Introduction to Physics I

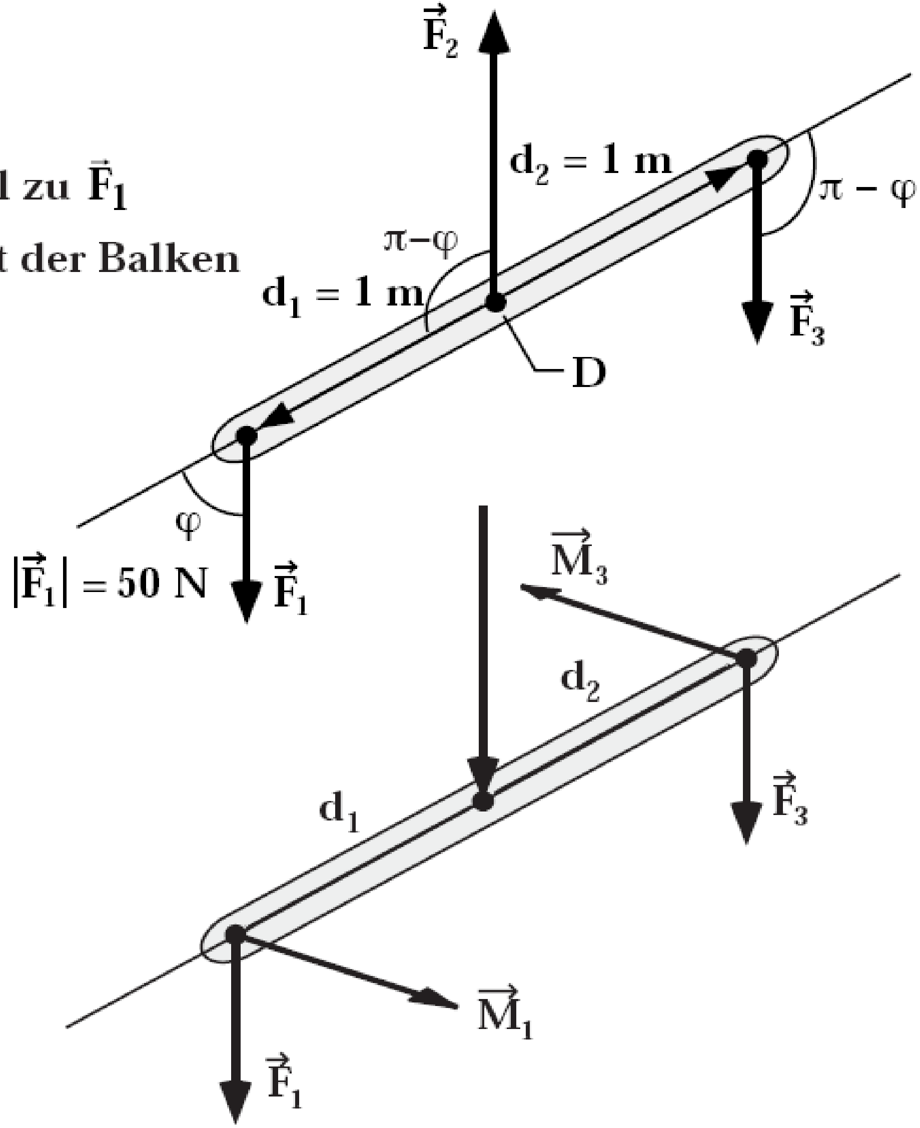
For Biologists, Geoscientists, & Pharmaceutical Scientists

Die Drehmomente der Kräfte $\vec{F}_i$ können bezüglich einer beliebigen Drehachse berechnet werden.

Beispiel:

Gegeben: $\vec{F}_1, d_1, d_2$, $\vec{F}_2$ und $\vec{F}_3$ sind parallel zu $\vec{F}_1$

Wie gross müssen $|\vec{F}_2|$ und $|\vec{F}_3|$ sein, damit der Balken im Gleichgewicht ist ?



