Introduction to Physics I

For Biologists, Geoscientists, & Pharmaceutical Scientists

L13, 02.11.2017

Mechanical Properties of Liquids

- Properties of liquids, surface tension; hydrostatics; surface tension, capillary forces.
- Pressure in fluids; hydrodynamics; continuity equation.
- Viscosity; laminar flow; turbulence.

buoyancy, Archimedes principle

(*a*) Crown and gold nugget have equal weight.

(*b*) Crown displaces more water than does the gold nugget.

FIGURE 13-13 (*a*) The crown and the gold nugget have equal weight. (*b*) The balance tips because the wreath displaces more water than the gold nugget.

Tipler

surface tension

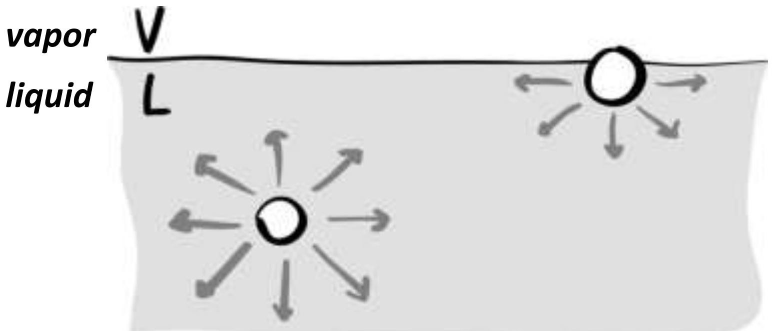
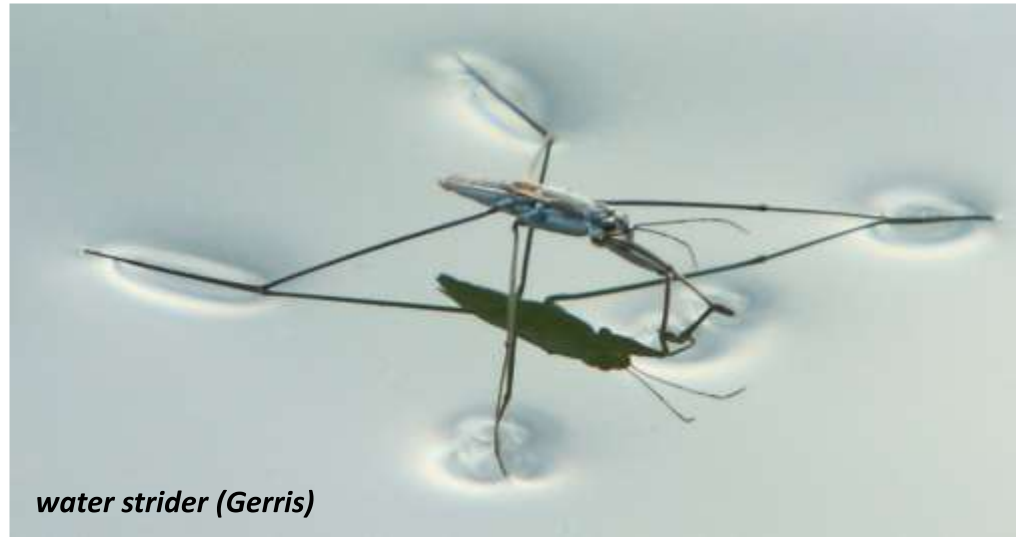
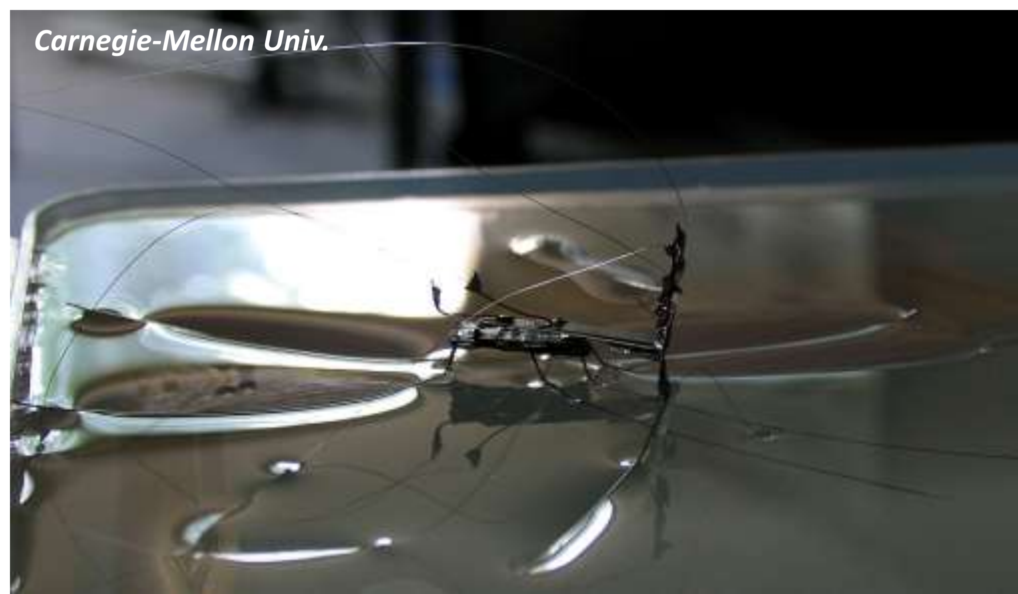


