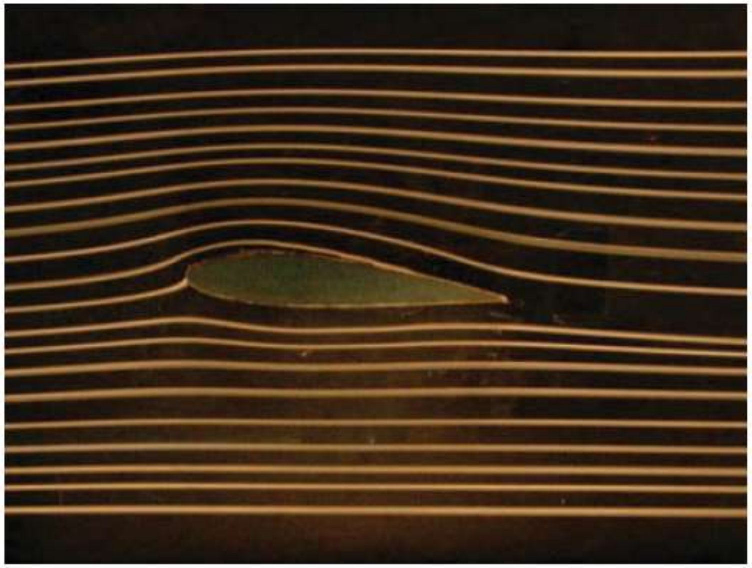
Introduction to Physics I

For Biologists, Geoscientists, & Pharmaceutical Scientists

L14, 03.11.2017

Bernouilli equation



The streamlines are made visible by using smoke trails. In streamlined flow the particles of the fluid follow smoothly curved lines. *(Holger Babinsky. 2003 Phys. Educ. 38 497-503.)*

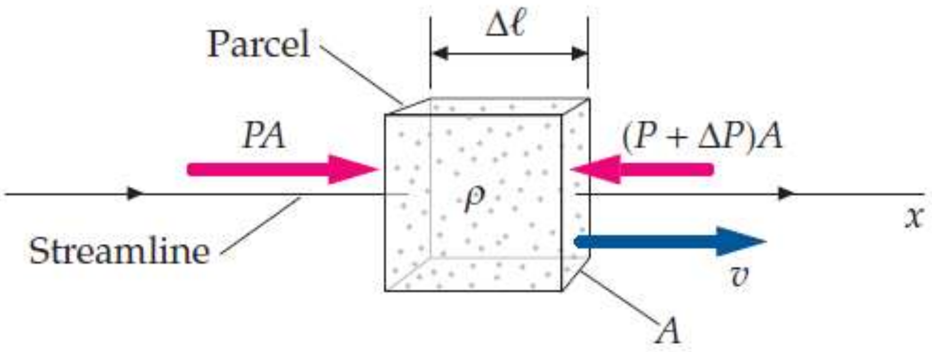


FIGURE 13-17 The small parcel moves along a streamline into a region of reduced pressure.

Tipler

wing and lift

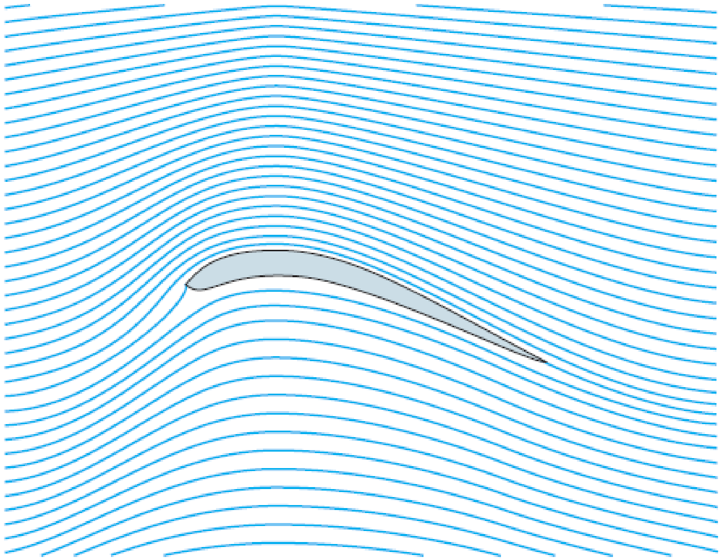


FIGURE 13-23 The purpose of an airfoil is to cause the streamlines to curve. Under normal conditions the streamlines will follow the curve of the airfoil. The airfoil shown is very thin, like the wing of a raptor. It is very efficient in creating lift.

