Waves I

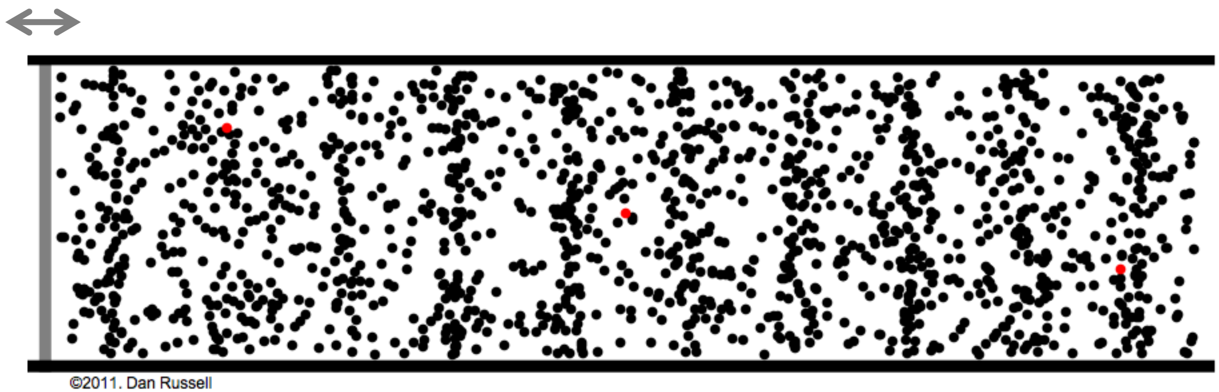
Introduction to Physics I

For Biologists, Geoscientists, & Pharmaceutical Scientists

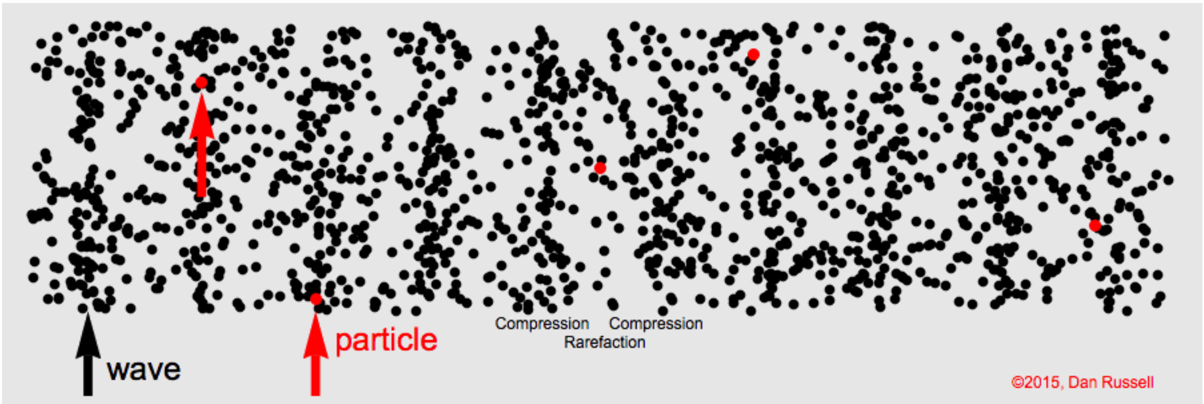
waves

waves

sound



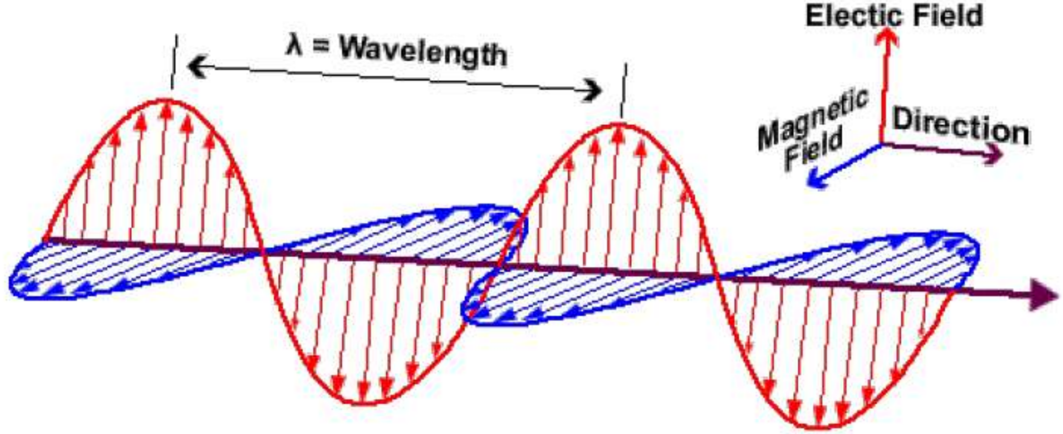
propagating compression & rarefaction of gas



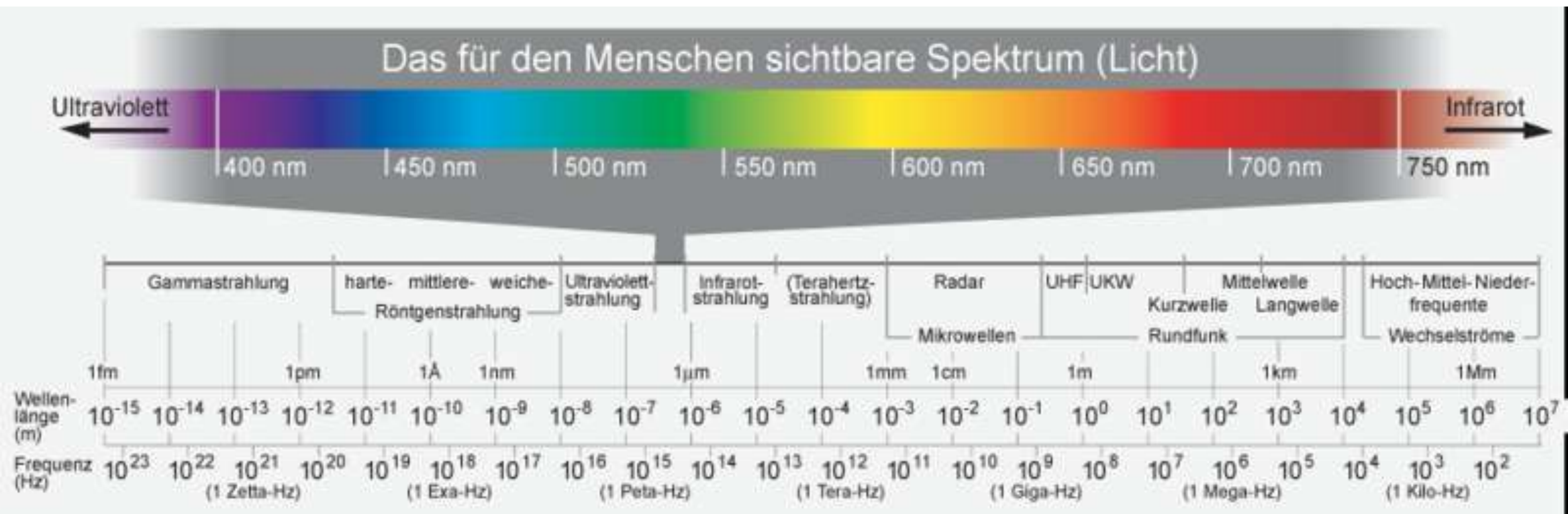
D. Russel, [*https://www.acs.psu.edu/drussell/demos/waves/wavemotion.html*](https://www.acs.psu.edu/drussell/demos/waves/wavemotion.html)