Fig. 1. Sketch showing the missing intermolecular bonds close to the liquid-vapor interface, giving rise to an increase in the free energy per unit area, that is, the **surface tension**.





Marchand et al., Am. J. Phys., 79(10), Oct. 2011

adhesion / cohesion

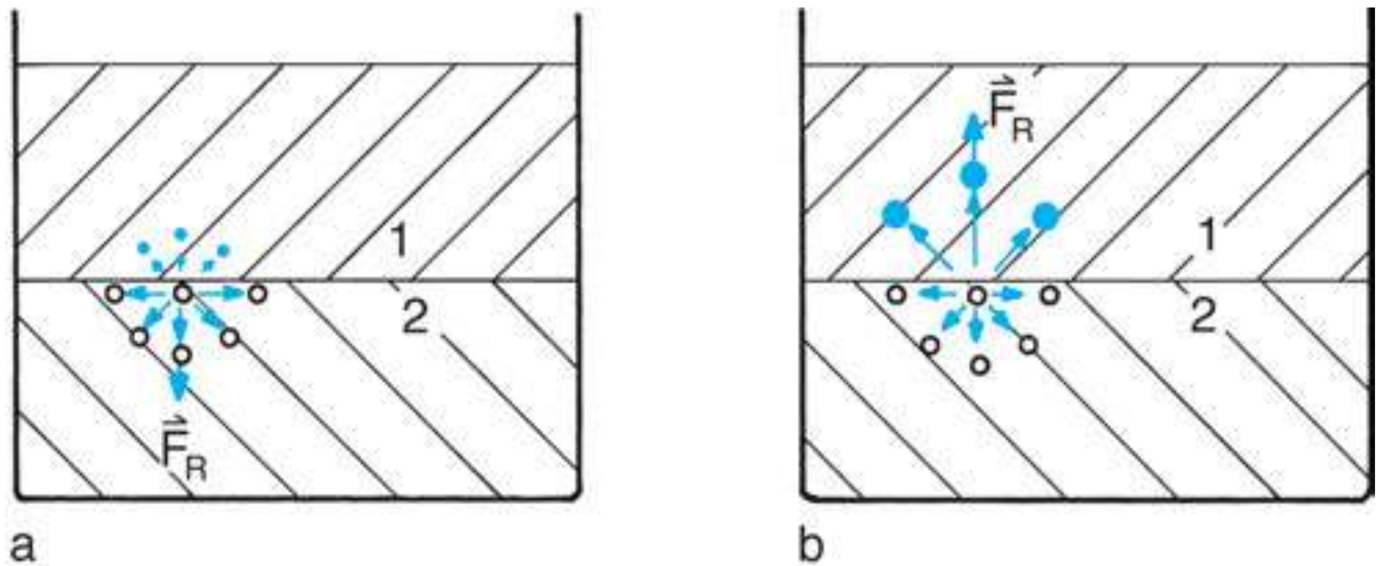


Abb. 5.14 Kohäsion und Adhäsion: (a) Kohäsion überwiegt Adhäsion, (b) Adhäsion überwiegt Kohäsion.