$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

$$\vec{F}_2 + \vec{F}_3 = -\vec{F}_1$$

wählen Drehpunkt D:

$$\vec{M}_1 + \underbrace{\cancel{\vec{M}_2}}_{0} + \vec{M}_3 = 0$$

Beträge:

$$d_1 F_1 \sin\varphi - d_2 F_3 \sin(\pi-\varphi) = 0$$

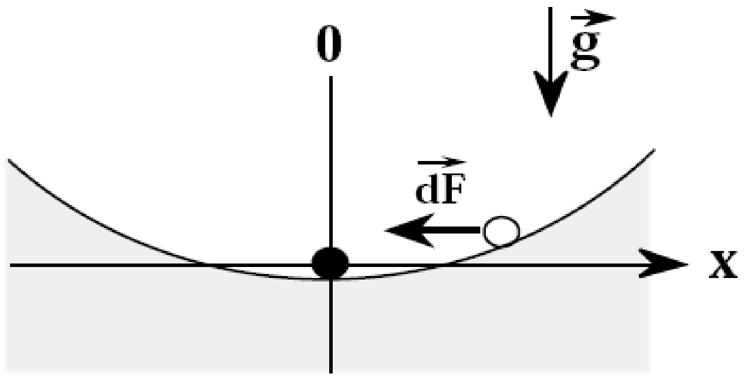
$$F_3 = \frac{d_1}{d_2} F_1$$

$$\rightarrow \vec{F}_2 = -\left(1 + \frac{d_1}{d_2}\right)\vec{F}_1 = -2\vec{F}_1 = -100\text{N} \quad , \quad \vec{F}_3 = \vec{F}_1 = 50\text{N}$$

3 Arten von Gleichgewicht

1. Stabiles Gleichgewicht

Modell

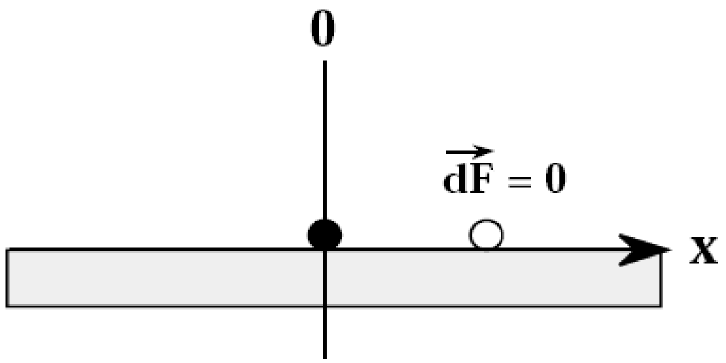


Gleichgewichtslage $x = 0$

Auslenkung um dx bewirkt eine rücktreibende Kraft $\vec{F}$
$dF = -\alpha\, dx$

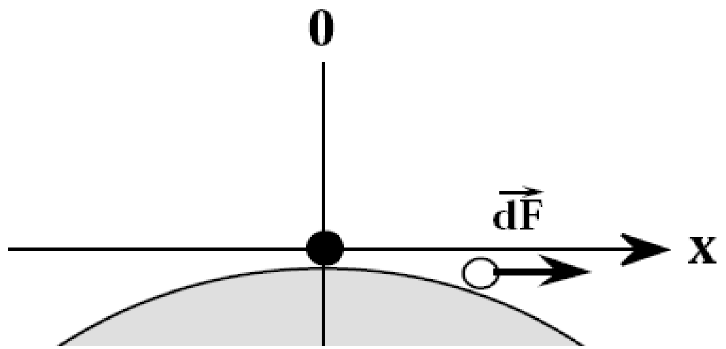
3 Arten von Gleichgewicht

2. indifferentes Gleichgewicht



es existieren unendlich viele Gleichgewichtslagen. Eine Auslenkung um Δx bewirkt keine Kraftwirkung.