Tipler

viscous fluid

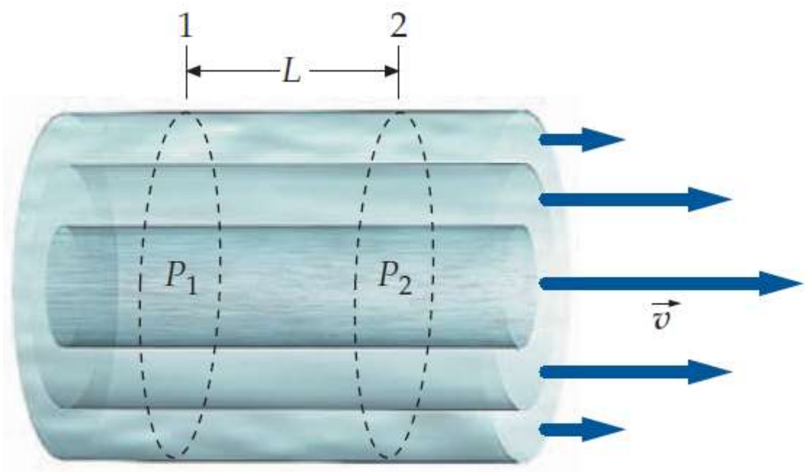


FIGURE 13-26 When a viscous fluid flows through a pipe, the speed is greatest at the center of the pipe. At the walls of the pipe, the speed of the fluid approaches zero.

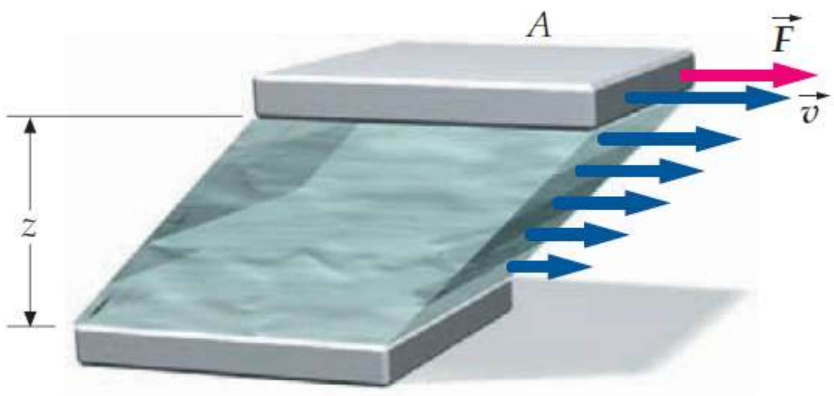


FIGURE 13-27 Two plates of equal area with a viscous fluid between them. When the upper plate is moved relative to the lower one, each layer of fluid exerts a drag force on the adjacent layers. The force needed to pull the upper plate is directly proportional to v and the area A, and inversely proportional to z, the separation between the plates.