Trautwein

hydrodynamics

basics of the description of fluids in motion

Daniel Bernoulli (CH)
1700-1782

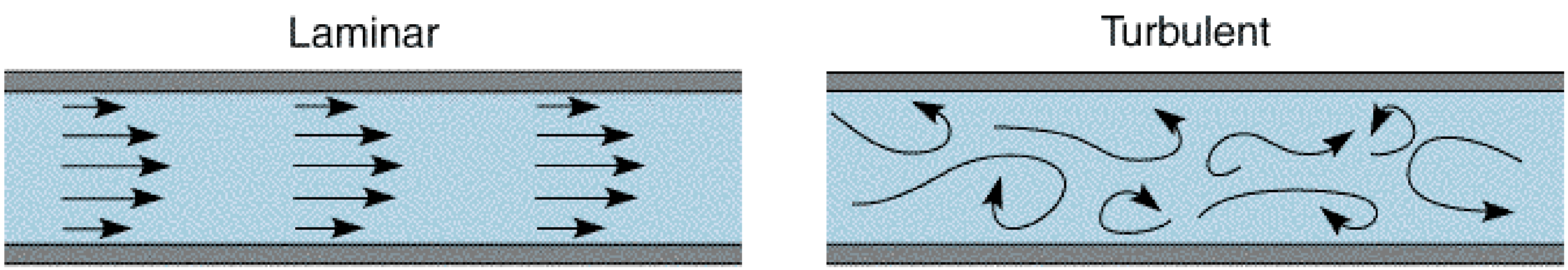
Leonhard Euler
1755-1825

Euler-Laterne während der Basler Fasnacht 2008

hydrodynamics

Wir unterscheiden :

• stationäre	und	nicht stationäre Strömungen
$\vec{v}$ ist zeitlich konstant in jedem Punkt P der Strömung		$\vec{v}$ ist zeitabhängig (turbulente Strömungen)
Beispiel :		
langsam fliessender Strom		Wasserfall



hydrodynamics

Wir unterscheiden :

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	$\vec{v}$ ist zeitlich konstant in jedem Punkt P der Strömung		$\vec{v}$ ist zeitabhängig (turbulente Strömungen)
	Beispiel :		
	langsam fliessender Strom		Wasserfall
•	inkompressible	und	kompressible Medien
	ρ = konstant		$\rho \neq$ konstant
	Beispiel :		
	Flüssigkeit (annähernd)		Gas
•	nicht viskose Medien	und	viskose Medien
	ohne innere Reibung		mit innnerer Reibung

Summary Hydrostatics

(mechanical properties of liquids)

(S2)
slide 2

- def Pressure P

$$P = \frac{F}{A} \quad \frac{\text{force}}{\text{area}}$$

$$[P] = \frac{N}{m^2} = Pa, \text{ Pascal}$$

P: scalar / depends on magnitude of force
Force: vector

Compressibility ?

change of volume (upon exerting a given change of pressure ΔP) ΔV

relative change of volume $\Delta V/V$

- def compressibility relative decrease of volume per unit pressure:

$$\text{compressibility} = -\frac{\Delta V/V}{\Delta P}$$

$$1/B = -\frac{\Delta V/V}{\Delta P}$$

$$B = -\frac{\Delta P}{\Delta V/V}, \quad \text{bulk modulus (def.)}$$

gases have large compressibility
liquids, solids have low compressibility

} microscopically: intermoleculare - atomic distance changes

- Pascal's principle:

A pressure change applied to confined fluid (in a container) at rest is transmitted (without loss) to every point in the fluid and to the walls of the container.