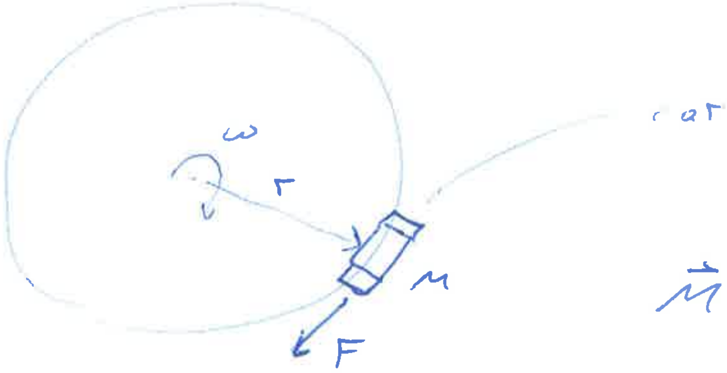
3. labiles Gleichgewicht



Gleichgewichtslage $x = 0$
Auslenkung um dx bewirkt eine Kraft in Richtung dx: $dF = \alpha\, dx$

Neben diesen drei Gleichgewichtsarten existiert noch das metastabile Gleichgewicht.

Torque (Drehmoment)



Car driving on circular track.

$$\vec{M} = \vec{r} \times \vec{F}$$

$$M = rF = rma = rm\frac{dv}{dt}$$

$$M = rm\frac{d(\omega r)}{dt}$$

$$\boxed{M = mr^2\frac{d\omega}{dt}}$$

or with vectors ...

$$\vec{M} = \vec{r} \times \vec{F} = \vec{r} \times m\vec{a} = \vec{r} \times m\frac{d\vec{v}}{dt}$$

$$\vec{M} = \vec{r} \times m\frac{d}{dt}(\vec{\omega} \times \vec{r})$$

$$\boxed{\vec{M} = mr^2\frac{d\vec{\omega}}{dT}}$$

(1)

$$\vec{p} = m\vec{v}$$

$$\vec{M} = \vec{r} \times \vec{F} = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \frac{d(m\vec{v})}{dt}$$

$$= \vec{r} \times m\frac{d\vec{v}}{dt} \qquad \vec{v} = \vec{\omega} \times \vec{r}$$

$$= \vec{r} \times m\frac{d}{dt}(\vec{\omega} \times \vec{r})$$

$$= \vec{r} \times m\left(\frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}\right)$$

$$= \vec{r} \times m\left(\frac{d\vec{\omega}}{dt} \times \vec{r} + \underbrace{\vec{\omega} \times \vec{v}}\right)$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\cancel{(\vec{\omega}\cdot\vec{r})}^{0}\vec{\omega} - (\vec{\omega}\cdot\vec{\omega})\vec{r}$$

$$= \vec{r} \times m\left(\frac{d\vec{\omega}}{dt} \times \vec{r} - (\vec{\omega}\cdot\vec{\omega})\vec{r}\right)$$

$$= m\left(\vec{r} \times \left(\frac{d\vec{\omega}}{dt} \times \vec{r}\right) - \omega^2 \cancel{\vec{r} \times \vec{r}}^{0}\right)$$

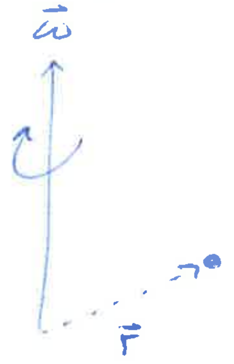
$$= m\left[(\vec{r}\cdot\vec{r})\frac{d\vec{\omega}}{dt} - \cancel{\left(\vec{r}\cdot\frac{d\vec{\omega}}{dt}\right)}^{0}\vec{r}\right]$$

$$\boxed{\vec{M} = mr^2\frac{d\vec{\omega}}{dt}}$$

Translation	Rotation
$\vec{F} = m \dfrac{d\vec{v}}{dt}$	$\vec{M} = \overbrace{mr^2}^{J} \dfrac{d\vec{\omega}}{dt}$
mass, acceleration	moment of inertia, angular acceleration
→ 2nd Law	2nd Law for Rotation

