Phys I : Waves III

Standing waves or stationary waves

Particular case of wave where the maxima and minima of the amplitude are always located at a given spatial location/position

positions where the amplitude is max : antinode

" " " is min. : node.

(Slide) 1/ standing waves : antinode, node

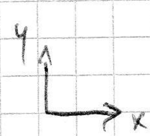
below (2/ " " : sound wave forced in semi-open tube)

Exp. Rubber tube

- Standing wave results from the superposition (sum) of 2 identical waves travelling in opposite directions.

For rubber tube or a cavity, the superposition occurs between waves reflected back and forth from the ends of the system (tube anchors, walls of cavity)

(Slide) 2) standing waves: sound wave forced in semi-open tube



- Wave in positive direction: (left to right) $\quad y_1(x,t) = y_0 \cdot \sin(\omega t - kx)$

Wave in negative direction: (right to left) $\quad y_2(x,t) = y_0 \cdot \sin(\omega t + kx)$

More generally, for reflected wave:

$$\psi_1(x,t) = \psi_0 \cdot \sin(\omega t - kx) \qquad \text{positive direction}$$

$$\psi_2(x,t) = \psi_0 \cdot \sin(\omega t + kx + \varphi_0) \qquad \text{negative direction}$$

(dephasing) phase constant

Superposition:

$$\psi(x,t) = \psi_1(x,t) + \psi_2(x,t)$$

$$= 2\psi_0 \underbrace{\sin(\omega t + \varphi_0/2)}_{\text{only time dependent}} \cdot \underbrace{\cos(kx + \varphi_0/2)}_{\text{only position dependent}}$$

<u>nodes</u>: positions x where amplitude is minimum

$$\Rightarrow \cos(kx + \varphi_0/2) = 0$$

$$k \cdot x_n + \varphi_0/2 = \frac{\pi}{2} + n \cdot \pi \quad , \quad n = 0, 1, 2, \dots$$

$$2k \cdot x_n + \varphi_0 = \pi + 2n \cdot \pi = \pi \cdot (1 + 2n)$$

and $$\| \quad x_n = \frac{(2n+1) \cdot \pi - \varphi_0}{2k} \quad ,$$ positions of nodes, $n = 0, 1, 2, \dots$

<u>anti-nodes</u> positions x where amplitude is max.:

$$\Rightarrow \cos(kx + \varphi_0/2) = \pm 1$$

$$k \cdot x_n + \varphi_0/2 = n \cdot \pi \quad , \quad n = 0, 1, 2 \dots$$

and $$\| \quad x_n = \frac{2 \cdot n \cdot \pi - \varphi_0}{2 \cdot k}$$

$\sin(\alpha-\beta) + \sin(\alpha+\beta) = 2 \sin\alpha \cdot \cos\alpha$

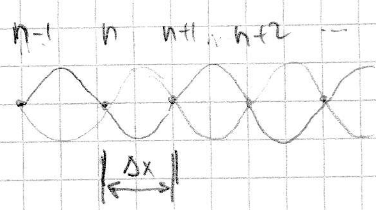
$\alpha-\beta = \omega t - kx$

$\alpha+\beta = \omega t + kx + \varphi_0$

$2\alpha = 2\omega t + \varphi_0$, $\alpha = \omega t + \varphi_0/2$

$\beta = kx + \varphi_0/2$

distance between nodes



$$\Delta x = x_{n+1} - x_n$$

$$x_{n+1} = \frac{(2(n+1)+1)\cdot\pi - \varphi_0}{2\cdot k}$$

$$x_n = \frac{(2n+1)\pi - \varphi_0}{2k}$$

$$\Delta x = \frac{2\pi}{2k} = \frac{\pi}{k} = \frac{\pi}{2\pi}\cdot\lambda = \frac{\lambda}{2}$$

$k = 2\pi/\lambda$

$\|\ \Delta x = \frac{\lambda}{2}$, distance between nodes

$\boxed{\text{Exp.}}$ Lecher line (Lecher leitung)

parallel wires supporting standing radio waves at microwave freq. (27 MHz)

propagation speed : c light speed , $c = 3\cdot10^8$ m/s

$c = \lambda\cdot\nu$, $\lambda = \frac{c}{\nu} = \frac{3\cdot10^8\ m/s}{27\cdot10^6\ Hz} \approx 11\ m$

