Exp Freier Fall quantitativ:

◦ ball falling from ceiling, meas. g

$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

$v_0 = 0 \qquad x_0 = 4.92\,m$

$$-4.92 = \frac{1}{2}\cdot(+a)\cdot t^2 \qquad , \quad t \sim 1s$$

$$\Rightarrow a \cong -9.84\ \frac{m}{s^2} \qquad , \quad \text{gravitation}$$

Exp freier fall, qualitativ

plume and ball in tube
a) air
b) no air

Phys I: Mass and force — Dynamics of motion

So far: motion / kinematics considered, but not the causes of motion

Why do objects start to move?
What causes a moving object to change speed and/or direction?

→ Newton (Galileo) → 3 basic laws of motion
(classical mechanics)

Slide: Tipler Fig 4-1: Tennis ball on table in plane

a) const speed (plane): ball remains at its position

b) plane accelerates ($a \neq 0$): ball does not gain speed as quickly as plane ... accelerates towards back of plane (relative to plane)

for object to move: needs "action"; force ↗ this force ... to a ch...

Exp LkB
(1) Luftkissenbahn
(J)

Slide: **Law of inertia**, Newton's 1st law

force: external influence (action) acting on an object

$\vec{F}$: vector quantity

- contact forces (hitting a ball, shoes on ground)
- action at a distance forces (gravitation, electrical forces em ↗)

Principle of superposition:

$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \sum_i \vec{F}_i$$

net force

$[F] = N$
Newton

Tue 2 force

11

Exp. LKB (Luftkissen Bahn) for 1st law (inertia)

cart moving on air cushion (no friction)
→ will not stop or change direction without a force acting on it

Mass

observation: objects resist being accelerated

mass = measure of an object's inertia; property of matter

$[m] = kg$

density: $\rho = \frac{m}{V}$, $[\rho] = \frac{kg}{m^3}$

→ relation between force, mass and acceleration?

(Slides) Newton 2nd law

Newton 3rd law

Exp Flasche, Kraftzerlegung (2)

(3) Seilziehen

Second law, Newton:

$$\| \ \vec{F} = \frac{d}{dt}(m \cdot \vec{v}) \overset{m = const}{=} m \cdot \vec{a}$$

net force acting on an object

~~momentum~~ is a vector -- (Hockey!)

units: $N = kg \cdot \frac{m}{s^2}$

· Note: momentum $\vec{p} = m \cdot \vec{v}$, $[\vec{p}] = kg \cdot m/s$

and $\vec{F} = \frac{d\vec{p}}{dt}$, change of momentum, vector quantity

velocity change
or mass change ...

$\sum \vec{F} = 0 \Rightarrow \sum \dot{\vec{p}} = 0$ and $\sum \vec{p} = const$ ‖ in a closed system (no external force), the total momentum is constant

- from Object perspective : $\vec{F}$: net force on an object $\vec{F} = \sum \vec{F}_i$

$\vec{a} \propto \vec{F}$ $\quad |\vec{F}| \nearrow \Rightarrow |\vec{a}| \nearrow$

prop. to

mass effect ? $\quad$ if m large $\nearrow$ (for given $|\vec{F}|$), $|\vec{a}|$ small $\searrow$

$\Rightarrow |\vec{a}| \propto \frac{1}{m}$

$\Rightarrow \quad \vec{a} \stackrel{?}{=} \frac{\vec{F}}{m}$, vectors, $\vec{a}$ along $\vec{F}$ direction, same sign.

- example of masses : bike ~ 10 kg
  car ~ 10^3 kg
  train ~ 10^6 kg
  earth $\gtrsim 10^{24}$ kg
  red blood cell ~ 10^{-12} kg
  (small) atom ~ 10^{-26} kg

densities : air (20°C) ~ 1.3 kg/m3
  water (20°C) ~ 10^3 kg/m3
  wood 400 - 800 kg/m3
  steel ~ $8 \cdot 10^3$ kg/m3
  Pt ~ $21 \cdot 10^3$ kg/m3

  atom nucleus ~ 10^{17} kg/m3

  best vacuum on earth / lab ~ 10^{-16} kg/m3

  matter in interstellar space ~ 10^{-21} kg/m3

- unified atomic mass unit $\quad 1\,u = 1.66 \; 10^{-27}$ kg $= \frac{1}{12}$ Carbon atom mass