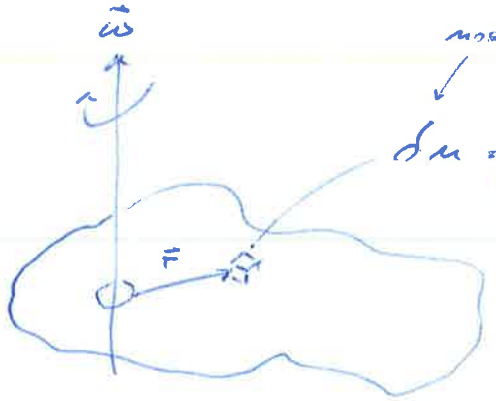
Moment of Inertia (Trägheitsmoment)

Point mass:



$$J = m r^2$$

Continuous Object:



mass density

$$dm = \rho\, dV$$

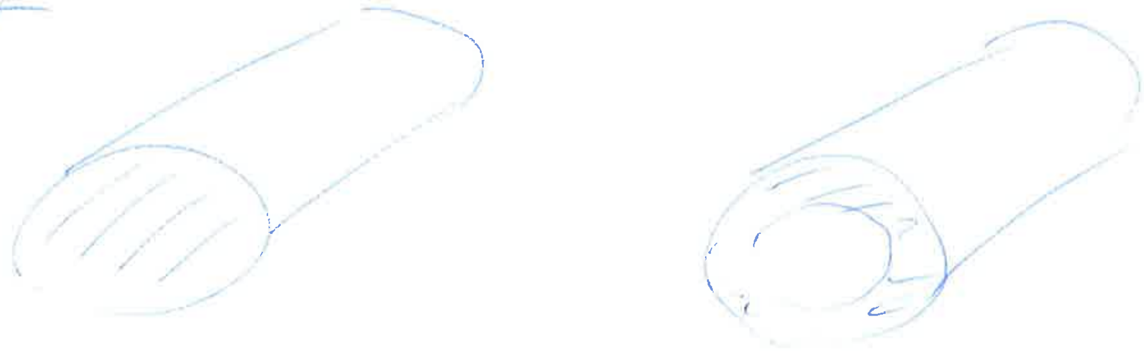
volume

$$dJ = r^2 dm = r^2 \rho\, dV$$

Total mass: $M = \int_B dm = \int_B \rho\, dV$

Total moment of inertia: $J = \int_B dJ = \int_B r^2 dm = \int_B r^2 \rho\, dV$

$\boxed{\text{Expl}}$



Filled Cylinder

M_1

J_1

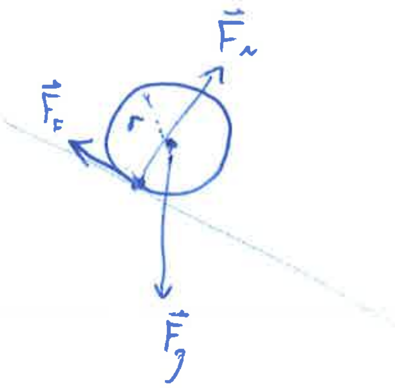
Hollow Cylinder

M_2

J_2

$\longrightarrow \quad M_1 = M_2 \quad , \quad \int_{B_1} dm = \int_{B_2} dm$

$\longrightarrow \quad J_1 \neq J_2 \quad , \quad \int_{B_1} r^2 dm \neq \int_{B_2} r^2 dm$



$$\vec{M} = \vec{r} \times (\vec{F}_N + \vec{F}_g + \vec{F}_F)$$

$$M = r F_F$$

$$M = J \frac{d\omega}{dt} = r F_F$$

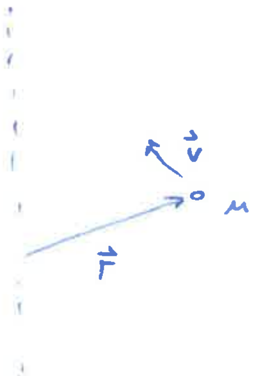
$$\frac{d\omega}{dt} = \frac{r F_F}{J}$$

