Phys I ; L3 : Space and time

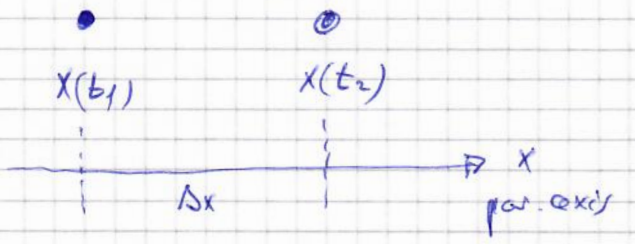
28.9.2017

1

## Motion in 1 dimension

$[x] = m$
$[t] = s$

- position $x(t)$




displacement $\Delta x = x(t_2) - x(t_1)$
during time interval $\Delta t = t_2 - t_1$

- average velocity (def) : (change of position)

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}, \quad [v_{avg}] = \frac{m}{s}$$

- instantaneous velocity (def) {

(slide) Tipler, Fig 2.5 : show, $v$ is the slope of the line tangent to the $x(t)$ curve

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \underset{\text{notation}}{\dot{x}}$$

, derivative of x with respect to t.

- average acceleration (def) (change of speed)

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v(t_2) - v(t_2)}{t_2 - t_1}$$

- instantaneous acceleration

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \underset{\text{notation}}{\ddot{x}}$$

later

~~slide Tipler, Fig falling apple stroboscope~~

~~(Slides) 2 Fragen : Geschw. & Beschleunigung~~

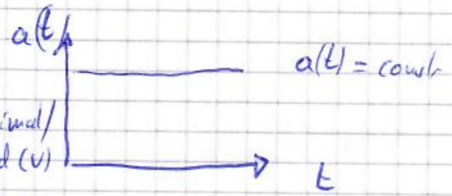
- motion with constant acceleration:

$$a = \text{const}$$

$$a = \frac{dv}{dt}$$

dv: small (infinitesimal) change of speed (v)

dt: small (infinitesimal) change of time (t)



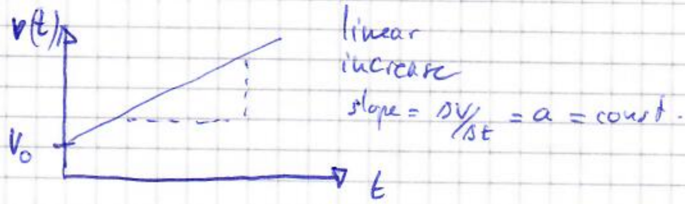

$$a \cdot dt = dv$$

$$\int_0^t a \cdot dt = \int_{v(0)}^{v(t)} dv$$

$$a \cdot t - a \cdot 0 = v(t) - v(0)$$

$$a \cdot t = v(t) - v_0$$

and $\| \underline{v(t) = a \cdot t + v_0}$



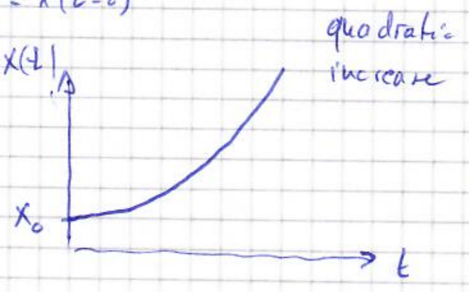

Speed: $v(t) = \frac{dx(t)}{dt}$ $\left( v = v(t) \right)$

$$v \cdot dt = dx$$

$$\int_0^t v \cdot dt = \int_{x(0)}^{x(t)} dx$$

$$\int_0^t (a \cdot t + v_0) \cdot dt = \int_{x_0}^{x(t)} dx \quad , x_0 = x(t=0)$$

$$\frac{1}{2} a t^2 + v_0 \cdot t = x(t) - (x_0)$$




and $\| \underline{x(t) = \frac{1}{2} a t^2 + v_0 \cdot t + x_0}$

(Slide) Tipler, falling apple: visualize acceleration

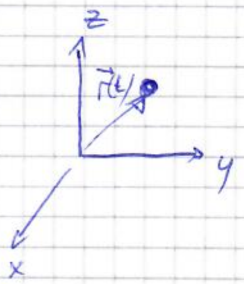
def: motion with $v = \text{const} \overset{!}{\equiv}$ uniform motion

(Slide) 2 questions, velocity acceleration

# motion in two and three dimensions

$x(t)$ $\rightarrow$ $\vec{r}(t) = (x(t), y(t), z(t))$

position $\qquad$ position vector




here: cartesian coordinates
(also cylindrical, spherical, ...)