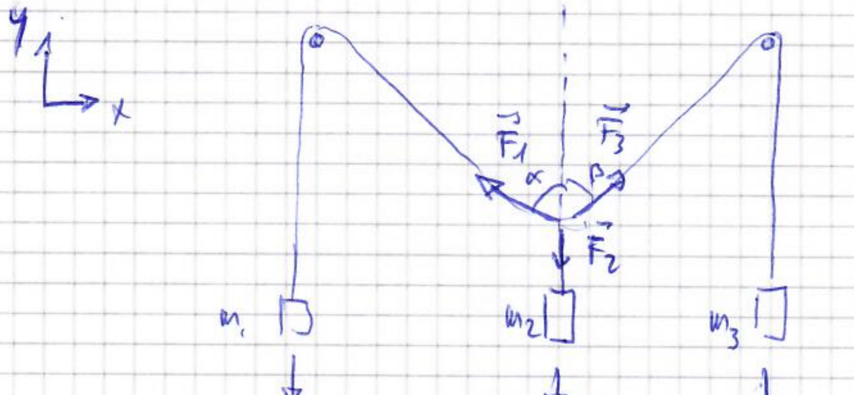
Exp: Flasche, Kegel, Münze

remove cone fast (push at top)

only $\vec{F}_G$, no additional force,

=> coin falls in bottle

Exp: zero net force, suspended masses, with ~~wire~~ wire



masses chosen

$m_1 = 5N$

$m_2 = 10N$

$m_3 = 8.7N$

Equilibrium ... ?
- angles
- masses

Superposition of vectors: $\sum \vec{F}_i = 0$ at rest (equilibrium)

$$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$$

along x: $F_1 \cdot \sin\alpha = F_3 \cdot \sin\beta$

" y: $F_2 = F_1 \cdot \cos\alpha + F_3 \cdot \cos\beta$

$$\Rightarrow \quad \frac{F_3}{F_1} = \tan(\alpha) = \frac{m_3}{m_1} \quad \text{(angle found)}$$

$F_i = m_i \cdot g$

Observe: $\alpha + \beta = 90°$

for this choice of masses

$\beta = 90 - \alpha$

$\sin\beta = \sin(90-\alpha) = \cos\alpha$

$\cos\beta = \cos(90-\alpha) = \sin\alpha$

Exp suspended mass, continued

also vector addition ($\sum \vec{F} = 0$):

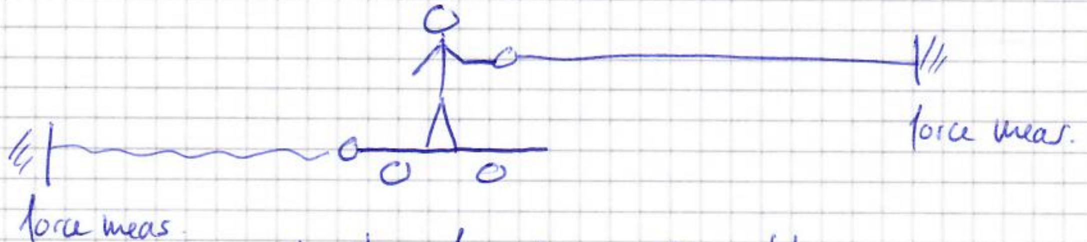
and $F_1^2 + F_2^2 = F_3^2$

$\| \; m_1^2 + m_2^2 = m_3^2$ relation between masses for equilibrium

%

(with $\tan \alpha = \frac{m_3}{m_1}$)

Exp. Seilziehe, car and pulling on cords



a) show force sensors separately

b) at rest, no force (on cord, no pulling)

c) pull on rope: show both forces ~~idem~~ identical

Note: we measure the module/amplitude of the force.

So in summary:

- net force = $\sum \vec{F_i}$, addition of vectors
- that means also: a force acting on an object can be compensated by counteracting forces

-> levitation, for instance: magnetic of Phys II, levitating frog
- in space: why astronauts "float"?

(ask) Fundamental interactions in nature

- gravitational interaction : long range
  due to mass

  (exchange of hypothetical particles: graviton?)

