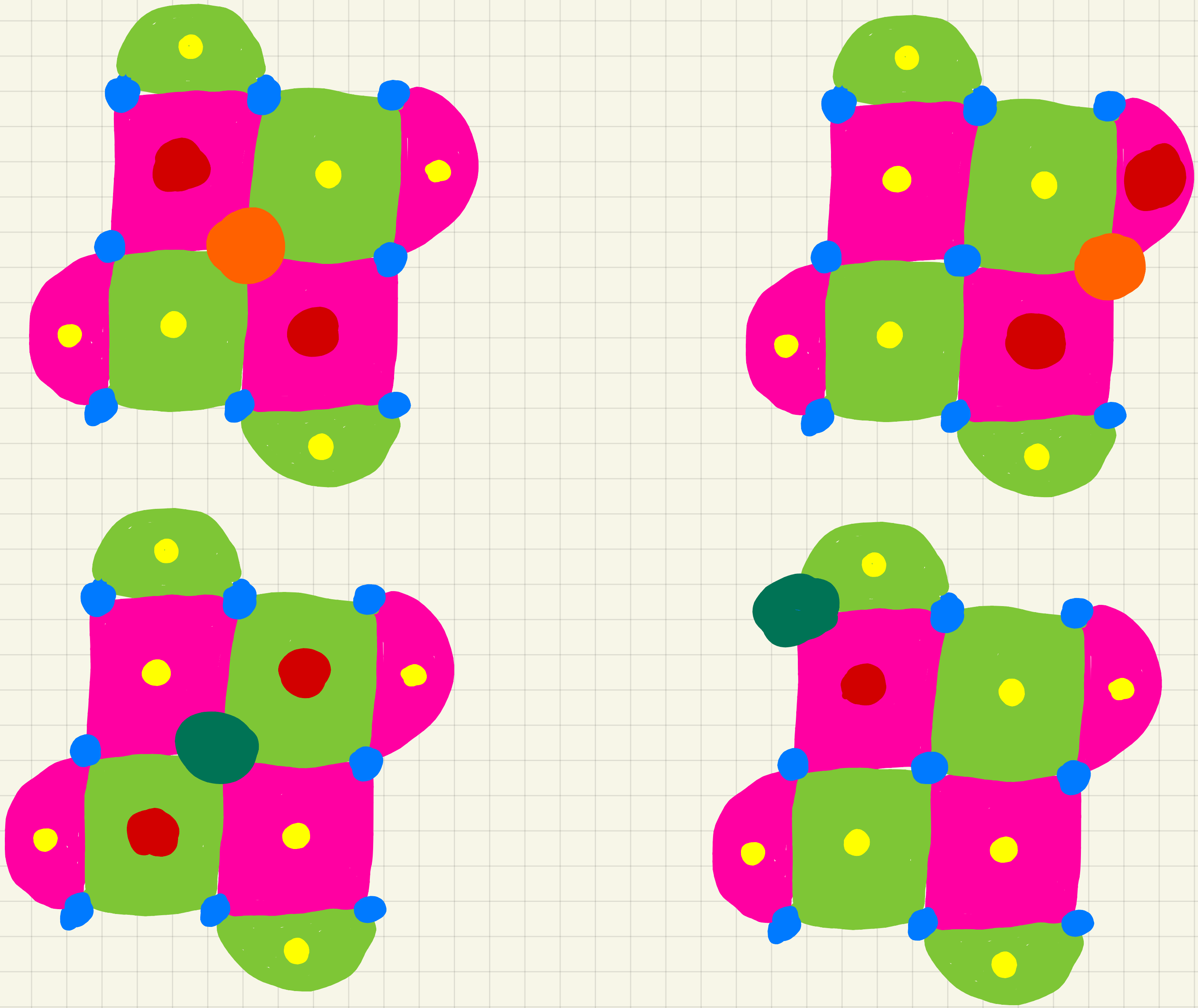
L_Z & L_X mutually anticommute on same qubit but commute with stabilizers S_Z & S_X respectively. Note $L_i^2 = 1$

This example has 9 qubits, hence:

- distance $d = 3$, can correct almost $d/2$ errors
- $k = 1$: # logical qubits
- $n = 9$: # physical qubits

ERRORS

alarmed ancilla   Z error   X error



Yaneda et al.