Angular Momentum (Drehimpuls)

Momentum (Impuls) : $\vec{p} = m\vec{v}$

$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$ ← Angular Momentum

$$\vec{L} = \vec{r} \times m\vec{v}$$



For circular motion :

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\therefore \quad \vec{L} = \vec{r} \times m(\vec{\omega} \times \vec{r})$$

$$\vec{L} = m\,\vec{r} \times (\vec{\omega} \times \vec{r}) = m(\vec{r}\cdot\vec{r})\,\vec{\omega} - \cancel{(\vec{r}\cdot\vec{\omega})}\,\vec{r}$$

$$\vec{L} = \underbrace{m r^2}_{J}\,\vec{\omega}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}\cdot\vec{C})\,\vec{B} - (\vec{A}\cdot\vec{B})\,\vec{C}$$

$$\boxed{\vec{L} = J\,\vec{\omega}}$$

moment of inertia

angular velocity

Rotation

$$\boxed{\vec{p} = m\,\vec{v}}$$

mass

velocity

Translation

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$\frac{d\vec{L}}{dt} = \cancel{\vec{v} \times (m\vec{v})} + \vec{r} \times \vec{F}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\mathcal{M}}$$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{\mathcal{M}}}$$

Rotation

$$\boxed{\frac{d\vec{p}}{dt} = \vec{F}}$$

translation

Translation

$\vec{r}$

$\vec{v} = \dot{\vec{r}}$

$\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}}$

m

$\vec{p} = m\,\vec{v}$

$\vec{F} = \dot{\vec{p}}$

Rotation

ϕ

$\omega = \dot{\phi}$

$\dot{\omega} = \ddot{\phi}$

J

$\vec{L} = J \cdot \vec{\omega}$

$\vec{M} = \dot{\vec{L}}$

Equilibrium (Gleichgewicht)

A body is in equilibrium (not accelerating or rotationally accelerating) when the sum of all forces and the sum of all torques is zero.

$$\rightarrow \quad \vec{F}_{tot} = \sum_{i=1}^{N} \vec{F}_i = 0$$

$$\frac{d\vec{p}_{tot}}{dt} = 0$$

$$\frac{d}{dt}\left(m_{tot}\vec{v}_{tot}\right) = 0$$

$$m_{tot}\frac{d\vec{v}_{tot}}{dt} = 0$$

$$m_{tot}\vec{a}_{tot} = 0 \quad \longrightarrow \quad \vec{a}_{tot} = 0$$

$$\rightarrow \quad \vec{M}_{tot} = \sum_{i=1}^{N} \vec{M}_i = 0$$

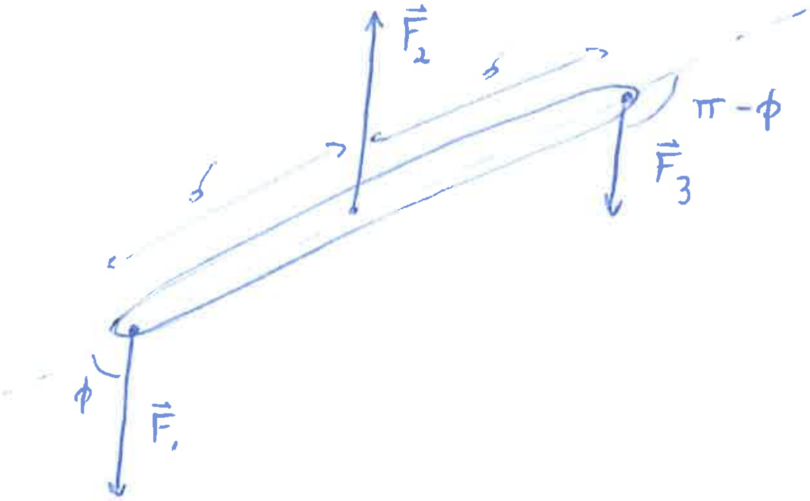
$$\frac{d\vec{L}_{tot}}{dt} = 0$$

$$\frac{d}{dt}\left(J_{tot}\vec{\omega}_{tot}\right) = 0$$

$$J_{tot}\frac{d\vec{\omega}_{tot}}{dt} = 0 \quad \longrightarrow \quad \frac{d\vec{\omega}_{tot}}{dt} = 0$$