Tipler

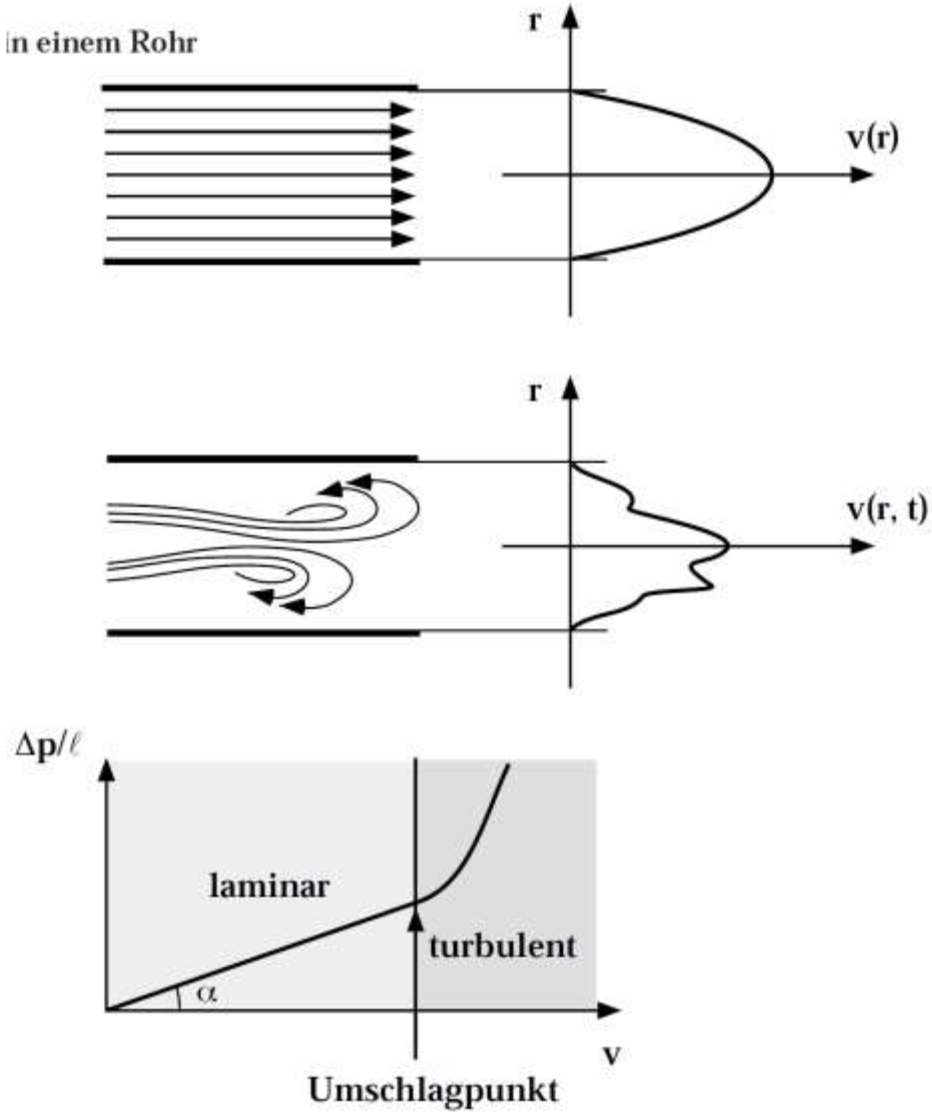
viscous fluid

Tab. 5.4 Viskosität η einiger Flüssigkeiten und von Luft

Substanz (bei 20 °C)	Öle	Glyzerin	Blut ♂ Mittelwert	Blut ♀ Mittelwert	Äther	Hg	H_2O	Luft
η (Pa s)	1	0,83	0,0047	0,0044	0,0018	0,0015	0,001	$1{,}8 \cdot 10^{-5}$

Trautwein

turbulent flow



Biological systems & Reynolds nb

dynamical systems in aqueous environment at 310K
constituted by small-size components

Friction (Stokes)

$F_S = 6\pi\eta a v \sim 10^{-12} N$

for a 1μm particle moving at 50μm/s

Stochastic (Brownian)

$F_B \propto \eta^2/\rho \sim 10^{-9} N$

η: viscosity, 10^{-3} Ns/m^2
ρ: density, 10^3 kg/m^3

essentially
no inertial forces

Living systems

- hydrodynamically speaking, we deal with fluids at low Reynolds number *(Purcell, 1976)*