Note: detection of gravitational waves, Kavli prize 2016, R. Drever, K. Thorne, R. Weiss

$\frac{\Delta L}{L} = 10^{-21}$; 1 atom size difference over 10^8 km (Sun-Venus; Sun earth: $150 \cdot 10^6$ km)

electro-weak interaction (unified interaction):

- electromagnetic interaction : long range
  due to charges (and charges in motion)

  (exchange of photons)

- weak interaction : very short range
  between sub-nuclear particles

  exchange/production of (W and Z) bosons

  (Nobel 2013 P. Higgs, F. Englert) origin of mass of subatomic particles / CERN experiments Atlas, & CMS

(

- strong interaction : long-range
  between hadrons* (made of quarks), binding together protons & neutrons to form atomic nuclei

  (exchange of mesons between hadrons, gluons a quarks)

)

* mesons = quark/antiquark

baryons: 3 quarks

Note: ◦ inertial forces are apparent forces ~~arising due to~~ acting on masses whose motion is described in a non-inertial frame of reference (e.g. rotating frame of ref.)

◦ not due to physical interaction between objects (like gravitation, electrostatics) but due to acceleration of ref. frame mass & force

Force due to gravity : weight

- gravitational force : $\vec{F}_g = m \cdot \vec{g}$, $g = |\vec{g}| = 9.81 \frac{m}{s^2}$

weight of m := $|\vec{F}_g|$

acting on m

- distinction mass and weight

mass: inertia

weight: force due to gravity

e.g. a) inertia of ball thrown on moon or earth, identical

momentum : $m \cdot \vec{v}$, depends on v and m

force to launch m horizontally and reach v will be the same

b) weight will be different, as $\vec{g}_{moon} \approx \frac{1}{6} \vec{g}_{earth}$

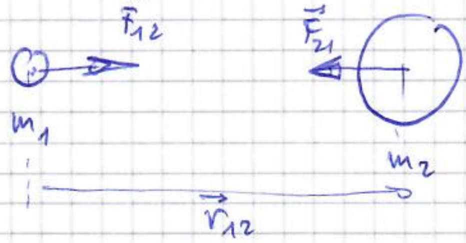
- origin of force : ~~mass~~ attractive interaction between masses

Gravitationswaage (Cavendish Pendulum)

Exp. Cavendish Pendulum (Gravitations Waage)

Slide setup, explain

interaction (force) between masses: $\vec{F}_{12} = -\vec{F}_{21}$ (Newton's 3rd law)



∘ Newton's gravitation law:

$$F = G \cdot \frac{m_1 \cdot m_2}{r_{12}^2} \quad , \quad G = 6.67 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}$$

universal gravitation constant

NB: m_i denote here gravitational masses (cf Tipler p 370)

equivalence of inertial mass (opposing acceleration) and gravitational mass demonstrated to ~ 1 part in 10^{12} ($5 \cdot 10^{13}$)

∘ gravitational field : def "field": associate to each point in space a value for a given physical quantity (scalar or vector)
e.g. Temperature (scalar)
air speed (wind) (vector)

$\vec{F} = m \cdot \vec{g}$

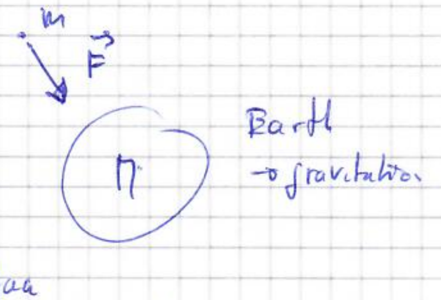
$\vec{g} = \frac{\vec{F}}{m} = \frac{\text{grav. force on test mass } m}{\text{test mass } m}$

gravitational field

with Newton's grav. law

$$|\vec{g}| = \frac{\frac{G \cdot M \cdot m}{r^2}}{m} = \frac{G \cdot M}{r^2}$$

$\simeq 9.81 \frac{m}{s^2}$, at Earth surface
r = Radius Earth ~ 6'371 km

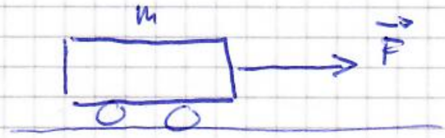


∘ Weight of a 1 kg mass

$\vec{F} = m \cdot \vec{g}$, $F = 1 \cdot 9.81 = \underline{9.81}$ N

inertial forces

Consider mass m, accelerated by force $\vec{F}$



a) observer at rest (ref frame is ground):

we have $\vec{F} = \dfrac{d\vec{p}}{dt} = \dfrac{d(m\vec{v})}{dt} = m \cdot \vec{a}$, $m = \text{const.}$

$\vec{F} \uparrow\uparrow \vec{a}$

b) observer on car (ref frame is car)

observer feels a force (inertial force, or pseudo force) opposed to the direction of the acceleration

$\sum \vec{F} = 0 \Rightarrow$ inertial force $= -$ force exerted on car

note: observer cannot differentiate the inertial force from a "true" gravitational force for instance

Exp.