18

$\boxed{\text{Exp}}$



① $\vec{F}_1 \parallel \vec{F}_2 \parallel \vec{F}_3$

② Body is in equilibrium

③ $d = 1\,m$, $F_1 = 50\,N$

④ How big are F_2 and F_3?

Forces:

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

$$\cancel{F_1 + F_2 + F_3 =}$$

$$F_2 + F_3 = -F_1$$

Torques:

$$\vec{M}_1 + \vec{M}_2 + \vec{M}_3 = 0$$

$$d \cdot F_1 \sin\phi + 0 - d \cdot F_3 \sin(\pi - \phi) = 0$$

$$\cancel{d}\; F_1 \cancel{\sin\phi} = \cancel{d} \cdot F_3 \cancel{\sin(\pi-\phi)}$$

$$F_1 = F_3$$

Together:

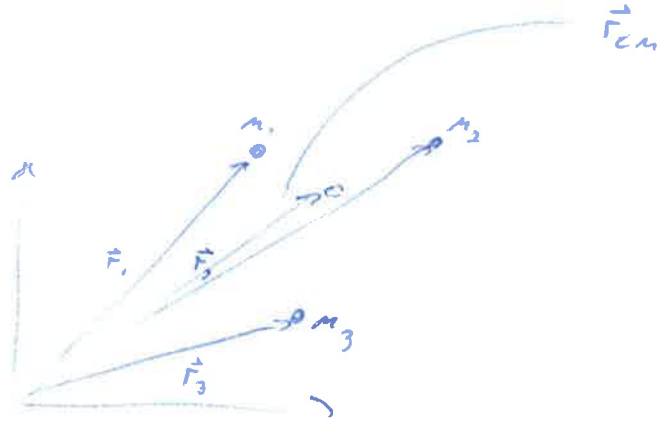
$$\boxed{\begin{aligned} F_3 &= F_1 = 50\,N \\ F_2 &= -F_1 - F_3 = -2F_1 = -100\,N \end{aligned}}$$

$\boxed{\text{Slides}}$

19

Center of Mass (Schwerpunkt)

$$\vec{r}_{\cancel{CM}} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{m_{tot}}$$



$$\vec{r}_s = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{m_{tot}}$$

$$m_{tot}\vec{r}_s = \sum_{i=1}^{n} m_i \vec{r}_i$$

$$\frac{d}{dt}\left(m_{tot}\vec{r}_s\right) = \sum_{i=1}^{n} \frac{d}{dt}\left(m_i \vec{r}_i\right)$$

$$m_{tot}\frac{d\vec{r}_s}{dt} = \sum_{i=1}^{n} m_i \frac{d\vec{r}_i}{dt}$$

$$m_{tot}\vec{v}_s = \sum_{i=1}^{n} m_i \vec{v}_i$$

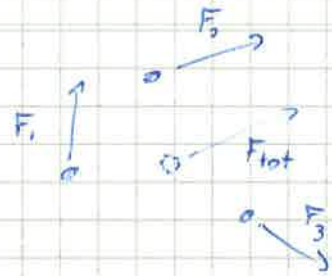
$$\left.\begin{aligned} \vec{p}_{tot} &= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 \\ \vec{p}_s &= m_{tot}\vec{v}_s \end{aligned}\right\} \vec{p}_{tot} = \vec{p}_s$$