$$R \cong v L \rho / \eta$$

$\rho \sim 1\ g/cm^3$, $\eta \sim 1\ g/(cm \cdot s)$

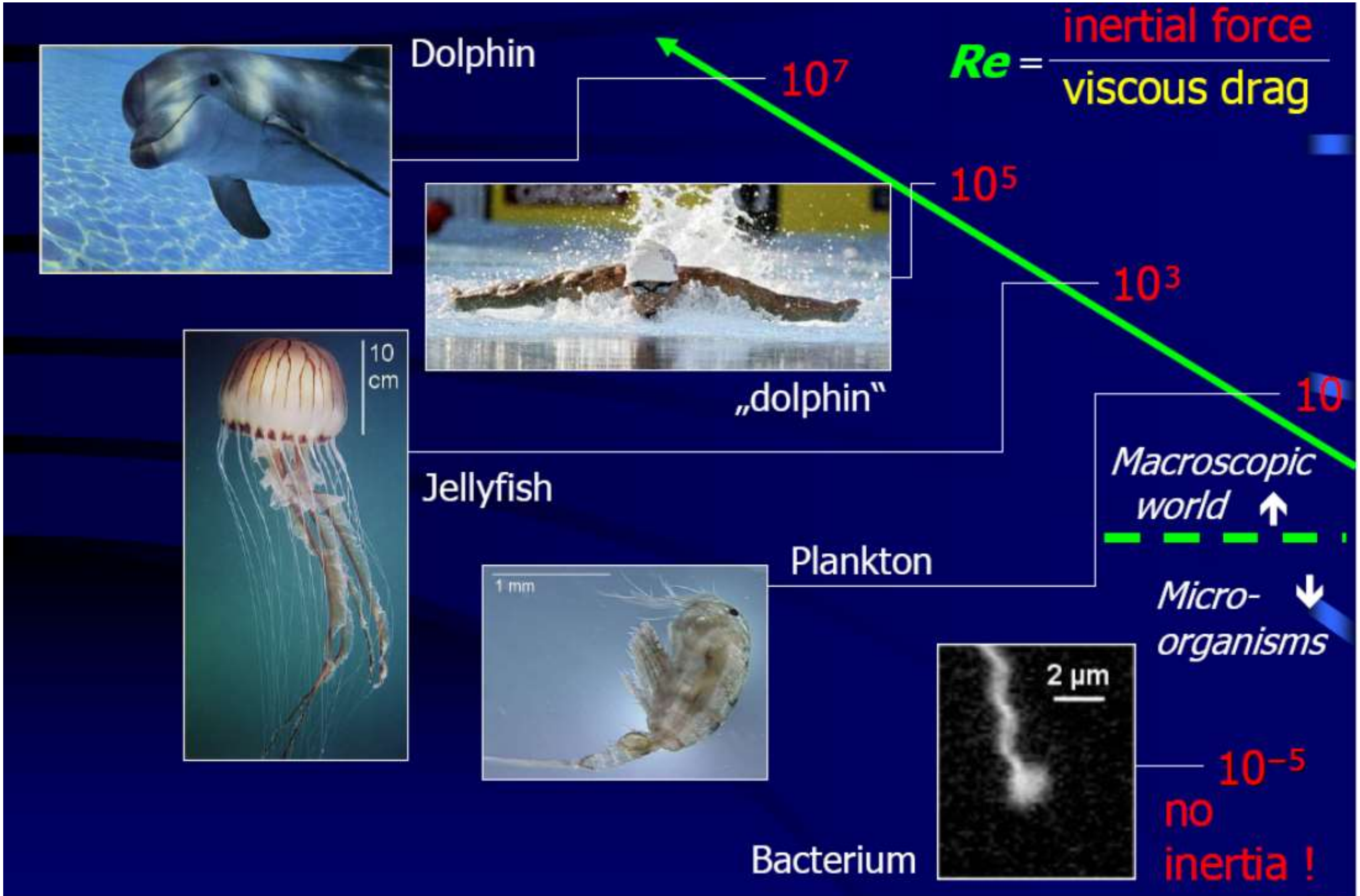
fish $\quad v \sim 1\ m/s$, $L \sim 10\ cm$ $\Rightarrow$ $R \sim 10^5$

(accelerates water to propel itself)

bacteria $\quad v \sim 10\ \mu m/s$, $L \sim 1\ \mu m$ $\Rightarrow$ $R \sim 10^{-5}$

(uses viscous shear to move)

Living systems



Stark 2007

Living systems

⇨ bacteria thus know nothing about inertia

what's the distance it can coast when it stops swimming ?

$m(-dv/dt)=6\pi\eta av$ ⇨ $v(t)=v(0)\cdot e^{-t/\tau}$

with $\tau=2a^2\rho/9\eta \sim 2\cdot10^{-7}$ s

the bacteria thus stops in about a micro-second coasting a distance $v(0)\cdot\tau \sim 0.04$ Å (using $v(0)=20$ μm/s).

So bacteria swimming in water is like us swimming in honey

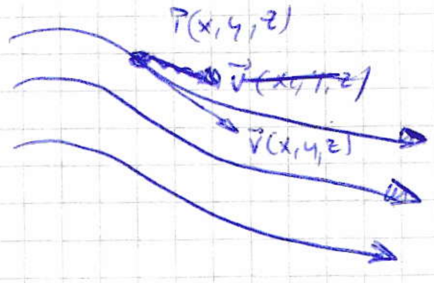
Further reading

- E. M. Purcell, *Life at low Reynolds number* Am. J. Phys. 45, 3 (1977)
- H. C. Berg, *Motile Behavior of Bacteria* Physics Today, January (2000)

hydrodynamics

slide S7 Euler (Leonhard), Bernoulli (Daniel)

- fluid in motion.



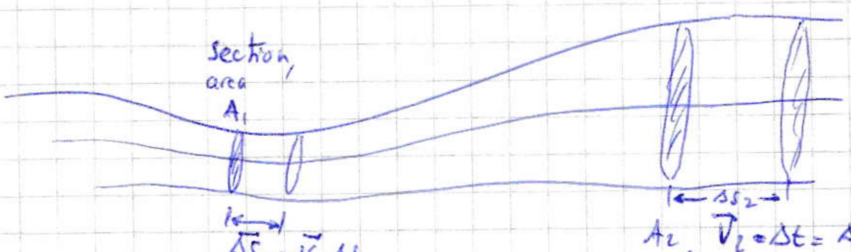
attribute at time t a density $\rho(x,y,z)$ and velocity $\vec{v}(x,y,z)$

$\forall P(x,y,z)$, point in a liquid in motion

slide S8/S9 defs: stationary / non-stationary, laminar / turbulent, etc...

