Physik IV FS 2010
Blatt 7 Abgabe: **Donnerstag, 22.04.2010**

1. Doppler cooling of free atoms (see figure 1)*

Consider an atom of mass m traveling at a non-relativistic velocity v. It possesses two internal electronic states, Ψ_g (ground) and Ψ_e (excited), which are separated by an energy $\Delta E = E_e - E_g = \hbar\omega_0$.

a) If we irradiate the traveling atom with photons of energy $\hbar\omega$, what (angular) frequency ω should we choose to efficiently drive the Ψ_g to Ψ_e transition? [You can assume the photons are anti-parallel to v, for simplicity.]

b) Apply the concepts of energy and momentum conservation for the absorption of a single, resonant photon. Show that the frequency of the incoming photon can be expressed in the following way:

$$\omega_{abs} = \omega_0 + \mathbf{k}_{abs} \cdot \mathbf{v} + \frac{\hbar k_{abs}^2}{2m}, \tag{1}$$

where $\mathbf{k}_{abs}$ is the wave vector of the incoming photon.

c) The lifetime of Ψ_e is finite, therefore spontaneous emission will eventually occur. Apply energy and momentum conservation once again, this time to the re-emission process, in order to find the following:

$$\omega_{em} = \omega_0 + \mathbf{k}_{em} \cdot \mathbf{v}' - \frac{R}{\hbar}, \tag{2}$$

where $\mathbf{k}_{em}$ is the wave vector of the emitted photon, $\mathbf{v}'$ is the atom's velocity while in the excited state, and $R = \hbar^2 k^2/2m$ is the recoil energy. [Hint: remember that the radiative wave vector is invariant under Galilean transformations, such that $|\mathbf{k}_{em}| = |\mathbf{k}_{abs}| = k = \omega_0/c$.]

d) The change in atomic kinetic energy per scattering event is equal, but opposite, to the average difference between the photon energies:

$$\Delta E_K = -(\hbar\omega_{em} - \hbar\omega_{abs}) = \hbar\mathbf{k}_{abs} \cdot \mathbf{v} + 2R \tag{3}$$

We have neglected the $\mathbf{k}_{em} \cdot \mathbf{v}'$ term because the direction of each emitted photon is random, and thus averages out after many scattering events. However, the $\mathbf{k}_{abs} \cdot \mathbf{v}$ and R terms are always present. Since $R > 0$ it increases E_K and causes heating. In contrast, if $\mathbf{k}_{abs} \cdot \mathbf{v}' < 0$ (anti-parallel for maximum effect), the laser beam will cool the atom. For counter-propagating lasers in three dimensions, the highest cooling rate (in one of these dimensions) can be shown to be:

$$\frac{dE_x}{dt} = 2\frac{dN}{dt}\left(\frac{-\hbar k^2 \langle v_x^2 \rangle}{\gamma} + R\right), \tag{4}$$

where dN/dt is the rate at which photons scatter off the atom and γ ($\gg$ Doppler width in this example) is the natural linewidth of Ψ_e. Starting from Eq. (4), find the minimum kinetic energy and temperature that are attainable with Doppler cooling.

2. Sideband cooling of trapped atoms (see figure 2)

Consider a two-level atom confined to a harmonic oscillator potential:

$$\sum_{i=x,y,z} \hbar\Omega_i \left(N_i + \frac{1}{2}\right), \tag{5}$$

where the number operator $N_i = a_i^\dagger a_i$ is the product of the creation and annhilation operators. Each vibrational energy level is characterized by an occupation number n_i (Eigenvalue of N_i). We are interested in detuning the laser to drive transitions between Ψ_g and Ψ_e of different vibrational levels. Some atoms and ions can be efficiently cooled below the Doppler limit using this technique. The regime of interest is shown in figure2(b) ($\Omega \gg \gamma$).

a) We begin with the time derivative of the average occupation number $\langle n \rangle$, which can be derived from rate equations for the populations P_{n+1}, P_n and P_{n-1}. Determine its equilibrium value $\langle n \rangle_{eq}$ using:

$$\frac{d\langle n \rangle}{dt} = A_+ + A_+\langle n \rangle - A_-\langle n \rangle, \tag{6}$$

where A_- and A_+ are the transition rate coefficients related to decreases and increases of n, respectively. [Hint: simply express $\langle n \rangle_{eq}$ in terms of A_- and A_+.]

b) The ratio of the above coefficients can be approximated by:

$$\frac{A_+}{A_-} = \frac{W(\delta - \Omega) + W(\delta)/3}{W(\delta + \Omega) + W(\delta)/3}, \tag{7}$$

where δ is the detuning, and the lineshape function is given by:

$$W(q) = \frac{1}{(\frac{2q}{\gamma})^2 + 1}. \tag{8}$$

Assuming a Maxwell-Boltmann distribution, approximate the equilibrium temperature that corresponds to $\langle n \rangle_{eq}$ when the laser is red-detuned by the trap frequency ($\delta = -\Omega$)? [Hint: note that for steady state conditions, $A_-P_{n+1} = A_+P_n$.]

c) What dominates the atomic kinetic energy at the lowest temperature?