$$\vec{p}_s = \sum_{i=1}^{n} \vec{p}_i = \vec{p}_{tot}$$

Schwerpunktsimpuls

Gesamtimpuls des System

$$\vec{F}_{tot} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$



$$\vec{F}_{tot} = m_{tot}\vec{a}_s$$

$$\vec{a}_s = \frac{\vec{F}_{tot}}{m_{tot}}$$

$$\frac{d}{dt}\left(m_{tot}\vec{v}_s\right) = \frac{d}{dt}\sum_{i=1}^{n} m_i \vec{v}_i$$

$$m_{tot}\frac{d\vec{v}_s}{dt} = \sum_{i=1}^{n} m_i \frac{dv_i}{dt}$$

$$m_{tot}\vec{a}_s = \sum_{i=1}^{n} m_i \vec{a}_i = \sum_{i=1}^{n} \vec{F}_i = \vec{F}_{tot}$$

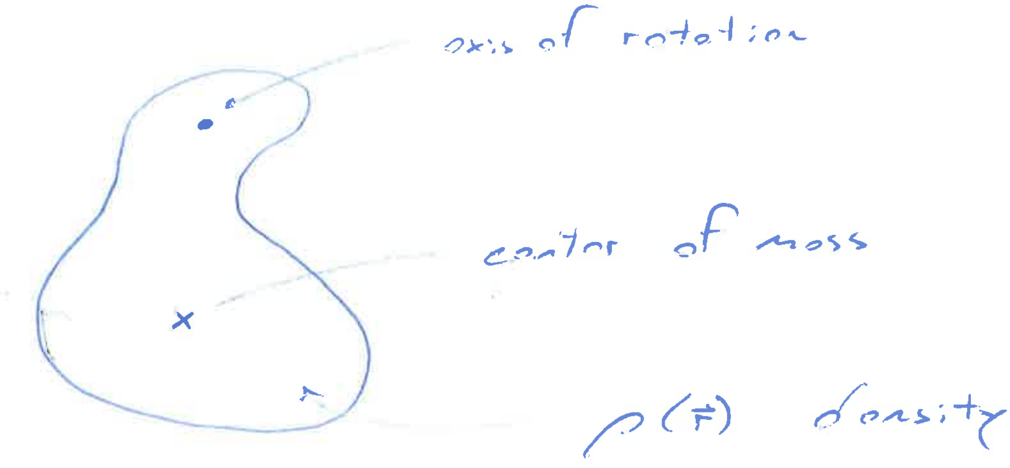
Summe der ~~...~~ aussen einwirkende Kraft.

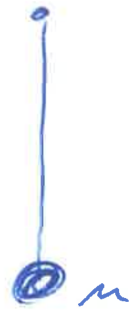
Wenn der aussen einwirkende Kraft $= 0$, bewegt sich der Schwerpunkt mit konstanter Geschwindigkeit

$\boxed{Exp}$

[Exp]

