# Introduction to Physics I

Ideal gases

Kinetic gas theory

L22, 07.12.2017

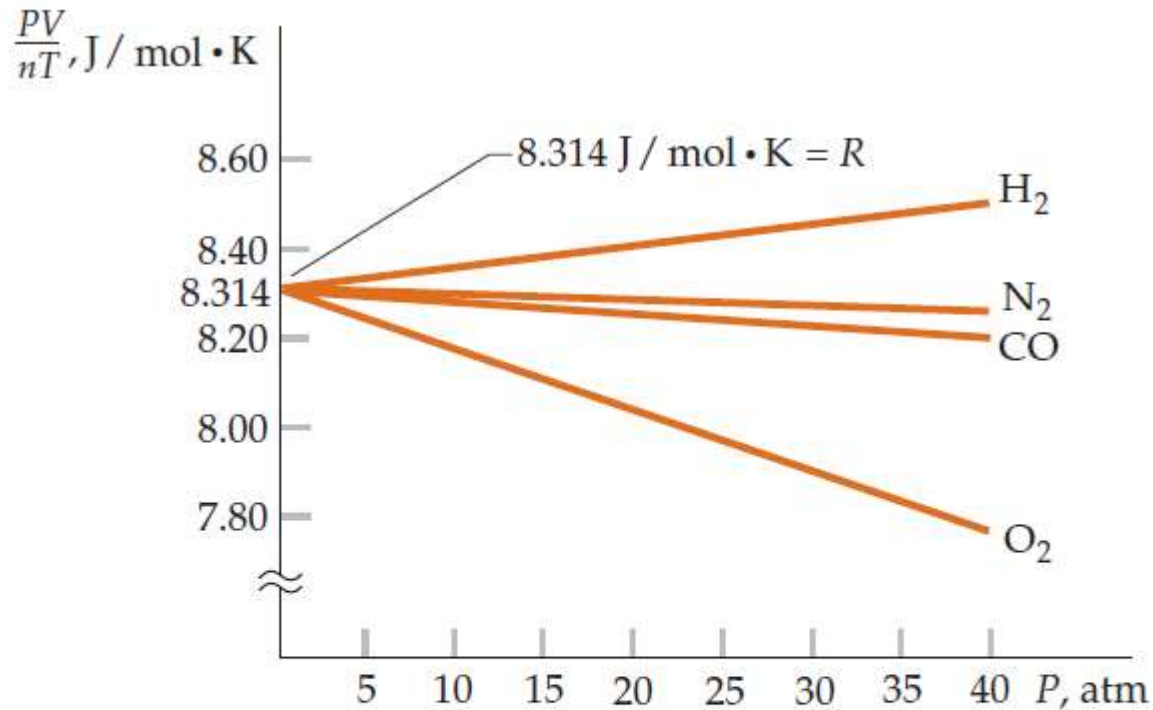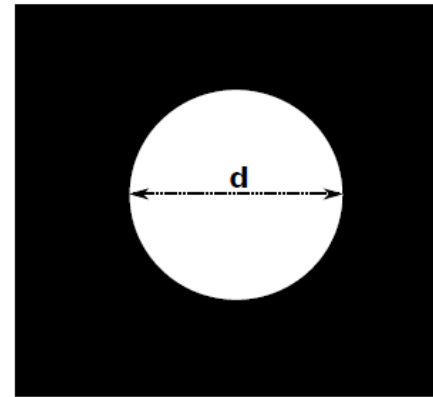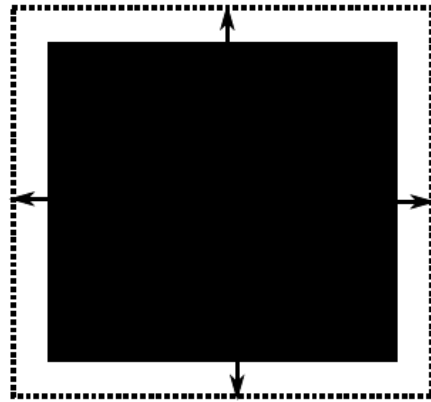# universal gas constant

$$PV = nRT$$



**FIGURE 17-8** Plot of $PV/nT$ versus $P$ for real gases. In these plots, varying the amount of gas varies the pressure. The ratio $PV/nT$ approaches the same value, 8.314 J/(mol·K), for all gases as we reduce their densities, and thereby their pressures, of the gases. This value is the universal gas constant $R$.