. Schnur zerreißen

. Tischlein Deck

29.09

Summary : motion / kinematics : interdependance of quantities

$$
\begin{array}{ccc}
 & \text{position vector } \vec{r}(t) & \\
\dfrac{d\vec{r}(t)}{dt} \downarrow & & \uparrow \int \vec{v}(t)\cdot dt \\
 & \text{velocity vector } \vec{v}(t) & \\
\dfrac{d\vec{v}(t)}{dt} \downarrow & & \uparrow \int \vec{a}(t)\cdot dt \\
 & \text{acceleration vector } \vec{a}(t) &
\end{array}
$$

e.g. : $\vec{v}(t) = \dfrac{d\vec{r}(t)}{dt}$ $\qquad$ $d\vec{r}(t)$: infinitesimal displacement vector

hence $\quad d\vec{r}(t) = \vec{v}(t)\cdot dt$

and $\quad \int d\vec{r}(t) = \vec{r}(t)$

$\quad\parallel\quad \vec{r}(t) = \int \vec{v}(t)\cdot dt$

Similarly $\quad \vec{v}(t) = \int \vec{a}(t)\cdot dt$

29.9.

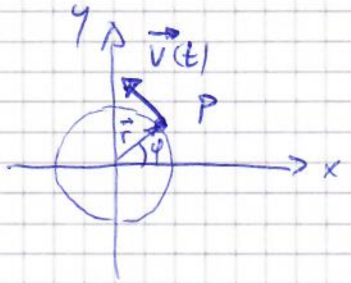
8

Circular motion (and harmonic oscillation)

particular case, present in many situations

(Slide) carousel, centrifuge

Trajectory of a point following a circular motion (2D)



uniform : $v = \text{const}$

but circle trajectory $\Rightarrow$ acceleration

def : angular velocity $\omega = \frac{d\varphi}{dt}$ ||

$[\omega] = \frac{\text{rad}}{s}$

(Slide) Geometry of motion (Tipler Fig 3-24)

black triangle ($\vec{r}$ vectors)
is similar to blue triangle (velocities)
(homothetic)

hence we can compare lengths, e.g.:

$$\frac{|\Delta \vec{v}|}{|\Delta \vec{r}|} = \frac{|\vec{v}|}{|\vec{r}|} = \frac{v}{r}$$

multiply by $\frac{|\Delta \vec{r}|}{\Delta t}$:

$$\frac{|\Delta \vec{v}|}{\Delta t} = \frac{v}{r} \frac{|\Delta \vec{r}|}{\Delta t}$$

lim $\Delta t \to 0$, $\frac{|\Delta \vec{v}|}{\Delta t} \to a$; $\frac{|\Delta \vec{r}|}{\Delta t} \to v$

$$\| \quad a_c = \frac{v^2}{r}$$

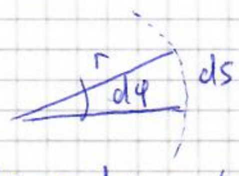
centripetal acceleration

acceleration $\perp$ to motion $v = \text{const}$, but circular trajectory

frequency of motion:

$\nu = \frac{\omega}{2\pi}$, 2π : full circle in rad. ($= 360°$)

distance along circle : ds



$ds = r\, d\varphi$ (circle: perimeter $= 2\pi \cdot r$)

$$\frac{ds}{dt} = r \cdot \frac{d\varphi}{dt}$$

$$\| \quad v = r \cdot \omega$$

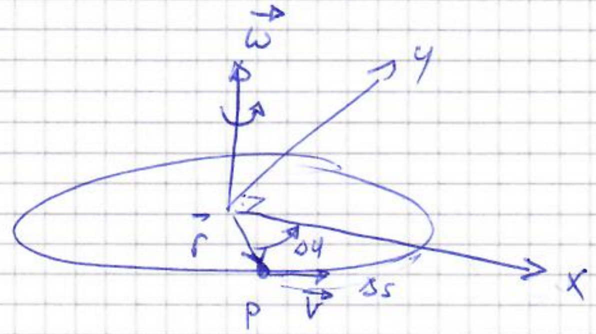
$$= 2\pi \cdot \nu \cdot r$$

$$= \frac{2\pi \cdot r}{T}$$

T: period of rotation

$T = \frac{1}{\nu}$

in 3D:



- velocity

$$V = \omega \cdot r$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$ vector product (Slide) thumb rule (show corkscrew, Finger for rule)

- acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\vec{\omega} \times \vec{r}\right)$$

$$\vec{a} = \vec{\omega} \times \frac{d\vec{r}}{dt} = -\omega^2 \cdot \vec{r} = -\frac{v^2}{r^2} \cdot \vec{r} \quad , \quad v = \omega \cdot r$$

$\omega = \text{const}$, uniform circular motion

$$= \vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right) \quad \left(= -\omega^2 \cdot \vec{r}\right)$$

$$\frac{d\vec{r}}{dt} = \vec{v} = \vec{\omega} \times \vec{r}$$

% demo vector product

Note: projection of circular motion to x or y axis represents a harmonic motion

def harmonic oscillator

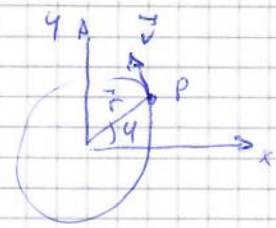
$$A(t) = A_0 \cdot \cos\left(\omega t + \varphi_0\right) \quad \left(\text{or } = A_0 \cdot \sin\left(\omega t + \varphi_0\right)\right)$$

amplitude of motion

$$T = \frac{2\pi}{\omega} = \frac{1}{\nu}$$ period of ~~motion~~ oscillatory motion

Exp. rot. Scheibe, Pendel

circular motion (continued)



Coordinates of moving point on trajectory $\vec{r}(t) = \left(x(t), y(t)\right)$

$$x(t) = r \cdot \cos\left(\varphi(t)\right) \qquad \left(r = |\vec{r}| = \text{const}\ ;\ \text{circular motion}\right)$$

$$y(t) = r \cdot \sin\left(\varphi(t)\right)$$

$\uparrow$ time depend.

$$\omega = \frac{d\varphi}{dt}\ ,\ \text{circular freq.}$$

$$[\omega] = \frac{\text{rad}}{s}$$

frequency $\nu = \frac{\omega}{2\pi}$, $[\nu] = \frac{1}{s} = Hz$, Hertz

period $T = \frac{1}{\nu} = \frac{2\pi}{\omega}$, $[T] = s$

Velocity: $\vec{v}(t) = \left(\dot{x}(t), \dot{y}(t)\right)$

$$v_x(t) = \frac{d}{dt}\left(r \cos\varphi(t)\right) \underset{\varphi = \omega \cdot t}{=} \frac{d}{dt}\left(r \cdot \cos(\omega t)\right) = -\omega r \cdot \sin(\omega t)$$

$$v_y(t) = \frac{d}{dt}\left(r \cdot \sin(\varphi(t))\right) = \frac{d}{dt}\left(r \cdot \sin(\omega t)\right) = \omega r \cdot \cos(\omega t)$$

$$|\vec{v}| = \left(v_x^2 + v_y^2\right)^{1/2} = \left(\omega^2 r^2 \cdot \left(\sin^2(\omega t) + \cos^2(\omega t)\right)\right)^{1/2}$$

$$\| \ v = \omega \cdot r$$

acceleration

$$\vec{a}(t) = \dot{\vec{v}}(t) = \left(\ddot{x}(t) ; \ddot{y}(t)\right)$$

$$= \left(-\omega^2 r \cdot \cos(\omega t) ; -\omega^2 r \cdot \sin(\omega t)\right)$$

$$\| \ |\vec{a}(t)| = \omega^2 \cdot r$$

Note:

$$\vec{v}(t) \perp \vec{r}(t)$$

$$\vec{a}(t) \uparrow\downarrow \vec{r}(t)$$

antiparallel