- equation of continuity: assumption (for later): "ideal" liquid, i.e.:
  - incompressible
  - inviscid (no viscosity, or internal friction)

consider tube with a tapered section: constant fluid flow (laminar)



- mass flowing through area A_1: $\Delta m_1 = \rho_1 . \Delta V_1 = \rho_1 . A_1 . v_1 \Delta t$
  ↑ density

- mass flowing through area A_2: $\Delta m_2 = \rho_2 . \Delta V_2 = \rho_2 . A_2 . v_2 . \Delta t$

no sources or sinks of fluid $\Rightarrow$ $\Delta m_1 = \Delta m_2$

or $\| \rho . A . v = const$, continuity equation

or $\| A . v = const$ for an incompressible medium ($\rho = const$)

Bernoulli equation

incompressible & inviscid liquid
(ρ = const) (no viscosity)

Consider a parcel of fluid along a streamline laminar flow
(show on slide)

slide Fig 13-17 Tipler

air parcel (or cube) : $m = \rho \cdot A \cdot \Delta l$, pressure difference ΔP (behind / ahead of parcel) $\Rightarrow$ force F

Newton : $F = m \cdot \frac{dv}{dt} \left(\approx m \cdot \frac{\Delta v}{\Delta t} \right)$

force : $F = P \cdot A - (P + \Delta P) \cdot A = - A \cdot \Delta P$

i.e. with Newton $- \cancel{A} \cdot \Delta P = \rho \cdot \cancel{A} \cdot \Delta l \cdot \frac{\Delta v}{\Delta t}$ $\frac{\Delta l}{\Delta t} \approx v$

$$\Delta P = - \rho \cdot v \cdot \Delta v$$

for $\Delta P \to 0$, $dP = - \rho \cdot v \cdot dv$

integrate

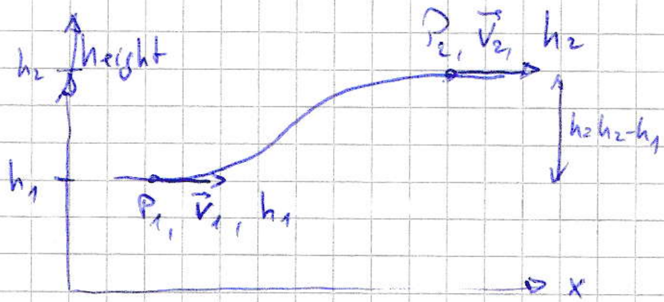
$$\int_{P_1}^{P_2} dP = - \rho \int_{v_1}^{v_2} v \cdot dv \quad , \quad \rho = \text{const}$$

$$P_2 - P_1 = \frac{1}{2} \rho v_1^2 - \frac{1}{2} \rho v_2^2$$

and

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

and ($\forall P_1, P_2$) $\quad \| \; P + \frac{1}{2} \rho v^2 = \text{const}$



If the streamline is not horizontal :
add gravitation term : $\rho g h$

i.e. $\quad \| \; p + \rho \cdot g \cdot h + \frac{1}{2} \rho \cdot v^2 = \text{const}$

Bernoulli eq.

$p + \rho g h$ represent a "static" pressure

$\frac{1}{2} \rho v^2$: " " "dynamic" pressure

units: $[p] = \frac{N}{m^2}$, pressure

$[\frac{1}{2} \rho v^2] = \frac{kg}{m^3} \cdot \frac{m^2}{s^2} = \frac{kg}{m^2} \cdot \frac{m}{s^2} = \frac{1}{m^2} \cdot N = \frac{N}{m^2}$

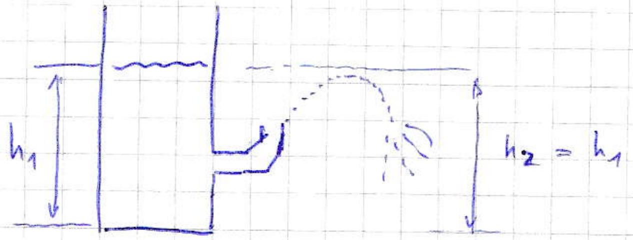
$F = m \cdot dv/dt$

$[\rho \cdot g \cdot h] = \frac{kg}{m^3} \cdot \frac{m}{s^2} \cdot m = \frac{N}{m^2}$

exp 107.12 Springbrunnen, fountain

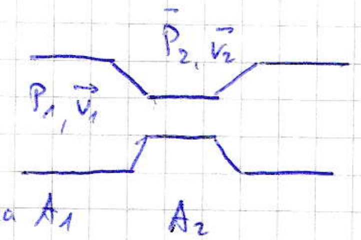
$h_2 = h_1$,

$h_1 \searrow$, $h_2 \searrow$



exp 107-13 **Venturi effect**

a) constriction in a pipe:



Bernoulli: $P + \frac{1}{2} \rho v^2 = \text{const}$

eq. of continuity $\Rightarrow$ $v_2 > v_1$ as $A_2 < A_1$ (eq. of continuity)

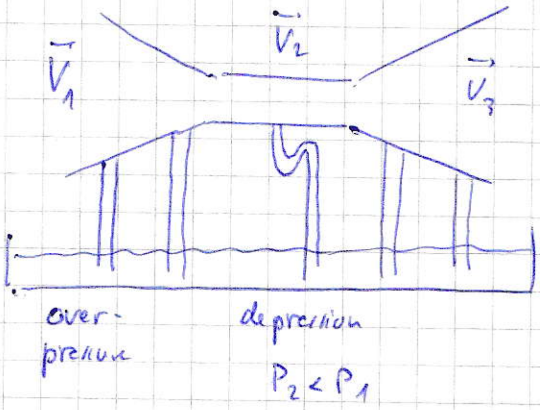
$\rho = \text{const}$

$A \cdot v = \text{const}$

and if $v_2 > v_1 \Rightarrow \boxed{P_2 < P_1}$

Bernoulli

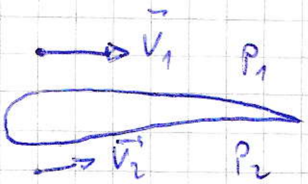
b) show Venturi tube



$\Rightarrow$ if geometry known (A_1, A_2) and density known, Venturi tube to measure fluid velocity

- **Creating lift using a wing**

(slide) wing : air foil forces streamlines to curve and $v_1 > v_2$



Bernoulli

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

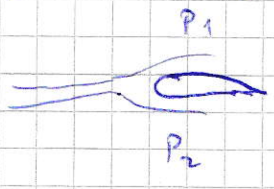
$$P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2$$

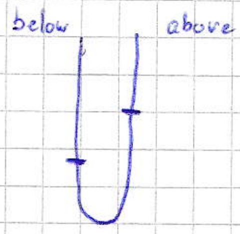
$$v_1 > v_2 \Rightarrow P_1 < P_2$$, lift (upward force)

NB: air is compressible, hence Bernoulli cannot be strictly applied, and air pressure change due to gravity neglected

L13 6

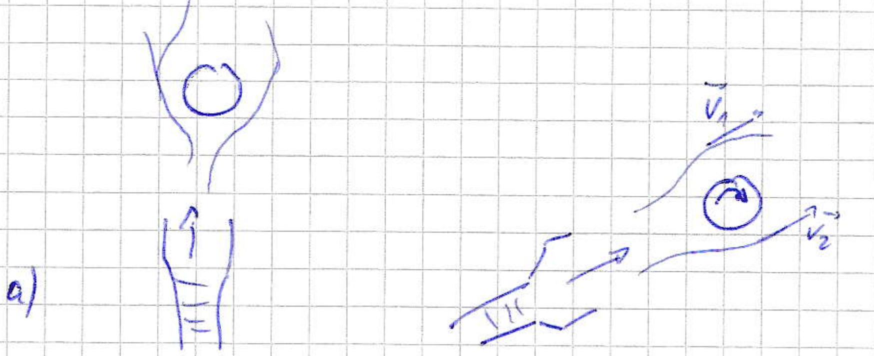
exp a) ~~Strömung / Apparat~~ Strömlinien, wing and pressure difference meas.
107-14 Flugzeug fliege

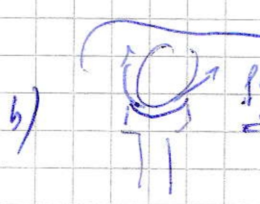




→ change angle to emphasize effect.

exp ball in air flow



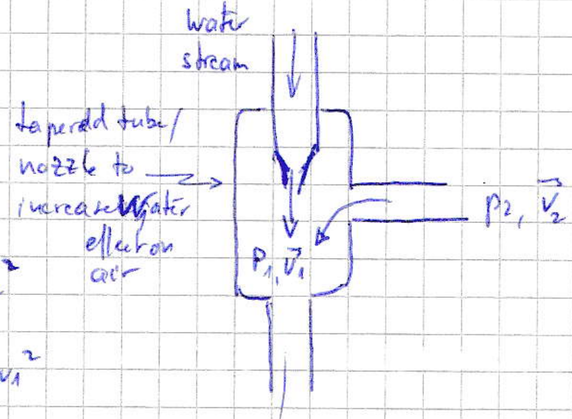


$v_1 > v_2$, friction at air-ball interface
$F_1 > F_2$, rotation
fast flow
=> low p (compared to atmosphere)
=> "stick" to outlet

corollary: rotating ball with horizontal ~~flight~~ speed, e.g. start
(football) will/can gain height due to rotation effect and the resulting different flow speed above and below

exp water tap pump
107-15

Bernoulli



$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$p_1 - p_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