waves

light



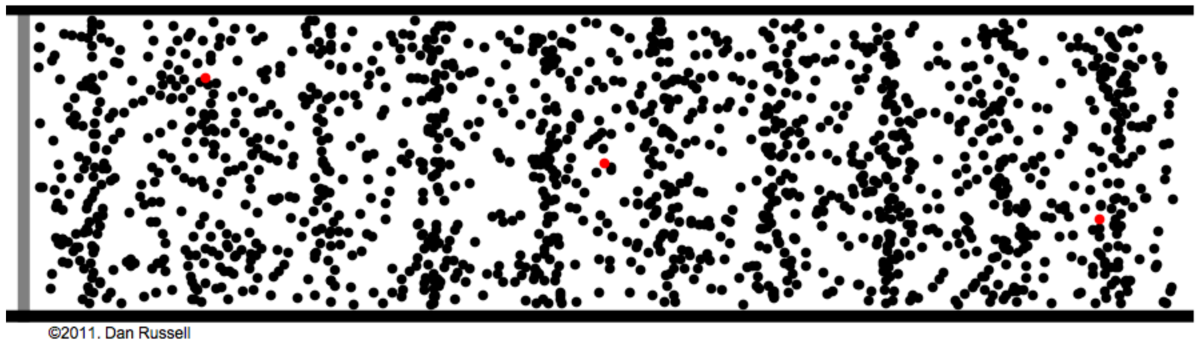
λ = Wavelength
Electic Field
Magnetic Field
Direction



Das für den Menschen sichtbare Spektrum (Licht)
Ultraviolett
Infrarot
400 nm
450 nm
500 nm
550 nm
600 nm
650 nm
700 nm
750 nm
Gammastrahlung
harte- mittlere- weiche-
Röntgenstrahlung
Ultraviolett-strahlung
Infrarot-strahlung
(Terahertz-strahlung)
Radar
Mikrowellen
UHF UKW
Kurzwelle
Mittelwelle
Langwelle
Rundfunk
Hoch-Mittel-Nieder-frequente
Wechselströme
1fm 1pm 1Å 1nm 1µm 1mm 1cm 1m 1km 1Mm
Wellen-länge (m)
10^{-15} 10^{-14} 10^{-13} 10^{-12} 10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7}
Frequenz (Hz)
10^{23} 10^{22} 10^{21} 10^{20} 10^{19} 10^{18} 10^{17} 10^{16} 10^{15} 10^{14} 10^{13} 10^{12} 10^{11} 10^{10} 10^{9} 10^{8} 10^{7} 10^{6} 10^{5} 10^{4} 10^{3} 10^{2}
(1 Zetta-Hz)
(1 Exa-Hz)
(1 Peta-Hz)
(1 Tera-Hz)
(1 Giga-Hz)
(1 Mega-Hz)
(1 Kilo-Hz)