27 MHz for microwave

$\frac{\lambda}{2} \approx 5.5\ m$ distance between nodes (also antinodes : $\Delta x = \frac{\lambda}{2}$)

$\boxed{\text{Exp.}}$ Chladni figures

loud speaker , 290 Hz

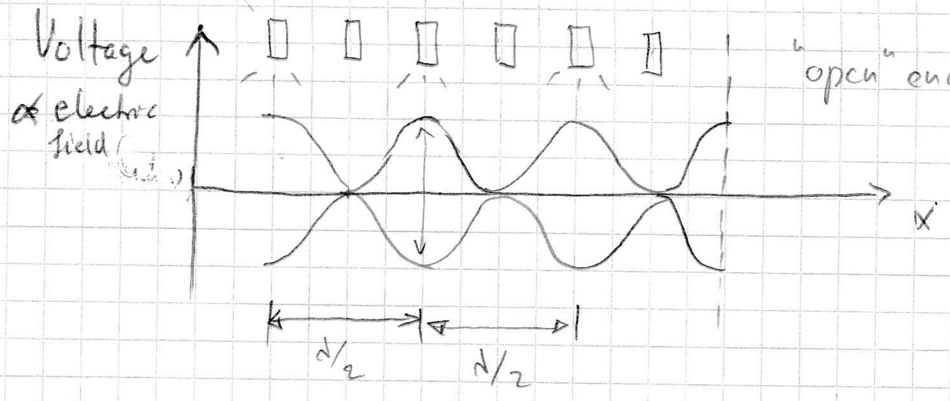
740 Hz

1308 Hz

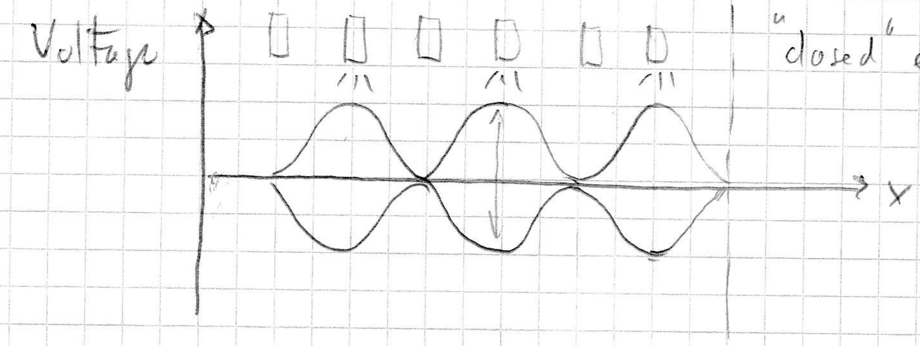
1520 Hz

1960 Hz

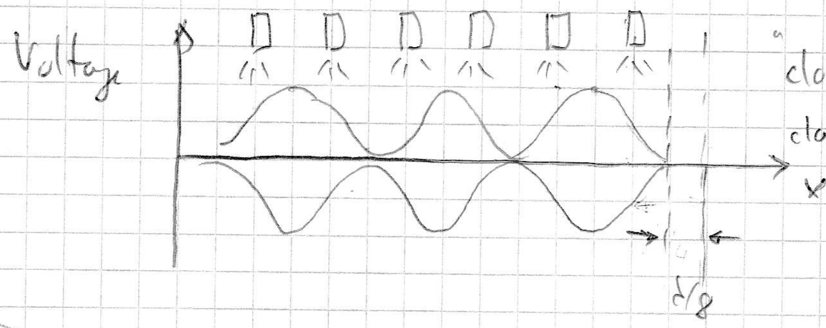
Standing wave : oscillating Electric field