Viscous flow

slide

viscous flow

Bernoulli : $P + \frac{1}{2}\rho v^2 = \text{const}$, horizontal tube

laminar flow : $v = \text{const}$ $\Rightarrow$ $P = \text{const}$

$\rho = \text{const}$

BUT : exp. show P decreases downstream

slide

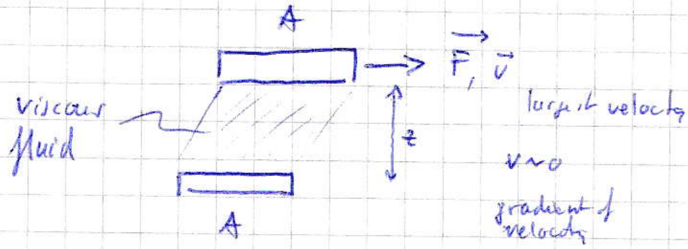
viscous flow $\Delta P = P_1 - P_2 \propto v \cdot A$, flow rate

"picture" : concentric cyclinders

speed, area

def viscosity parallel plates

slide



def : $\| \; F = \eta \frac{v \cdot A}{z}$, η : viscosity (dynam) (prop. constant)

$[\eta] = \frac{N \cdot s}{m^2} = Pa \cdot s$

Pascal

or better

$$\| \; F = \eta \cdot A \cdot \frac{dv}{dz}$$

velocity gradient.

- resistance to flow : R

start exp

Hagen - Poiseuille (p. 8)

seen above viscous flow : $\| \; \Delta P = P_1 - P_2 \propto v \cdot A$, volume flow rate

$\equiv R \cdot v \cdot A$

def resistance to flow $\| \; R = \frac{\Delta P}{v \cdot A} \propto \Delta P$

% unit

Poiseuille law (~~see~~ demo below) : $R = \frac{8\eta \cdot L}{\pi \cdot r^4}$ for a tube with radius r, length L

and

$$\| \; \Delta P = \frac{8\eta L}{\pi \cdot r^4} \cdot v \cdot A \quad , \quad \propto \frac{1}{r^4}$$

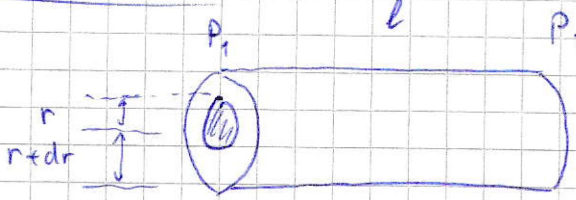
for a viscous flow $v \cdot A = I_v \propto r^4$

volume flow rate

NB : fluid is said to be "Newtonian" if η is indep. of $\frac{dv}{dz}$

slide typ. viscosity values,

demo Poiseuille — 413 7a



tube, radius R
(concentric cylinders)

per def viscosity $\quad F_F = \eta \cdot A \cdot \frac{dv}{dr}$, friction force

with $A = 2\pi r \cdot L$ contact surface between cylinders

force expressed via ΔP: $\quad F_p = \Delta P \cdot S = (P_1 - P_2) \cdot \pi \cdot r^2$

Equilibr: $\quad F_F + F_p = 0$

$$\Rightarrow \quad 2\pi r \cdot l \cdot \eta \frac{dv}{dr} = -\Delta P \cdot \pi r^2$$

$$\frac{dv}{dr} = -\frac{\Delta P}{2\eta l} \cdot r$$

$$\int_0^r dv = -\frac{\Delta P}{2\eta l} \int_0^r r \cdot dr$$

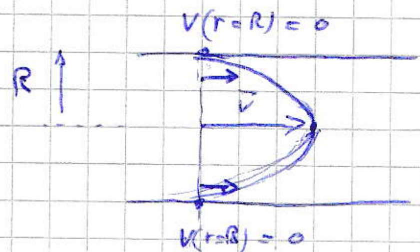
$$V(r) - V(0) = -\frac{\Delta P}{4\eta l} \cdot r^2$$

$V(r) = \underbrace{V(0)}_{\text{const}} - \frac{\Delta P}{4\eta l} \cdot \underline{\underline{r^2}}$
parabolic velocity distribution