waves

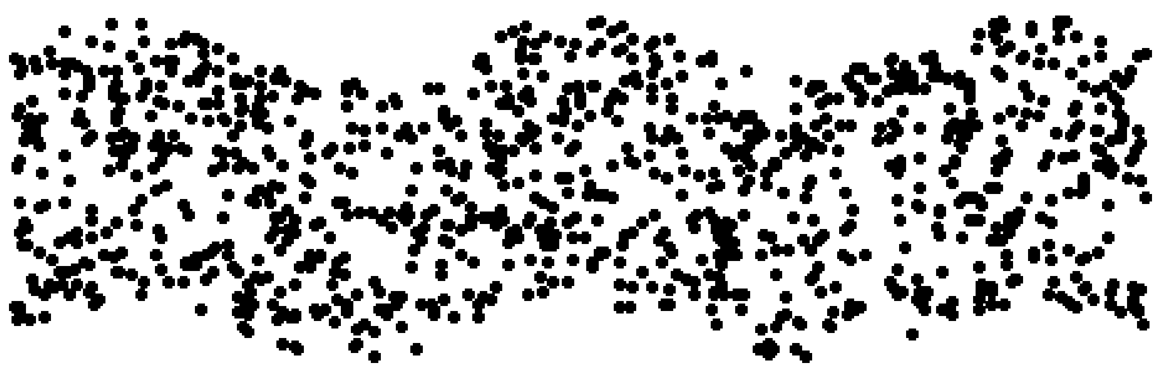
longitudinal wave



propagation →

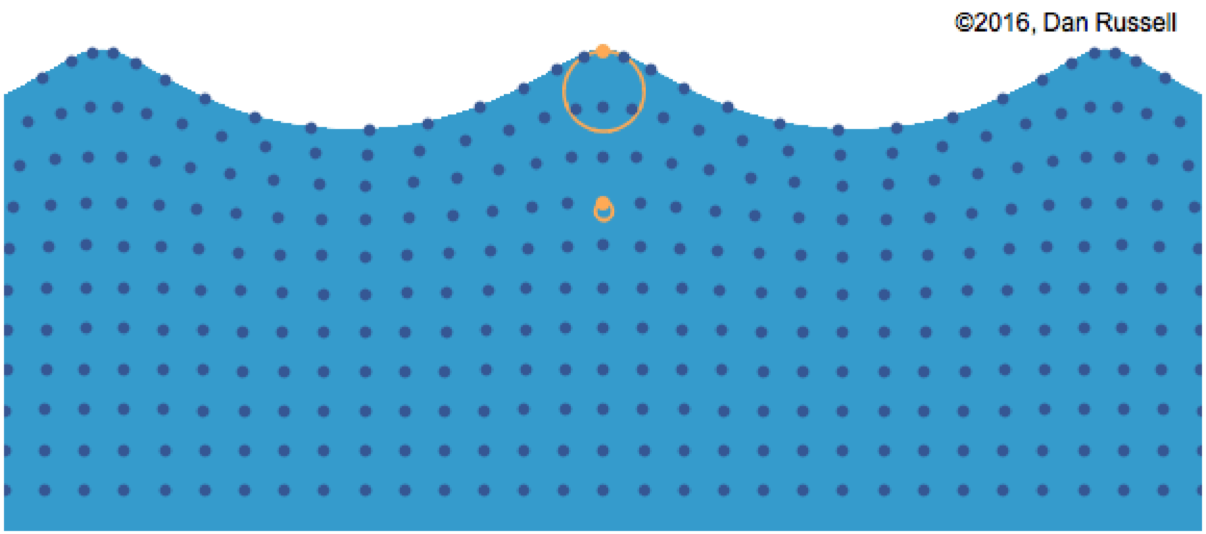
oscillation

transverse wave



D. Russel, https://www.acs.psu.edu/drussell/demos/waves/wavemotion.html

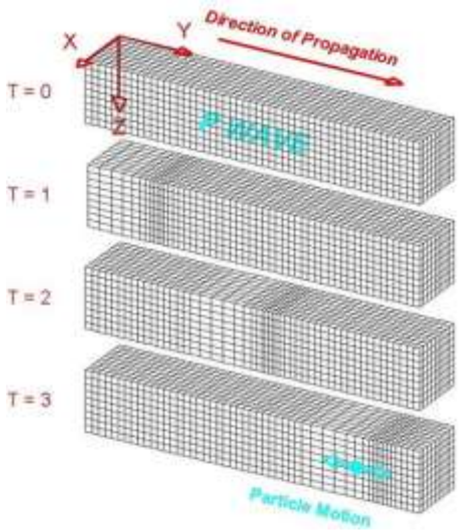
water waves

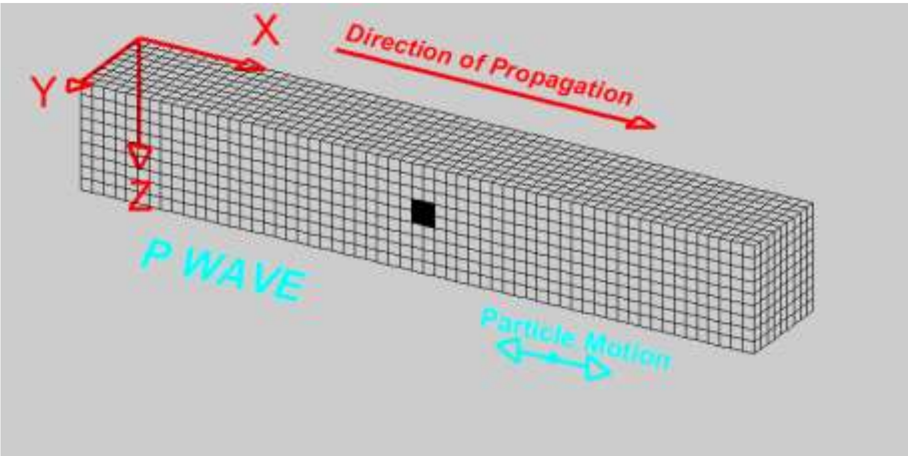


combination of longitudinal & transverse motion

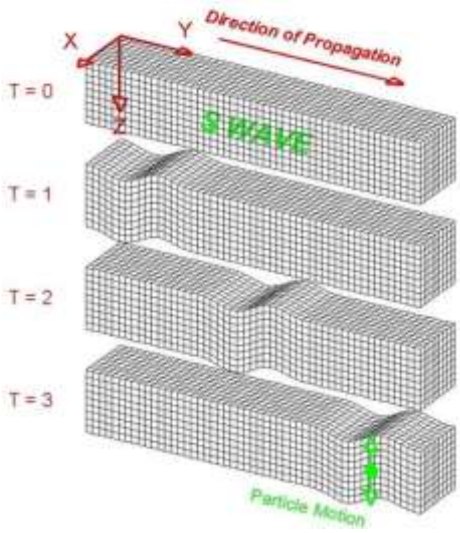
D. Russel, https://www.acs.psu.edu/drussell/demos/waves/wavemotion.html

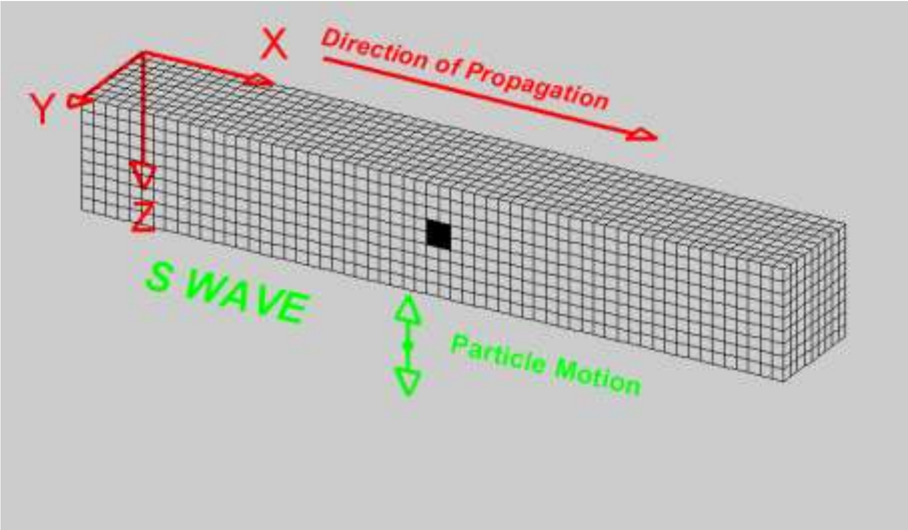
seismic waves





longitudinal wave

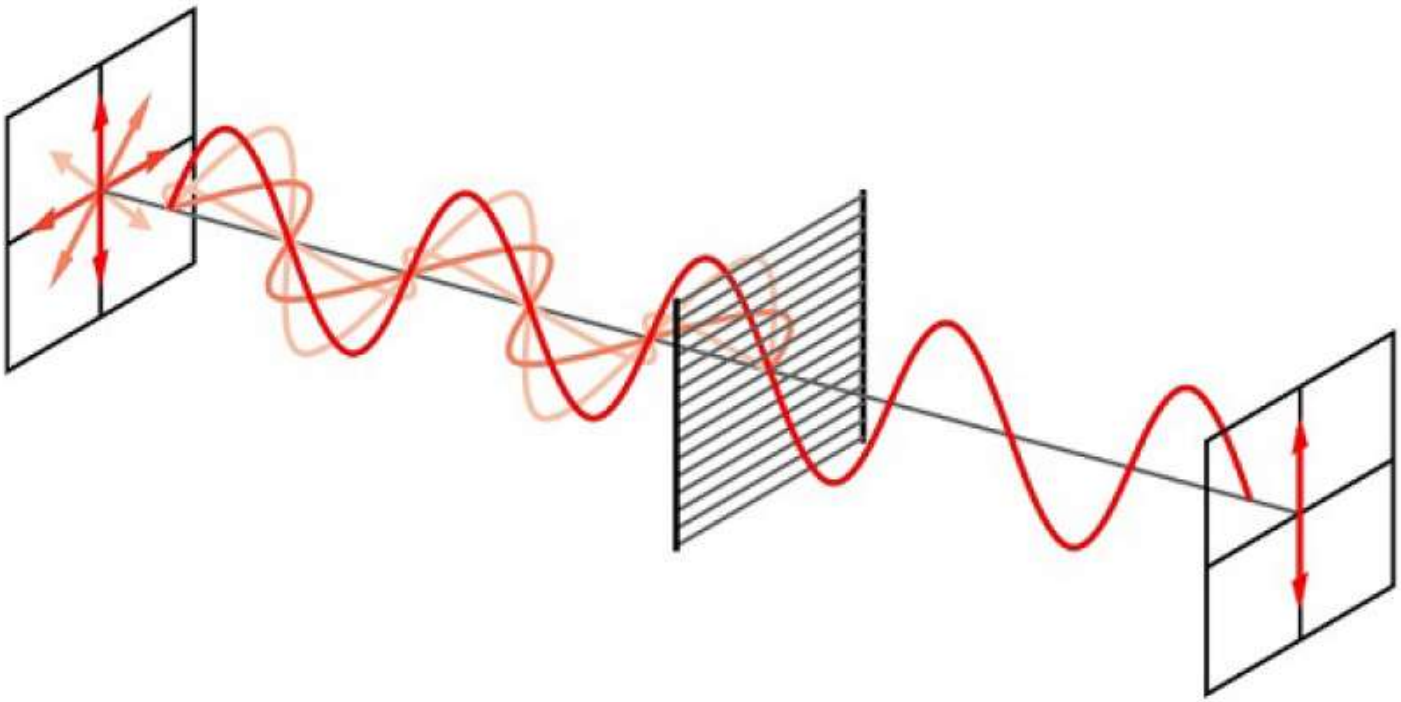




transverse wave

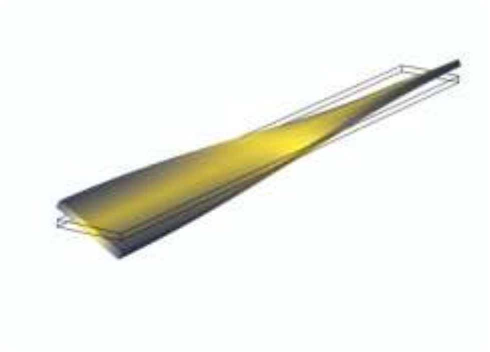
http://www.geo.mtu.edu/UPSeis/waves.html