1D: $\Delta x = x(t_2) - x(t_1) \overset{3D}{\Rightarrow} \Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1) = \left(x(t_2) - x(t_1);\ y(t_2) - y(t_1);\ z(t_2) - z(t_1)\right)$

displacement vector

Velocity: $\vec{v}(t) = \dfrac{d\vec{r}(t)}{dt} = \left(\dfrac{dx(t)}{dt}; \dfrac{dy(t)}{dt}; \dfrac{dz(t)}{dt}\right)$

$= \left(v_x(t); v_y(t); v_z(t)\right)$

Notes: avg velocity vector $\vec{v}_{avg} = \dfrac{\Delta\vec{r}}{\Delta t} = \left(\dfrac{x(t_2) - x(t_1)}{t_2 - t_1}; \ldots ; \quad\right)$

instantaneous velocity vector
$\equiv$ velocity $\qquad \vec{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{d\vec{r}}{dt}$

magnitude of velocity vector: $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

similarly for the acceleration,
(instantaneous)

$\vec{a}(t) = \dfrac{d\vec{v}(t)}{dt} = \dfrac{d^2\vec{r}(t)}{dt^2} = \left(\dfrac{d^2x(t)}{dt^2}; \dfrac{d^2y(t)}{dt^2}; \dfrac{d^2z(t)}{dt^2}\right)$

$= \left(a_x(t); a_y(t), a_z(t)\right)$

## relative velocity and reference frame

travel in train or airplane: what is your velocity

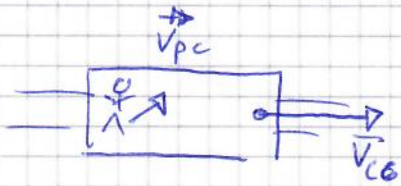
$v_1 = 0$ relative to train/airplane (if seated)

$v_2 \neq 0$ " to earth surface

$v_3 \neq 0$ " " " " and $v_3 \neq v_2$ if in a jet stream

→ a velocity makes sense only when specifying the reference frame.

example: person ~~walking~~ in (~~train~~) railroad car:
(P) (C)




$\vec v_{pc}$: velocity of person relative to car

$v_{CG}$: " " car " to ground

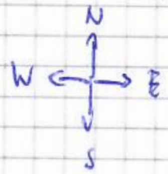
Velocity of person relative to ground:

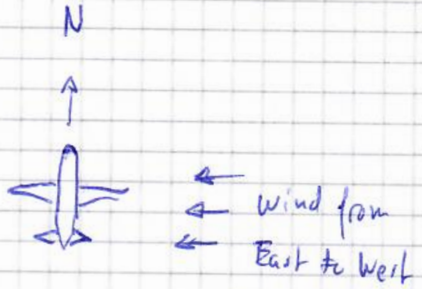
$$\vec v_{pG} = \vec v_{pc} + \vec v_{CG} \qquad \text{(additivity)}$$

note $\vec v_{pc} = -\vec v_{cp}$

↑ velocity of car relative to person

<u>example</u> plane + wind (Tipler, example 3-2)




Plane: $|\vec{V}_{pA}| = 600\ km/h$, airspeed of plane

Wind: $|\vec{V}_w| = |\vec{V}_{AG}| = 100\ km/h$, wind speed (air to ground speed)

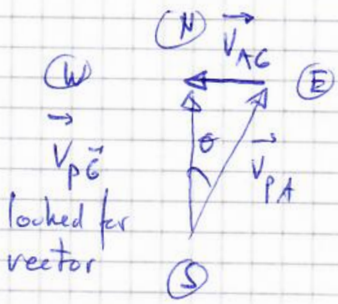
a) Pilot wants to fly North. In which direction shall the plane head?

b) What is the ground-speed of the plane?

- Wind => plane drifts off course towards West.

a) - velocity of plane relative to ground: $\vec{V}_{pG} = \vec{V}_{pA} + \vec{V}_{AG}$

- vector addition diagram:
(Wind direction is fixed w respect to ground)




|| $\sin\theta = \frac{V_{AG}}{V_{pA}} = \frac{100}{600}$ $\theta \simeq 10^0$ $(9.59°)$

(note: $V_{AG} = |\vec{V}_{AG}|$)

|| Plane shall fly with $10°$ angle East to North

b) $V_{pA}^2 = V_{pG}^2 + V_{AG}^2$ (triangle with ∟ angle)