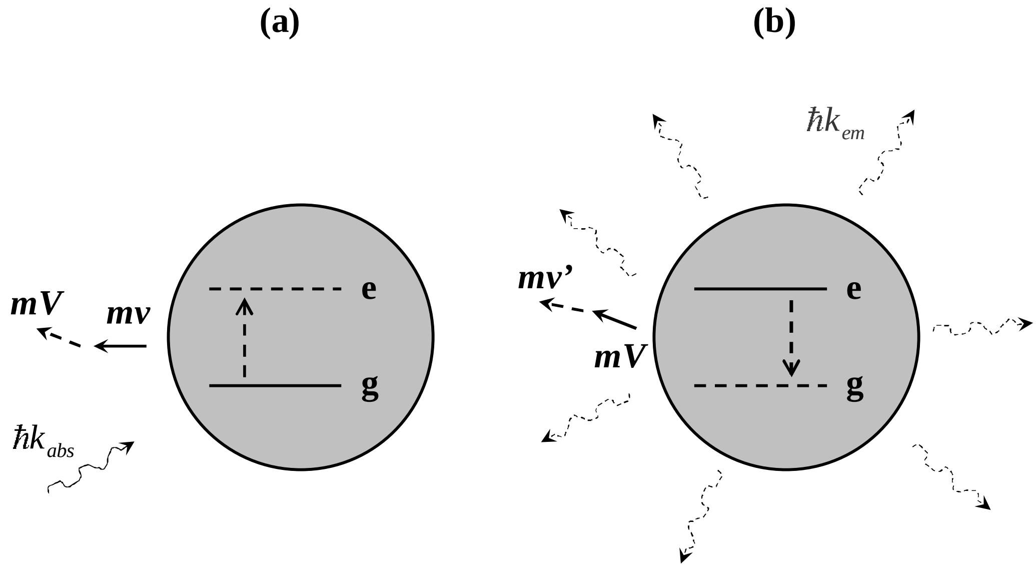

Figure 1: (a) Absorption of photon drives an electronic transition and causes the atom to recoil. (b) The photon is re-emitted following the lifetime of the excited state. Its direction is random. Again, a recoil energy is dissipated.

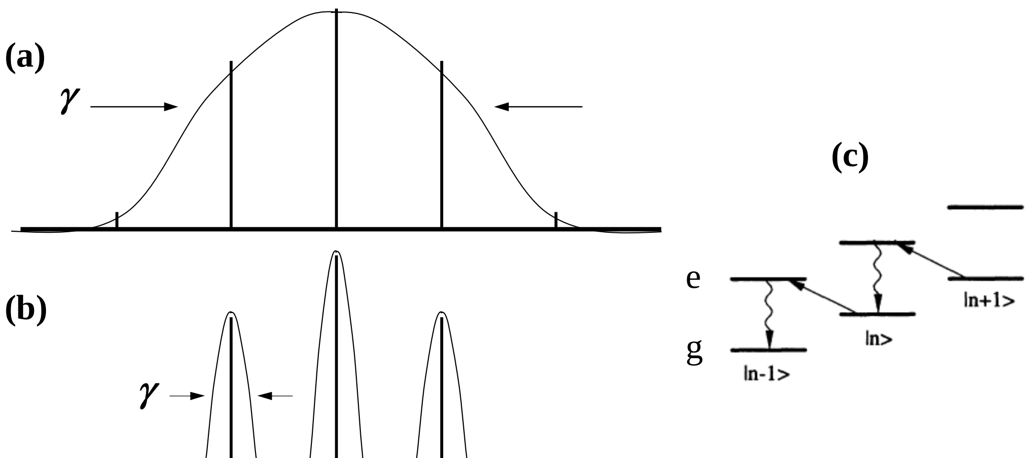

Figure 2: (a) Doppler broadened limit. (b) Sideband resolved limit where the trap frequency is much greater than the linewidth. (c) Energy level schematic showing transitions between successively lower sidebands.