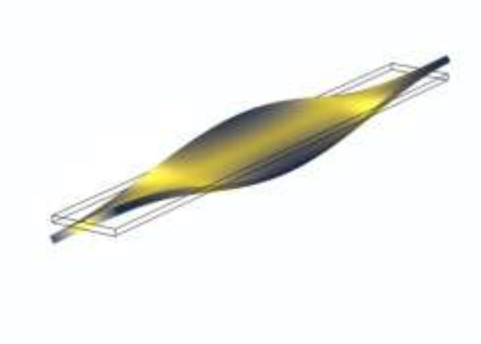
polarization



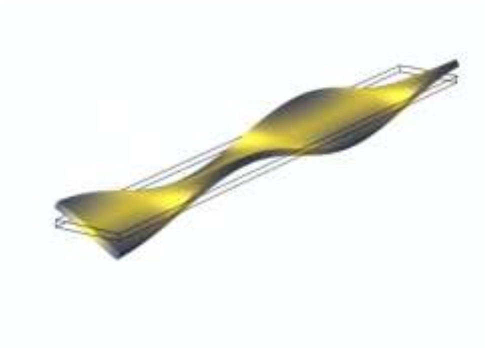
torsional modes

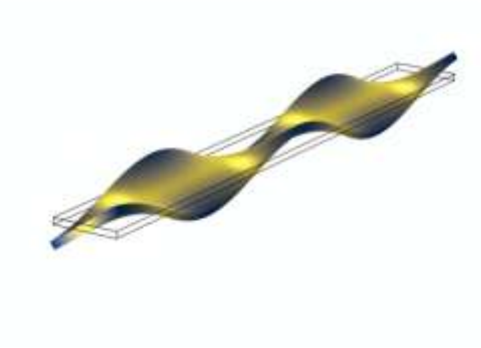
fundamental mode (1)

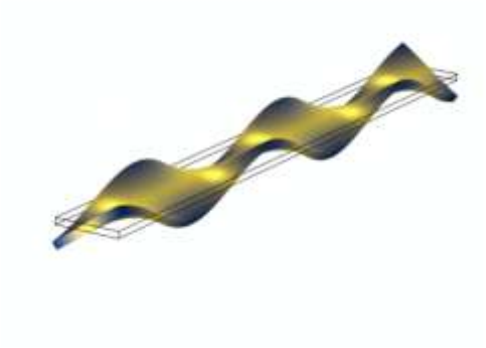




mode #	Frequency (Hz)	fn/f1
1	303.56	1.00
2	611.68	2.02
3	928.89	3.06
4	1259.30	4.15
5	1606.90	5.29