Physical Pendulum



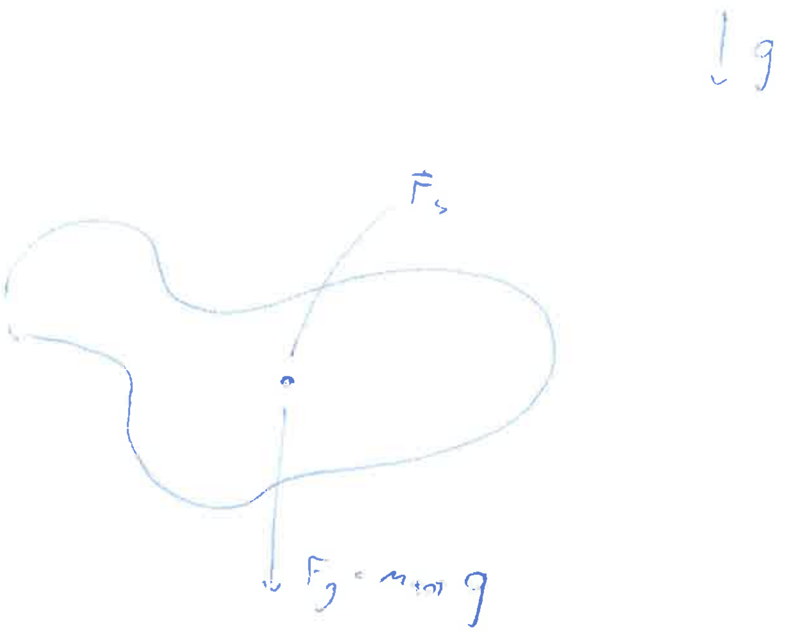


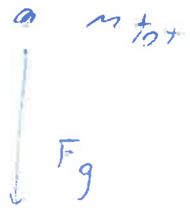
$$\vec{r}_s = \frac{1}{\int \rho(\vec{r})\, dV} \int \vec{r}\, \rho(\vec{r})\, dV$$

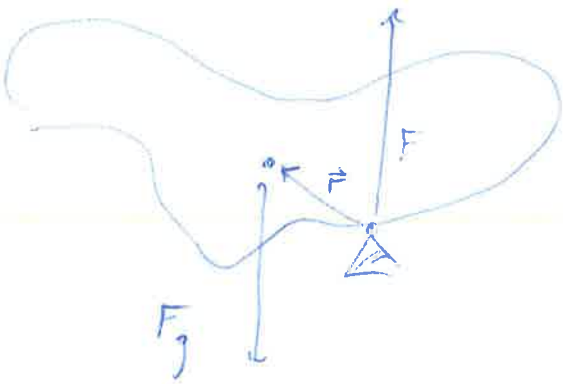
$$\vec{r}_s = \frac{1}{m} \int \vec{r}\, \rho(\vec{r})\, dV$$

Center of mass swings as if the total mass m were concentrated in it, as in the right drawing.

2d

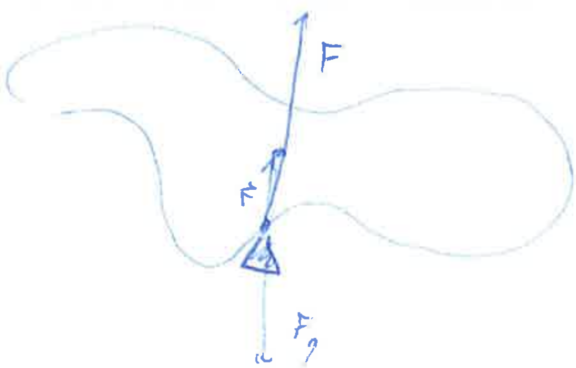






Force: $F + F_g = 0$

Torque: $\vec{r} \times \vec{F}_g + \cancel{0 \times \vec{F}}^{\,0} \neq 0$ ← not in equilibrium



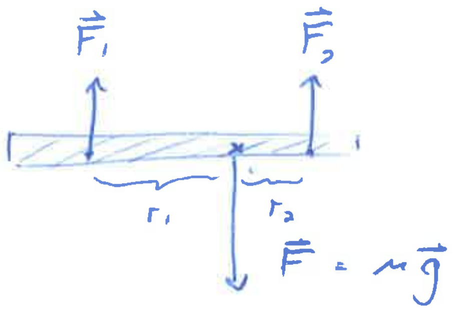
Force: $F + F_g = 0$

Torque

$\cancel{\vec{r} \times \vec{F}_g}^{\,0} + \cancel{0 \times \vec{F}}^{\,0} = 0$

In equilibrium

23 $\boxed{\text{Exp}}$



Forces: $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

$$\boxed{F_1 + F_2 = mg}$$

Torques: $\vec{M}_1 + \vec{M}_2 = 0$

$$-r_1 F_1 + r_2 F_2 = 0$$

$$r_1 F_1 = r_2 F_2$$

$\therefore$ For $r_1 > r_2$

$\hookrightarrow F_1 < F_2$

If $F_1 < F_2$ then the friction against finger 1 is smaller than against finger 2. There finger 1 slides more easily. This will bring finger 1 closer to the center of mass until $r_1 \leq r_2$. Then finger 2 will slide more easily... ultimately the two fingers will meet at the center of mass.