*Tipler*

# Konzeptfrage



Eine Metallplatte dehnt sich nach dem Erwärmen aus. Die gleiche Metallplatte, mit einem Loch in der Mitte, wird erwärmt. Wie verändert sich der Durchmesser des Lochs?

1. Der Durchmesser wird grösser.

2. Der Durchmesser wird kleiner.

3. Der Durchmesser bleibt gleich gross.

# Konzeptfrage

**Antwort: 1. Der Durchmesser wird grösser.**

Durch die Erwärmung werden die Gitterabstände im Festkörper grösser, und zwar in der ganzen Platte. Ein kleiner Durchmesser, bedeutet kleinere Molekülabstände auf der Innenseite des Lochs, was der Ausdehnung im ganzen Festkörper widersprechen würde.

# equipartition theorem
## degrees of freedom

**vibration modes $CO_2$ molecule**
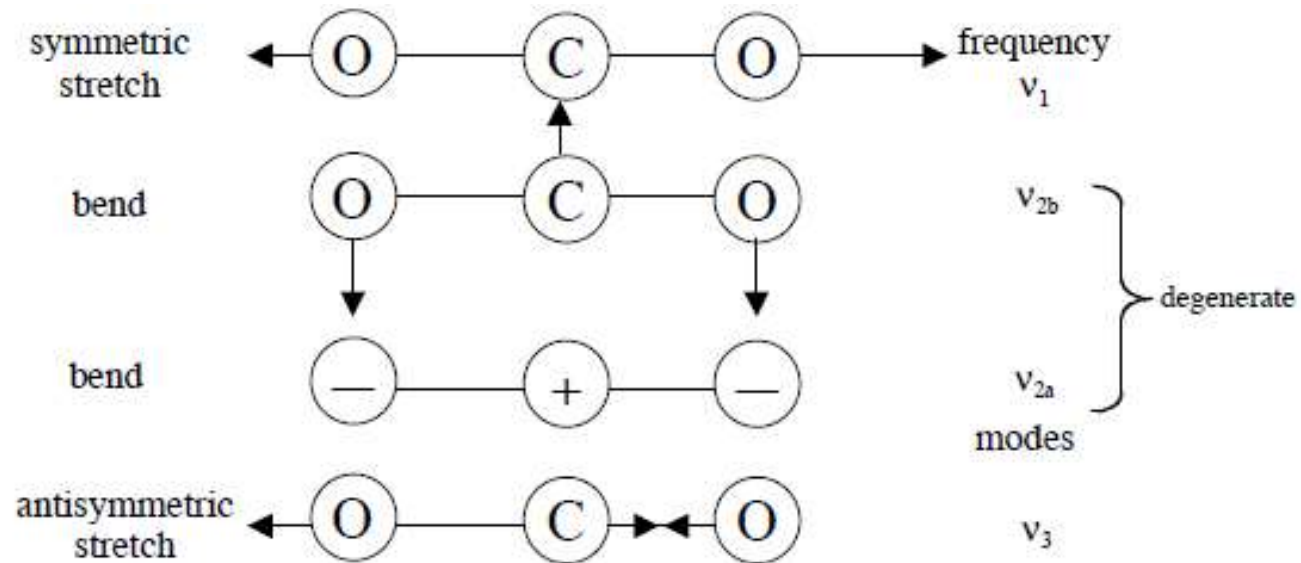
# equipartition theorem
## degrees of freedom

| Gas | $i = \dfrac{\text{Atome}}{\text{Molekül}}$ | $f_{trans}$ | $f_{rot}$ | $f_{vibr}$ | $f_{tot}\ (=3i)$ |
|---|---|---|---|---|---|
| He | 1 | 3 | 0 | 0 | 3 |
| $H_2$ | 2 | 3 | 2 | 1 | 6 |
| $CO_2$ | 3 | 3 | 2 | 4 | 9 |
| $NH_3$ | 4 | 3 | 3 | 6 | 12 |