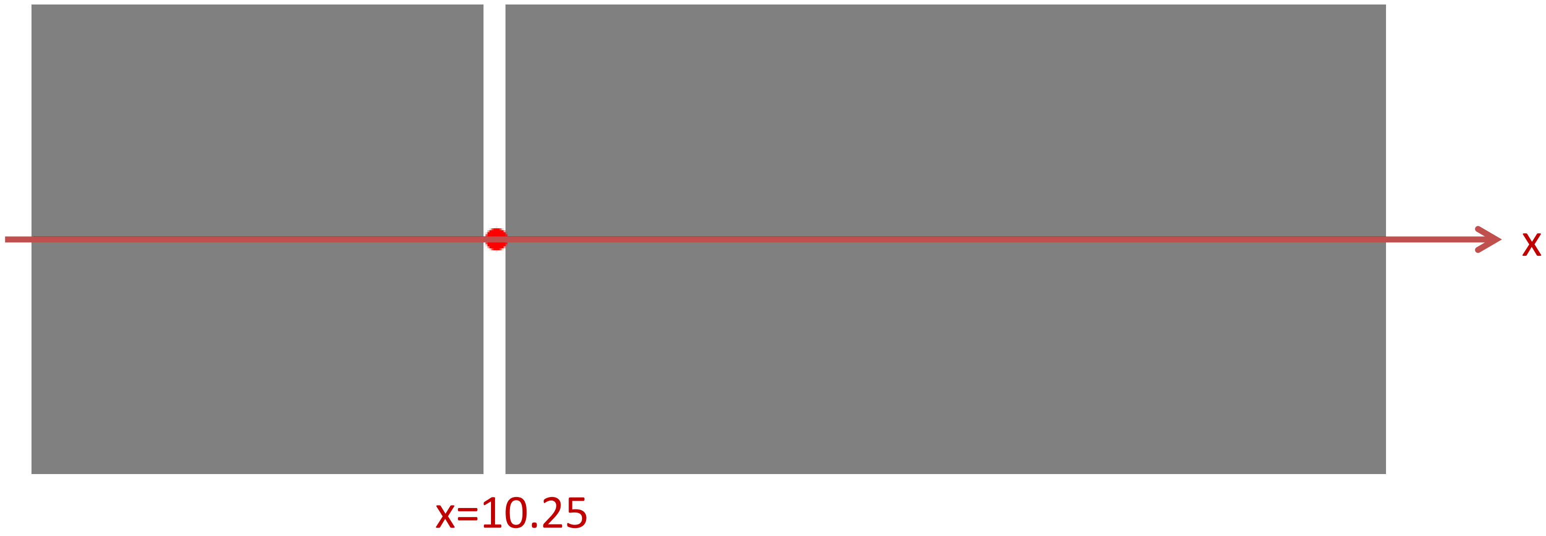




Aluminum Bar
0.1 m wide, 1 m long,
0.01m thick
free at both ends

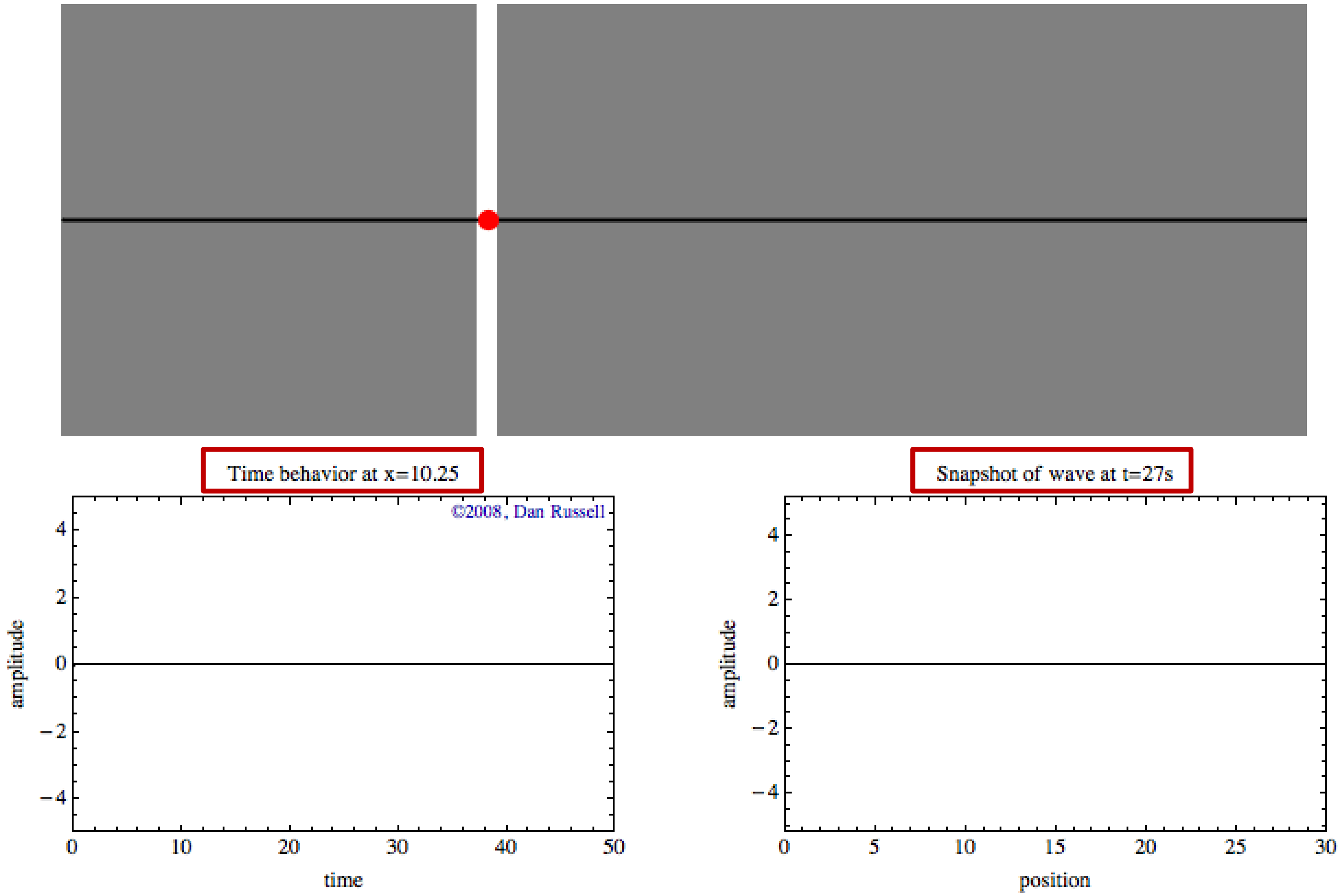
D. Russel, https://www.acs.psu.edu/drussell/Demos/Torsional/torsional.html

wave motion in space & time



https://www.acs.psu.edu/drussell/Demos/wave-x-t/wave-x-t.html

wave motion in space & time

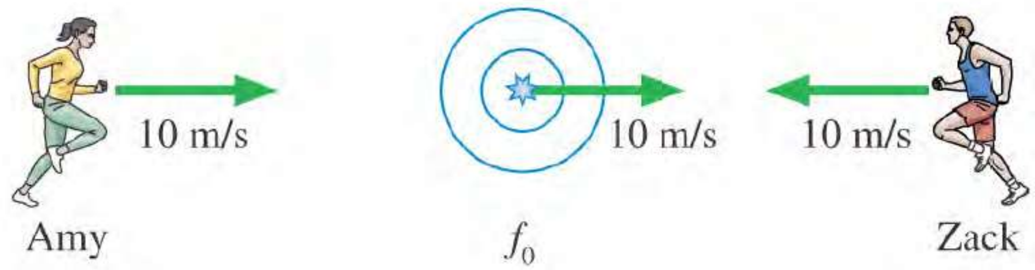


https://www.acs.psu.edu/drussell/Demos/wave-x-t/wave-x-t.html

wave speed

Medium	Ausbreitungsgeschwindigkeit von Schall (m / s)
CO_2	266
N_2	349
trockene Luft (20° C)	343
trockene Luft (0° C)	331
He	1007
H_2	1309
Blei	1300
Wasser	1485
Aluminium	5100
Kronglas	5400
Beryllium	12'900
Diamant	17'500

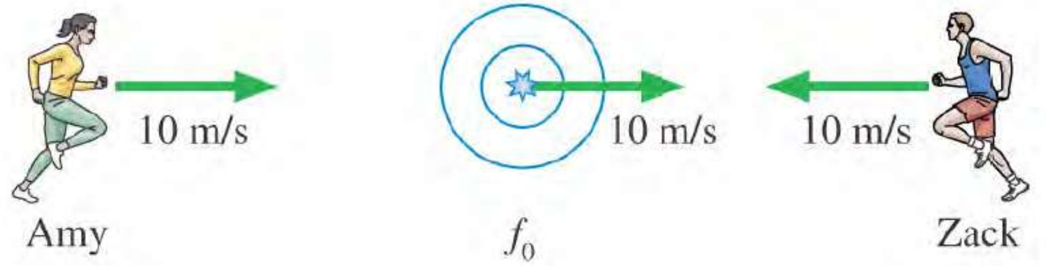
Question – Doppler effect



Zwei Jogger, hören die Schallwellen einer sich nach rechts bewegenden Quelle. Vergleichen sie die wahrgenommenen Frequenzen.

1. $f_{Amy} < f_{Zack}$
2. $f_{Amy} = f_{Zack}$
3. $f_{Amy} > f_{Zack}$

Question – Doppler effect



Antwort: **1. $f_{Amy} < f_{Zack}$**

Es bewegt sich sowohl der Beobachter als auch der Sender, so dass beide Effekte kombiniert werden müssen. Amy bewegt sich mit der gleichen Geschwindigkeit wie die Quelle, wodurch sie etwa die gleiche Frequenz hört, wie die Quelle aussendet. Zack bewegt sich in Richtung der Quelle, und die Quelle bewegt sich auf ihn zu. Beide Effekte erhöhen die von Zack gehörte Frequenz.