$V_{pG} = \left(V_{pA}^2 - V_{AG}^2\right)^{1/2} \cong 592\ km/h$

Note if plane would fly due East $V_{pG} = 500\ km/h$

# non uniform motion: adding acceleration ($a \neq 0$, $a = \frac{dV}{dt}$)

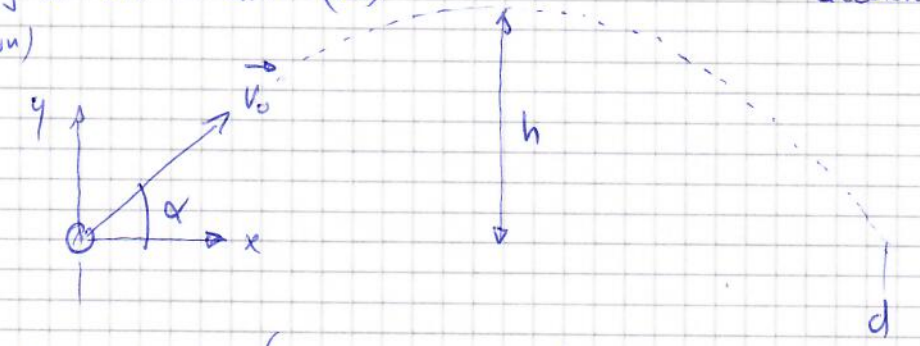
Seen eq. of motion (10):

$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0 \qquad (1)$$

$$v(t) = a \cdot t + v_0 \qquad (2)$$

launching a ball in air (20)
(no friction)

acceleration: $\downarrow \vec{g}$ (gravity)




$\vec{V}_0 = (V_{0x}, V_{0y}) = (V_0 \cdot \cos\alpha, V_0 \cdot \sin\alpha)$

$V_0 = 10$ m/s, $\alpha = 60°$

$\vec{a} = (0, -g)$, $g = 9.81$ m/s$^2$

**Find** $\vec{r}(t)$ and $\vec{v}(t)$ for $t = 1$ s

a) initial velocity vector : $t=0$ $\quad \vec{V}_0 = (V_0 \cdot \cos\alpha, V_0 \sin\alpha)$
$= (5, 8.67)$

b) velocity at time $t$: $\quad \vec{V}(t) = (V_x(t), V_y(t))$

$$V_x(t) = V_{0x} + \int_0^t a_x dt = V_{0x}$$

Eq. 2 above

$$V_y(t) = V_{0y} + \int_0^t a_y dt = V_{0y} - g\int_0^t dt = V_{0y} - g \cdot t$$

and $\vec{V}(t) = (V_{0x}, V_{0y} - g \cdot t)$

c) position at time $t$: $\quad \vec{r}(t) = (x(t), y(t))$, $dx = x_0 + V_x dt$

$$x(t) = x_0 + \int_0^t V_x dt \cancel{+ \int_0^t \frac{1}{2} a_x t \text{ (const acceleration)}}$$

$$= V_{0x} \cdot t \qquad (x_0 = 0, y_0 = 0)$$

$$y(t) = y_0 + \int_0^t V_y(t) \cdot t = y_0 + \int_0^t (V_{0y} - g \cdot t) \cdot dt$$

$$= V_{0y} \cdot t - \frac{1}{2} g t^2$$

$$\vec{r}(t) = \left(V_{0x} \cdot t, \; V_{0y} \cdot t - \frac{1}{2} g t^2\right)$$

28.9

6'

Exp Waterjet.

- project parabola on wall
- adjust waterjet, show overlap with parabolic curve

Exp schiefer Wurf + Platte

explain trajectory, then equation

ask assumptions!?

d) for $t = 1\,\text{sec}$

$$\vec{r} = (v_{0x} \cdot t \,,\; v_{0y} \cdot t - \tfrac{1}{2} g t^2)$$

$$= (5 \cdot 1 \,,\; 8.66 \cdot 1 - \tfrac{1}{2} \cdot 9.81 \cdot 1^2) = (5\,\text{m} \,,\; 3.76\,\text{m})$$

$$\vec{v} = (v_{0x} \,,\; v_{0y} - g \cdot t)$$

$$= (5\,\text{m/s} \,,\; -1.15\,\text{m/s})$$ going right and down

Slide Tipler Fig 3-12 : effect $v_{0x} \neq 0$
note, vertical motion is identical for both balls
i.e., vertical motion indep. of horizontal motion (here)

With eq. of motion, calculate precisely trajectory

Slide Cassini mission : include effect of planets, sun (additional acceleration effect)

<u>So far</u> : - the origin of the acceleration has not been discussed (gravitation : force due to earth mass)

- we assumed no other disturbances (forces) acting on motion of objects considered (no friction, no electrical forces)

Exp <u>Freier Fall quantitativ:</u> • ball falling from ceiling, meas. g

$x(t) = \frac{1}{2} a t^2 + v_0 t + x_0$

$v_0 = 0 \quad x_0 = 4.92\,m$

$-4.92 = \frac{1}{2} \cdot (+a) \cdot t^2 \quad , \; t \sim 1s$

$\Rightarrow a \cong -9.84 \; \frac{m}{s^2} \quad , \; \text{gravitation}$

Exp <u>freier fall, qualitativ</u>

plume and ball in tube a) air

b) no air