*NB: vibration modes contribute kinetic and potential energy to the total internal energy of the gas*

Ideal gases        low density gases :
- microscopic billiard/pool : elastic shocks between atoms (molecules)
- no intermolecular forces (well separated atoms)

• energy (internal energy) : kinetic energy of atoms/molecules
of gas
=> any change in energy result in a Temp change

(exp) • exp. observation : $P \cdot V = $ constant, for a given $T$

Boyle-          (R.Boyle, 17th Century)     ↑ volume
Rarriott                                    pressure

exp. observation :        $P \cdot V = const \cdot T$        ( const is >0 )

(exp)       ( ∫ . Charles        18th-19th )
Gay-        ( ∫ . Gay-Lussa     Cent )                        ↑ temperature
Lussac

• const = ... ?        take 2 containers    $P, V, T$  $P, V, T$   $\frac{P \cdot V}{T} = const$
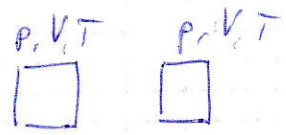
→ merge as one                              $\frac{P \cdot 2V}{T} = const'$
system

hence : const ∝ nb of gas molecules   ( const' = 2·const)
(amount of gas)

and, def :    const = $k_B N$
$R$ ↗  ↖ nb molecules
other constant ..(!)

and      $p \cdot V = N \cdot k_B \cdot T$

$k_B = 1.38 \cdot 10^{-23} \, J/k$ ,   Boltzmann constant
(value found exp.)

$N = n \cdot N_A$

$N_A = 6.022 \cdot 10^{23} \, mol^{-1}$ , Avogadro number
$n$ : nb of moles (mol)

def : 1 mol of a substance contains $N_A$ elements
(measure of the amount of a gas, here)        (atoms or molecules)

thus

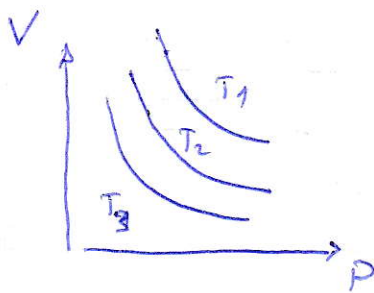$P \cdot V = N \cdot k_B \cdot T = n \cdot N_A \cdot k_B \cdot T$

ideal gas      ‖ $P \cdot V = n \cdot R \cdot T$ ,   $R = N_A \cdot k_B = 8.314 \, \frac{J}{mol \cdot K}$
law
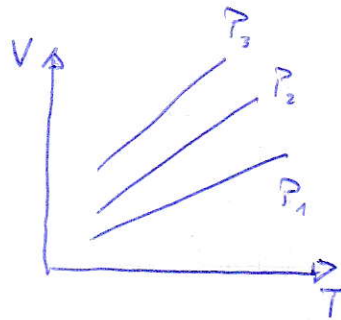(slide) univ. gas constant    note $[P \cdot V] = \frac{N}{m^2} \cdot m^3 = N \cdot m = J$     universal gas constant
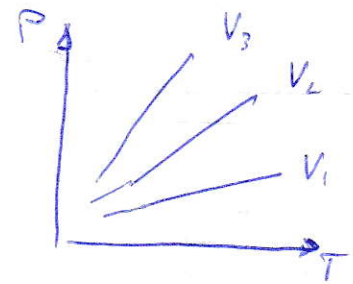energy/work

a) <u>schematic</u>
   <u>behavior</u>   $PV = n \cdot R \cdot T$      . take $n = const$   ( e.g $1$ mol )