Doppler effect

stationary source

moving source, $v_r = 0.7 \cdot v$

v: wave velocity

https://www.acs.psu.edu/drussell/Demos/doppler/doppler.html

shock wave, supersonic speed

$v_r = 0.7 \cdot v$

$v_r = v$

Mach 1

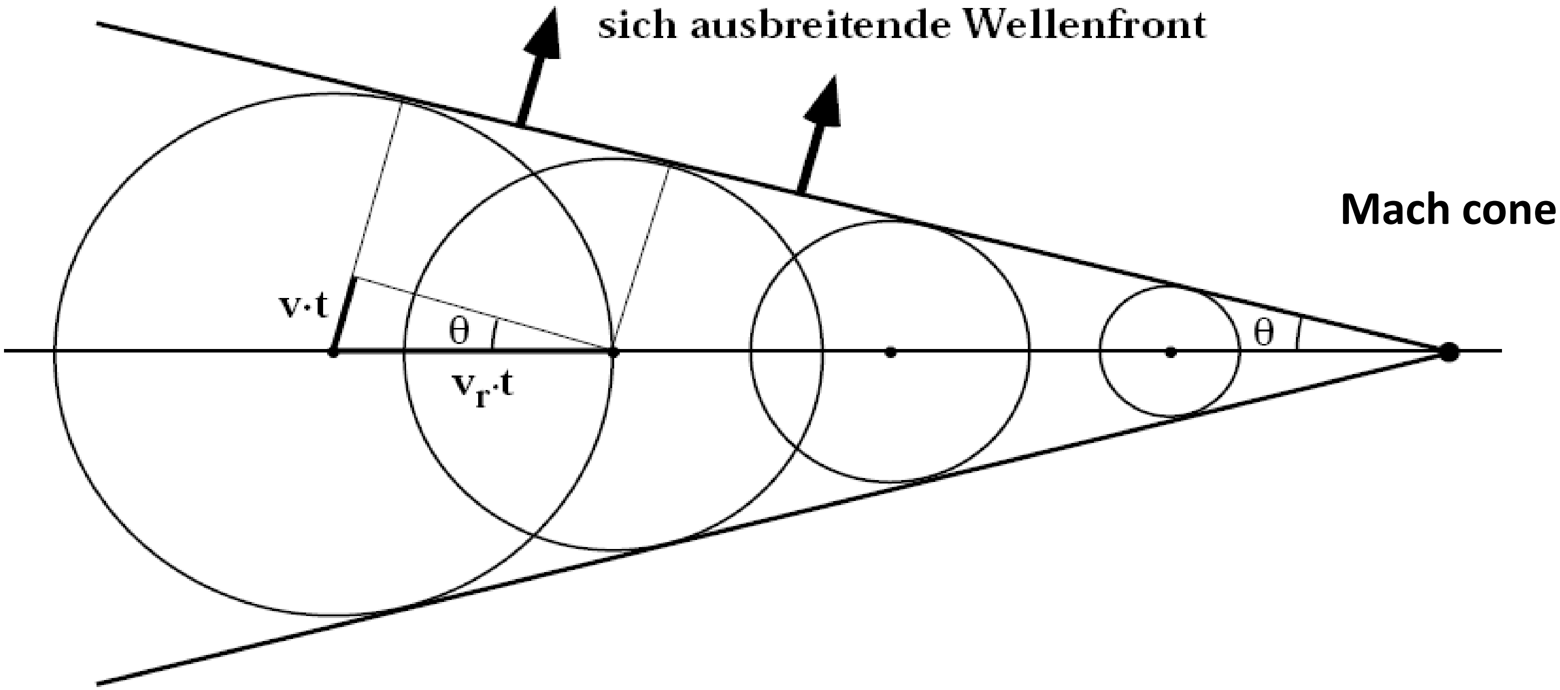
$v_r = 1.4 \cdot v$

Mach 1.4

v_r: source velocity
v: wave velocity

https://www.acs.psu.edu/drussell/Demos/doppler/doppler.html

Mach cone



v_r: source velocity
v: wave velocity

https://www.acs.psu.edu/drussell/Demos/doppler/doppler.html

supersonic speed

Mach ~ 1

supersonic aircraft *Tipler*

supersonic car

https://www.acs.psu.edu/drussell/Demos/doppler/doppler.html

Waves I

- Last lectures: oscillations
  → oscillations in matter, through media: physics of waves

(slides) e.g. travelling oscillation in
1) waves
2) sound
3) light

air: sound wave, music (scalar)
water: (water) waves, ocean
land/earth: earthquakes
(vectorial)
vacuum: ? → light, traveling electromagnetic wave
(does not require matter support / can also travel through matter)
+ gravitational waves

- Waves transport energy
  & momentum
  not mass (i.e. ~ propagating local oscillation)

- Description: function of position x and time t
  3D: (x, y, z)

$$A(x, y, z, t),\ A(\vec{r}, t),\ \vec{r} = (x, y, z)$$

origin: Waves ~~propagate~~ caused by a local disturbance in a medium ...
↑ mechanical (light: local disturbance of elec-mag. field, no medium)

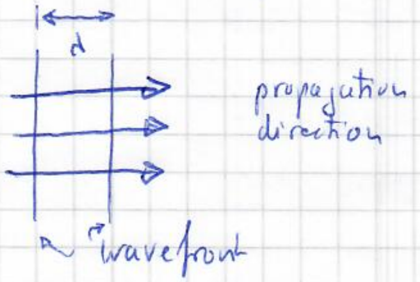
propagation ... and propagated by the coupling between matter constituents

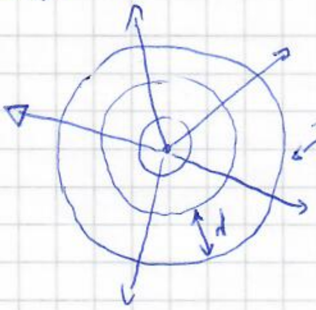
(Exp) coupled pendulum: a) 2 masses
b) 8 masses, longitudinal

(slide) - Transverse & longitudinal waves
1) →]
2) water
3) seismic

(oscillation transverse to propagation) (oscillation along propagation)

. Plane waves , circular waves





after exp.

. Polarized vs unpolarized light / wave

(slide)

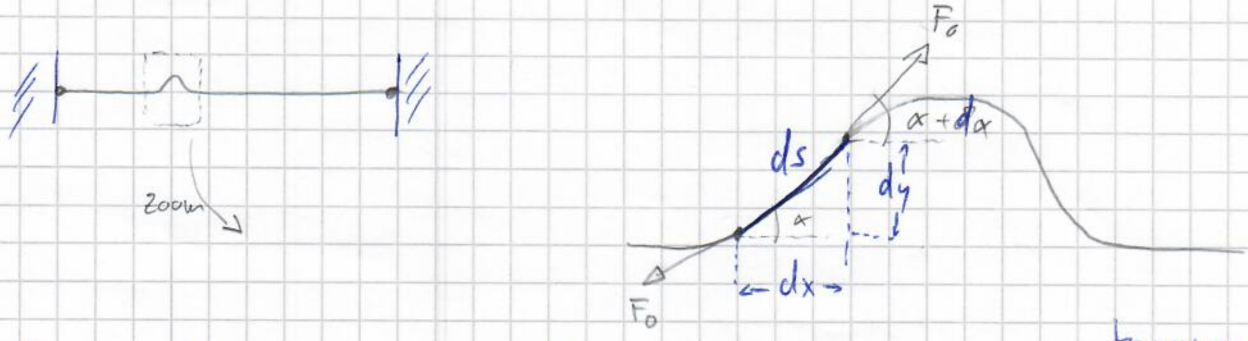
Exp Wave pool : a) planar wave , b) circular wave

(Exp. Torsion wave : a) 1 element (lamella, bar)
b) N elements

(Slide) Torsional vibration

Mathematical description of waves (wave equation)

Exp. Rope wave : on floor



- for a small rope element $d\vec{s} = (dx, dy)$

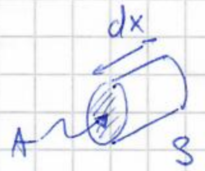
along y axis $dF_y = F_0 \cdot \sin(\alpha + d\alpha) - F_0 \cdot \sin(\alpha)$, ~~elementary~~ force acting on rope element along y axis