"open" end, voltage can vary at end



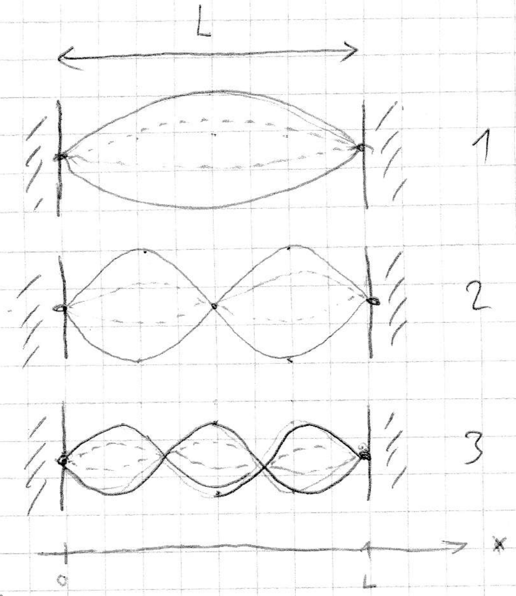
"closed" end:



"closed" end and λ/8 closer to source

slide with drawings

Standing waves and resonance



- Consider a cavity and first 3 modes:

⇒ find resonance freq. using boundary conditions?

- Standing wave:

$$y(x,t) = 2y_0 \cdot \sin\left(\omega t + \frac{\varphi_0}{2}\right) \cdot \cos\left(kx + \varphi_0/2\right)$$

Ends are fixed: $y(0,t) = 0$ for all modes
$y(L,t) = 0$

a) $y(0,t) = 0 = 2 \cdot y_0 \cdot \sin(\omega t + \varphi_0/2) \cdot \cos(\varphi_0/2)$

$$0 = \cos(\varphi_0/2)$$

$$\Rightarrow \quad \frac{\varphi_0}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$= (2n+1) \cdot \frac{\pi}{2}, \quad n = 0, 1, 2, \ldots$$

and $\varphi_0 = (2n+1) \cdot \pi$

b) $y(L,t) = 0 = 2y_0 \cdot \sin(\omega t + \varphi_0/2) \cdot \cos(k \cdot L + \varphi_0/2)$

$$0 = \cos(k \cdot L + \varphi_0/2)$$

$$k \cdot L + \underbrace{\frac{(2n+1) \cdot \pi}{2}}_{\varphi_0/2 \text{ from a)}} = (2m+1) \cdot \frac{\pi}{2} \qquad n, m = 0, 1, 2, \ldots$$

$$k \cdot L = (m-n) \cdot \pi = n' \cdot \pi$$

and $k = \dfrac{n' \cdot \pi}{L}$ n' integer

Using $k = \frac{2\pi}{\lambda}$

$$\frac{2\pi}{\lambda} = \frac{n' \cdot \pi}{L}$$

and $\| \; L = n' \cdot \frac{\lambda}{2}$, for $n' = 1, 2, 3$

cavity length, wavelength

($n' = 0$: no cavity, $L = 0$)
trivial case

also $\| \; \lambda = \frac{2 \cdot L}{n'}$, for $n' = 1, 2, 3$

($n' = 0$ not defined)

Frequency ν: $v = \lambda \cdot \nu$

$$\nu = \frac{v}{\lambda}$$

and $\nu_{n'} = \frac{v}{\lambda_{n'}} = \frac{v}{\frac{2L}{n'}} = \frac{n' \cdot v}{2L}$

$\| \; \nu_{n'} = \frac{v}{2L} \cdot n'$, for $n' = 1, 2, 3, \ldots$

resonance frequencies of cavity, $\propto \frac{1}{L}$, L: cavity dimension

Exp Chladni

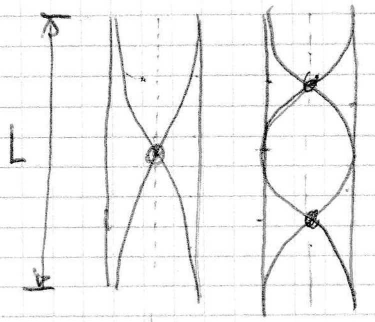
Standing waves : flute (acoustics)

we have seen the importance of particular spatial locations (max, min), in particular the "ends" of a system.

flute: (hollow cylinder)

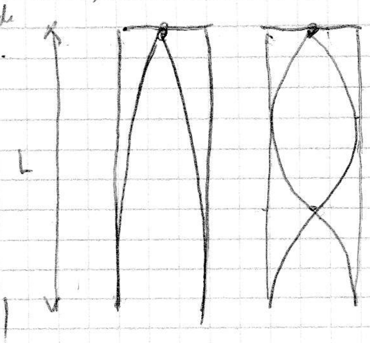
a) open ends

Amplitude is max.