At $r = R$, $V = 0$: no motion at interface tube / liquid (cf schematics)

$$V(R) = V(0) - \frac{\Delta P}{4\eta l} \cdot R^2 = 0$$

and $\quad V(0) = \frac{\Delta P}{4\eta l} \cdot R^2$



volume flow rate $\quad I_V = \int_0^R V(r) \cdot dA$

Rem. $I_V = v \cdot A$

$$= \int_0^R \left(v(0) - \frac{\Delta P}{4\eta l} \cdot r^2 \right) \cdot 2\pi r \cdot dr$$

$dA = 2\pi r \cdot dr$

$$= \frac{\cancel{2}\pi R^2}{\cancel{2}} v(0) - \frac{\cancel{2}\pi \cdot \Delta p}{\cancel{4}\eta l} \cdot \frac{R^4}{4}$$

$$I_V = \frac{\pi \cdot R^2 \cdot \Delta P \cdot R^2}{4\eta l} - \frac{\pi \cdot \Delta p}{\eta l} \cdot \frac{R^4}{8} = \underbrace{\frac{\pi}{8} \cdot \frac{R^4}{\eta \cdot l}}_{1/R} \cdot \Delta p$$

flow rate

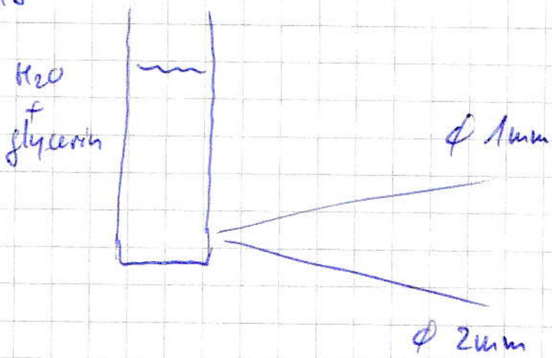
$v(0) = \frac{\Delta P \cdot R^2}{4\eta l}$

qed

exp Hagen-Poiseuille
107-18

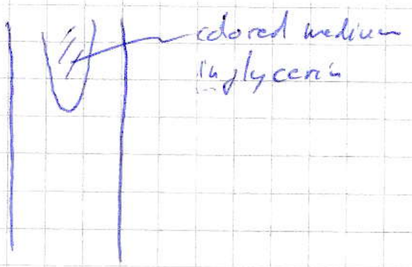
(glycerin : $C_3H_8O_3$

)



$J_V \propto R^4$

exp parabolic speed distribution



- Stokes friction & sedimentation
(drag force)

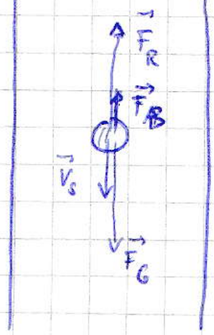
friction/drag force for a body moving in viscous liquid at velocity v (v not too large) :

$$\Big\| \ \vec{F}_R = -6\pi\eta R \cdot \vec{v}$$

, R: radius object

Stokes, $F_R \propto v$ for v small

- sedimentation :



$$\vec{F}_G + \vec{F}_B + \vec{F}_R = 0$$

gravitation, drag

buoyancy (Archimedes)

volume sphere

$$mg - \frac{m}{\rho_0}\cdot\rho_F\cdot g - 6\pi\eta R\cdot v_s = 0$$

density
ρ_0 : object
ρ_F : fluid

hence $$v_s = \frac{1}{6\pi\eta R}\cdot mg\left(1-\frac{\rho_F}{\rho_0}\right)$$, $m = \frac{4\pi}{3}R^3\cdot\rho_0$

$$\Big\| \ v_s = \frac{2}{9\eta}\cdot(\rho_0-\rho_F)\cdot g\ R^2 , \quad \underline{\underline{\propto R^2}}$$

centrifuge to collect small objects (DNA, cells, ...)

- **exp** sedimentation : 3 different ϕ
107-22

turbulent flow

when $v \nearrow$, crossover laminar to turbulent

slide

critical speed at which crossover take place:

$$v_R = \frac{\eta}{\rho \cdot R} \cdot N_R$$

R: radius tube

N_R: const.

when $v \nearrow$, and $\frac{\Delta p}{\ell}$, pressure drop (viscous fluid) per unit length $\nearrow$, → turbulence

exp: 107-23 turbulent flow: colored medium in tube

a) low v: laminar

b) $v \nearrow$, crossover to turbulent

NB: liquid mixing: turbulent regime!

exp: storm kinetic model: crossover / change laminar turbulent object shape

a) show wing, chg angle

Reynolds number N_R

$$N_R = \frac{\rho \cdot v \cdot d}{\eta}$$

d: typical dimension, characteristic length of system

$N_R \gtrsim 10^3$ → turbulent

slides life at low Reynolds nb.