α small, ➡ $\alpha = \frac{dy}{dx}$ and $\sin(\alpha) \sim \alpha$

$$dF_y = F_0 \cdot (\alpha + d\alpha) - F_0 \cdot \alpha = F_0 \cdot d\alpha$$

With $\frac{d\alpha}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$

we have $dF_y = F_0 \frac{d^2y}{dx^2} dx$,



- Newton's eq. of motion :

$$dF_y = dm \cdot a = \rho \cdot A \cdot ds \cdot a \simeq \rho \cdot A \cdot dx \cdot \frac{d^2y}{dt^2}$$

($ds \sim dx$ for small α; $\frac{d^2y}{dt^2}$: acceleration)

ρ: rope density
A : cross-sectional area

and

$$\rho \cdot A \cdot dx \frac{d^2y}{dt^2} = F_0 \frac{d^2y}{dx^2} \cdot dx$$

thus

$$\frac{d^2y}{dt^2} = \frac{F_0}{\rho \cdot A} \cdot \frac{d^2y}{dx^2}$$

Wave equation (differential eq.)

units : $\frac{N}{kg/m^3 \cdot m^2} = N \cdot \frac{m}{kg} = \frac{kg \cdot m}{s^2} \cdot \frac{m}{kg} = \left(\frac{m}{s}\right)^2 = (\text{velocity})^2$

Solution of wave eq.:

$$y = y(x, t) = f(v \cdot t - x)$$

(x: position, t: time, v: constant velocity)

Demo: $y = f(\underbrace{v \cdot t - x}_{u})$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt}$$

- $$\frac{\partial y}{\partial t} = f'(v \cdot t - x) \cdot v$$

$$\frac{\partial^2 y}{\partial t^2} = v \cdot \frac{\partial}{\partial t}\left(f'(v \cdot t - x)\right) = v \cdot f''(vt - x) \cdot v = v^2 \cdot f''(vt - x)$$

(v: const)

- $$\frac{\partial y}{\partial x} = f'(v \cdot t - x) \cdot (-1)$$

$$\frac{\partial^2 y}{\partial x^2} = (-1) \cdot \frac{\partial}{\partial x}\left(f'(v \cdot t - x)\right) = (-1) \cdot f'' \cdot (v \cdot t - x) \cdot (-1)$$
$$= f''(v \cdot t - x)$$

In the wave eq.:

$$v^2 \cdot f''(v \cdot t - x) = \frac{F_0}{\rho \cdot A} \cdot f''(v \cdot t - x)$$

and $v^2 = \frac{F_0}{\rho \cdot A}$, $v = \sqrt{\frac{F_0}{\rho \cdot A}}$ QED ☐ $y(x,t) = f(v \cdot t - x)$ satisfies wave equation

Wave equation solution of the form $y(x, t) = f(v \cdot t - x)$

and $v = \sqrt{\frac{F_0}{\rho A}}$, velocity

for the propagation of a rope wave

Harmonic waves

plane waves

Harmonic oscillator : cos or sin function

For harmonic waves :

$$y(x,t) = y_0 \cdot \sin\left(\frac{2\pi}{\lambda} \cdot (v \cdot t - x)\right)$$

↑ $\frac{rad}{m}$; units ok : $\frac{rad}{m} \cdot m = rad$; (sin of angle)

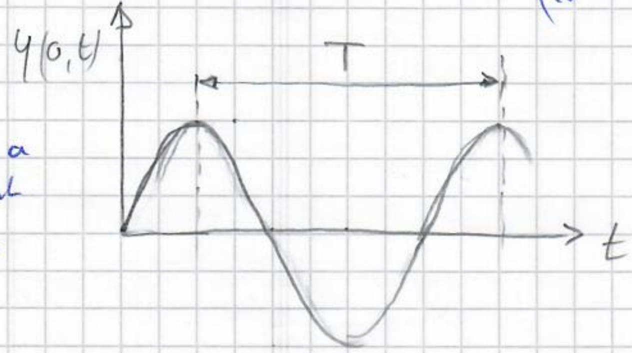
Or

$$y(x,t) = y_0 \cdot \cos\left(\frac{2\pi}{\lambda} \cdot (v \cdot t - x)\right)$$

with
- v : wave speed
- λ : wavelength

Representation of $y(x,t) = y_0 \cdot \sin\left(\frac{2\pi}{\lambda} (v \cdot t - x)\right)$

1/ at $x = 0$, $y(0,t) = y_0 \cdot \sin\left(\frac{2\pi}{\lambda} v \cdot t\right)$



motion of a rope segment at position $x = 0$

Exps: Heiter, Wheel model

period T : $t = T$

$$2\pi = \frac{2\pi}{\lambda} v \cdot T$$

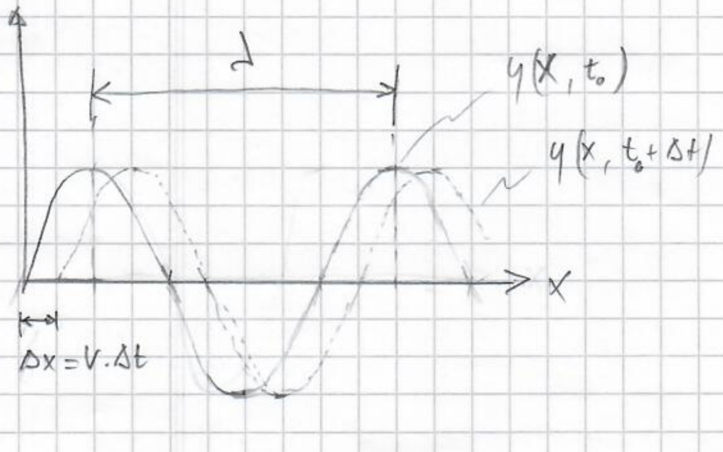
1 full cycle

thus $T = \frac{\lambda}{v}$

$$v = \lambda \cdot \frac{1}{T}$$
$$= \lambda \cdot \nu$$

↑ frequency

2/ at $t = t_0$ and $t_0 + \Delta t$



$$y(x, t_0) = y_0 \cdot \sin\left(\frac{2\pi}{\lambda}(v \cdot t_0 - x)\right)$$

$$y(x, t_0+\Delta t) = y_0 \cdot \sin\left(\frac{2\pi}{\lambda} \cdot \left(\underbrace{v(t_0+\Delta t) - x}\right)\right)$$

$$v \cdot t_0 + v \cdot \Delta t - x = \underline{v \cdot t_0 + \Delta x - x}$$

- for $\underline{\Delta t = T}$, (1 period), $\Delta x = v \cdot T = v \cdot \frac{\lambda}{v} = \lambda$ ($T = \frac{\lambda}{v}$)

$$y_0(x, t_0+T) =$$

($\Delta x = v \cdot T$)

$$= y_0 \cdot \sin\left(\frac{2\pi}{\lambda} \cdot (v \cdot t_0 - x) + \underbrace{\frac{2\pi}{\lambda} \cdot \lambda}_{2\pi}\right)$$

$$= y_0 \cdot \sin\left(\frac{2\pi}{\lambda} \cdot (v \cdot t_0 - x)\right)$$

$\sin(\alpha + 2\pi) = \sin(\alpha)$

and $y_0(x, t_0+T) = y_0(x, t_0)$

- Notation: introduce $\omega = 2\pi \cdot \nu = \frac{2\pi}{T}$ and $k = \frac{2\pi}{\lambda}$, wavenumber

angular frequency

$$y_0(x,t) = y_0 \cdot \sin\left(\underbrace{\frac{2\pi}{\lambda}}\cdot v \cdot t - \underbrace{\frac{2\pi}{\lambda}}_{k,\ \text{wave number}} \cdot x\right)$$

$$\frac{2\pi}{\lambda} \cdot v \cdot t = \frac{2\pi}{\lambda} \cdot \lambda \cdot \nu \cdot t = 2\pi \cdot \nu \cdot t = \omega \cdot t$$