$T_1 > T_2 > T_3$

isotherms

$P_1 > P_2 > P_3$

isobars

$V_1 > V_2 > V_3$

isochores

b) $PV = n \cdot R \cdot T$  ,   take $n = 1$,  $N_A$ molecules

<u>exp</u> show
molvolumen

$$V = \frac{R \cdot T}{P} \simeq 22.7 \, l = 2.27 \cdot 10^{-2} \, m^3$$

at standard pressure
& temperature  (STP)

$P = 100 \, kPa$

$T = 273.15 \, K$

$R = 8.314 \, \dfrac{J}{mol \, K}$

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$PV = n \cdot R \cdot T$

- relation between $P, V$ and $T$ for an ideal gas ,  <u>state equation</u>
- correction required ~~at~~ when gas density increases  (gas law)

-

<u>exp</u>  Boyle Mariott :  $PV = $ const

work really ?

<u>exp</u>  Gay Lussac :  $\dfrac{P}{T} = $ const    for $V = $ const

Measure $P$ for various $T$          $\cdot R.T$                    ⎰ write values
for fixed volume of gas $V$          $\cdot$ boiling water         ⎱ + plot
            (He)                     $\cdot$ 0°C
                                     $\cdot N_2$ liquid 77k

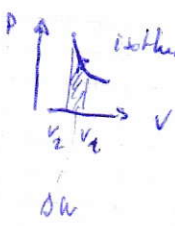examples of expansion of gas (ideal) from $V_1$ to $V_2$ at $T = const$

work: $\Delta W = \int_{V_1}^{V_2} P \cdot dV = \int_{V_1}^{V_2} n \cdot R \cdot T \cdot \frac{dV}{V} = n \cdot R \cdot T \cdot (\ln V_2 - \ln V_1)$
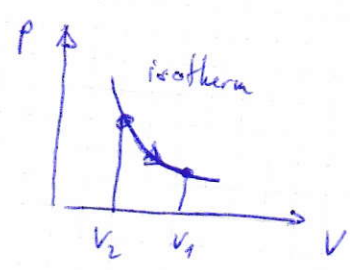
isotherm ($T = const$)

$PV = nRT$

$P = \frac{nRT}{V}$

nb of moles

$\|\ \Delta W = nRT \ln \frac{V_2}{V_1}$ 　　　　(1)

b) energy required to fill a nitrogen ($N_2$) gas bottle from ambient pressure (1 bar) to 200 bar (in the bottle)

$P_1 = 1\,bar$ 　　　　$P_2 = 200\,bar$

$V_1, T_1 = 300k$ 　　$V_2 = 50\,\ell$

　　　　　　　　$T_2 = 300k$

　　　　　　　　$= T_1 = T$

isotherm: $P_1 V_1 = P_2 V_2$, as $V_1 = \frac{P_2 V_2}{P_1}$

ideal gas : $P_2 V_2 = nR\overline{T}$, and $n = (\ ) = \frac{P_2 V_2}{R\overline{T}}$

from eq (1) above:

$$\Delta W = n \cdot R \cdot T \cdot \ln \frac{V_2}{V_1} = \frac{P_2 V_2}{R\overline{T}} \cdot R \cdot T \cdot \ln \frac{V_2}{\frac{P_2 V_2}{P_1}}$$

$$= P_2 \cdot V_2 \cdot \ln \frac{P_1}{P_2} \quad;\ indep.\ of\ T$$

$\|\ \Delta W = 2 \cdot 10^7 \frac{N}{m^2} \cdot 5 \cdot 10^{-2} m^3 \cdot \ln \left( \frac{1\,bar}{200\,bar} \right) = -5.3 \cdot 10^6\ J$

$1\,bar = 10^5\,Pa$

$= 10^5\,\frac{N}{m^2}$

→ energy required: $5.3 \cdot 10^6\ J$

$(\approx 1.47\ kWh)$

$1\,\ell = 10^{-3}\,m^3$

fridge ~ 200 kWh /year

~~computer~~ ~ 0.1 kWh