b) 1 closed end, 1 open end

Amplitude is min. (=0)



a) fundamental ($n=0$), 1 node: $L = \frac{\lambda}{2}$

1st overtone ($n=1$), 2 nodes: $L = \lambda$

b) fundamental ($n=0$), 1 node: $L = \frac{\lambda}{4}$

1st overtone ($n=1$), 2 node: $L = \frac{3\lambda}{4}$ $\quad \left(\frac{2}{3}L = \frac{\lambda}{2}\right)$

a) nth overtone

$n = 0, 1, 2, \ldots$

$$L = (n+1)\cdot\frac{\lambda}{2}$$

$v = \lambda \cdot \nu$

$$\Rightarrow \quad L = (n+1)\cdot\frac{v}{2\cdot\nu}$$

and $$\nu_n = (n+1)\cdot\frac{v}{2\cdot L} \propto \frac{1}{L}$$

b) nth overtone

$$L = (2n+1)\cdot\frac{\lambda}{4}$$

$$= (2n+1)\cdot v/4\nu$$

$v = \lambda\cdot\nu$

and $$\nu_n = (2n+1)\cdot\frac{v}{4L} \propto 1/L$$

$\boxed{\text{Exp.}}$

1/ PET tubes : make them rotate by hand

2/ (~~flute~~, ~~whistles~~) : organ pipe : air, CO_2, He $\quad v_{CO_2}: 258\,m/s$

3). He voice : sound speed $v_{air} = 343\,m/s$, $v_{He} = 930\,m/s$ (300K)

(4) Chladni figures : particles accumulate at amplitude minima (antinodes); standing waves)

(if not done before)

Acoustics

Sound waves : in gases, longitudinal pressure waves
in solids, longitudinal & transversal pressure waves

Amplitude A_0 : $A(x,t) = A_0 \sin(\omega t - kx)$

(Slide) standing sound wave

$A_0 \propto p$, local pressure (difference) in medium, or Δp, local pressure

Sound intensity : $I = \frac{P}{S} = \frac{\text{power}}{\text{area}}$, $\frac{W}{m^2}\left(= \frac{\text{Watts}}{m^2}\right)$

$$I \propto (\text{Amplitude})^2$$

with $\| \; I = \frac{(p)^2}{2\rho \cdot v}$

ρ: medium density, kg/m^3
v: speed of sound, m/s
p: pressure ~~difference~~, N/m^2
small p, pressure
(large P: power)

(units: %)

(Slide) Table sound intensities
$\boxed{\text{Exp}}$: dB 1
Note: values in dB, decibel

Loudness (in decibel) : (subjective for human being) perceived sound intensity ($\neq$ sound intensity!)
→ perception: twice as loud when ~~power~~ pressure increased by factor of 10

$$\| \; L = 10 \cdot \log_{10}\left(\frac{I}{I_0}\right), \text{ with } I_0 = 10^{-12}\, W/m^2, \text{ threshold of earing}$$

$$= 20 \cdot \log_{10}\left(\frac{p}{p_0}\right), \text{ with } p_0 = 2\cdot 10^{-5}\, \frac{N}{m^2}$$

$\to I \propto p^2$, pressure

$\boxed{\text{Exp}}$: 1) Hörgrenzen 20 – 15 kHz

NB typ. freq range for human ear ~ 20 Hz to 20 kHz
(sampling freq. CD's)

2) dB meter

(Slide) Loudness: freq. dependence / (Slide) human ear

Unit intensity $I = \frac{(\Delta p)^2}{2\rho \cdot v}$

units: $= \frac{N^2}{m^4} \cdot \frac{m^3}{kg} \cdot \frac{s}{m}$

$= \frac{N^2}{m^2} \cdot \frac{m}{N \cdot s^2} \cdot s$

$= \frac{N \cdot m}{m^2} \frac{1}{s} = \frac{J}{m^2 \cdot s} = \frac{Watt}{m^2}$

Newton: $F = ma$

$N = kg \cdot \frac{m}{s^2}$

$\Rightarrow kg = \frac{N \cdot s^2}{m}$