↑ angular frequency

$v = \frac{\lambda}{T} = \lambda \cdot \nu$

and

$$\| \; y_0(x,t) = y_0 \cdot \sin(\omega \cdot t - kx)$$

wave function, harmonic wave propagating along x-direction

(Slide) wave motion in space & time

Exp. Hamster wheel model

a) no cache
b) cache, show 1 mass only

- Wave speed and wavelength depend on material

$$v = \lambda \cdot \nu$$

$$\lambda = \frac{v}{\nu}$$

(slide) speed of sound in various materials

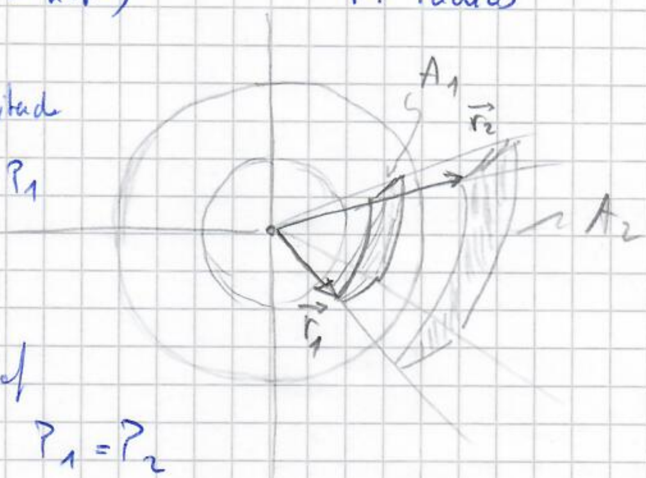
event - Exp: show rope again
a) floppy, on floor
b) tense, on floor
v larger

- Spherical harmonic waves

$$y(r,t) = \frac{y_0}{r} \cdot \sin(\omega t - k \cdot r)$$

r: radius

(distance dep. amplitude)



Power transmitted through A_1: P_1

power though A_2: P_2

In the absence of absorption of the wave by the medium: $P_1 = P_2$

Consider the intensity $I = P/A$, power per unit surface

$$\frac{I_1}{I_2} = \frac{P_1/A_1}{P_2/A_2} = \frac{P_1}{A_1} \cdot \frac{A_2}{P_2} = \frac{A_2}{A_1}$$

($P_1 = P_2$)

$$\frac{I_1}{I_2} = \frac{A_2}{A_1} = \frac{r_2^2}{r_1^2}$$

and $$I_2 = I_1 \cdot r_1^2 \cdot \frac{1}{r_2^2}$$

The intensity decreases as $1/r^2$ as r increases

As $I \propto \text{amplitude}^2 = y_0^2$, y_0 decreases as $1/r$ as r increases

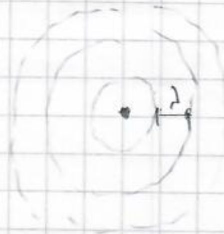
Exp: circular wave in pool

Doppler effect

(slide) Question Doppler Effect

importance of relative displacements of source, receiver and medium (not for light)

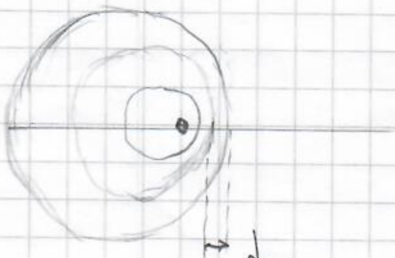
- Stationary source : $v = \lambda \cdot \nu$, $\nu = \frac{1}{T}$



$$\lambda = \frac{v}{\nu} = v \cdot T$$

(v: propagation speed)

- moving source : at $t \geq 0$, v_r: velocity of source, $v_r \neq 0$, $v_r < v$



$$\lambda_m = v \cdot T - v_r \cdot T = (v - v_r) \cdot T = \frac{v - v_r}{\nu}$$

$$= \frac{v}{\nu}\left(1 - \frac{v_r}{v}\right) = \lambda \cdot \left(1 - \frac{v_r}{v}\right) < \lambda$$

(rewrite; $\lambda = \frac{v}{\nu}$; $(v_r < v)$)

"Compression" of wavelength along propagation direction

For the frequency : $\lambda_m = \frac{v}{\nu_m}$

hence : $\nu_m = \frac{v}{\lambda_m} = \frac{v}{\lambda} \cdot \left(\frac{1}{1 - \frac{v_r}{v}} \right) = \frac{\nu}{1 - v_r/v}$

$$\nu_m = \frac{\nu}{1 - v_r/v} \quad , \quad \nu_m > \nu \text{ for } v_r < v$$

(Slide) stationary source + moving source

For $v_r \ll v$

$$\nu_m = \lim_{\frac{v_r}{v} \to 0} \frac{\nu}{1 - \frac{v_r}{v}}$$

$$= \lim_{x \to 0} \frac{\nu}{1 - x} = \nu \cdot \underbrace{\lim_{x \to 0} \frac{1}{1-x}}_{1+x} = \nu(1+x) = \nu\left(1 + \frac{v_r}{v}\right)$$

$x = v_r/v$

Thus,

$$\underbrace{\nu_m - \nu}_{\Delta\nu} = \frac{v_r}{v} \cdot \nu$$

and

$$\frac{\Delta \nu}{\nu} = \frac{v_r}{v} \quad , \text{ for } v_r \ll v$$

relative freq. change — ratio of ~~relative~~ source velocity to propagation velocity.

Exp.

- Wave pool
- Applet car

Shock waves (sonic boom)

- if v_r, source speed is larger than the wave propagation speed, $v_r > v$
  ⇒ no waves in front of source
  the waves "pile up" behind the source and form a shock wave
- the shock wave is heard as a sonic boom when it arrives at the receiver

(Slide)

1) $v_r = 0.7 v$, $v_r = v$, $v_r = 1.4 v$
2) Wave front at time t : $\sin\theta = \frac{v \cdot t}{v_r \cdot t} = \frac{v}{v_r}$, $v_r \geq v$
3) illustrations — Mach cone angle : 2θ

%

for $v = v_r$, Mach 1

$\sin\theta = 1$, $\theta = 90°$ and $2\theta = 180°$

Mach cone angle

Summary Waves I

- Wave: propagating oscillation due to local disturbance
- Wave equation:

$$\frac{d^2y}{dt^2} = v^2 \cdot \frac{d^2y}{dx^2}$$

- solutions of the form $y(x,t) = f(v \cdot t - x)$
- harmonic wave: $y(x,t) = y_0 \cdot \sin(\omega t - k \cdot x)$

$\omega = 2\pi\nu$ angular frequency, $\nu = \frac{1}{T}$

$k = \frac{2\pi}{\lambda}$ wave number

$v = \frac{\lambda}{T}$

$v = \lambda \cdot \nu$

- representation

↳ (slide) wave motion in space & time

- wave speed and wavelength depend on material

$v = \lambda \cdot \nu$, $\lambda = \frac{v}{\nu}$

(slide) speed of sound in various materials

→ continue p7 Waves I: Spherical harmonic waves