→ example of application hydraulic ~~press~~ press, or lift

Buoyancy, buoyant force (Auftrieb)

Archimedes' principle:

A body (dense, no water can get in) immersed in a fluid is buoyed up by a force equal to the weight of the displaced fluid

(S3) Crown & Au nugget (Tipler Fig. 13-13)

L13 **Hydrostatics** final part: surface tension & capillarity

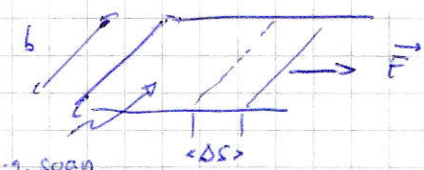
3.11.2016
2.11.2017

surface tension, ~~reminder~~

(slide): forces at surface, $\sum \vec{F}_i \neq 0$, cohesion forces in liquid

draw on board (S4) in liquid: $\sum \vec{F}_i = 0$ (⇓ net surface tension)

def:



e.g. soap film in frame

work: $\Delta W = F \cdot \Delta s$ (~~F = σ·b~~)

$= \sigma \cdot b \cdot \Delta s$

↗ def

"mechanical" point of view ⇒ ‖ $\sigma = \frac{F}{b}$, surface tension, (force per unit length) $[\sigma] = \frac{N}{m}$

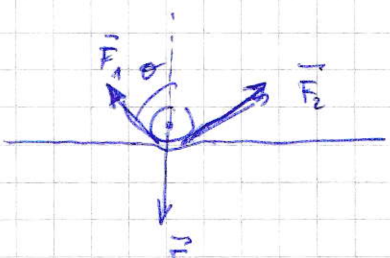
"thermodynamic" point of view or ‖ $\sigma = \frac{\Delta W}{\Delta A}$, $\Delta A = b \cdot \Delta s$, surface, $[\sigma] = \frac{J}{m^2} = \frac{N \cdot m}{m^2} = \frac{N}{m}$

(energy per unit surface) interfacial energy

(macroscopic manifestation of molecular interactions)

exp: floating needle (or trombone): probe interface (liquid-vapor) energy

symmetric



$\vec{F}_g = \vec{F}_1 + \vec{F}_2$

$m \cdot g = F_1 \cdot \cos\theta + F_2 \cdot \cos\theta$

$= 2\sigma \cdot l \cdot \cos\theta$

↗ $F = \sigma \cdot l$, $F_1 = F_2$

contact length

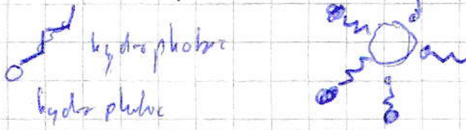
($\vec{F}_1, \vec{F}_2$: resultant forces due to intermolecular and interface forces
$\vec{F}_g$: gravitation)

⇒ ‖ $\cos\theta = \frac{mg}{2\sigma l}$, valid only if ($\cos\theta \leq 1$) $mg \leq 2\sigma l$

floating condition

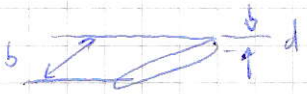
(slides) Gerris

| add surfactant / soap: amphiphilic molecule (→ wetting)



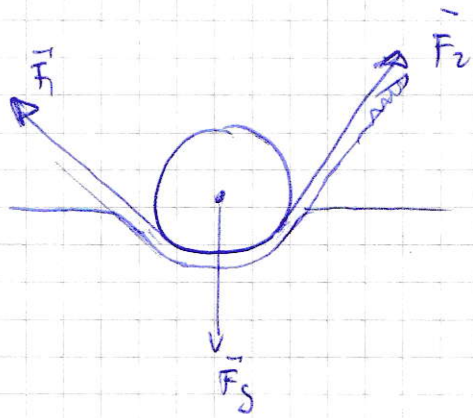
1) the surface tension is an attractive force, resulting from intermolecular interaction.

2) this argument assumes an infinitesimally thin liquid film (monoatomic)

⇒) if thickness d 

, take into account the perimeter of the interface $\Rightarrow 2b + 2d \approx 2b$ if $b \gg d$

3) estimate σ, thermo → cf notes next page



tangent to liquid

surface tension : $\sigma = \frac{\Delta W}{\Delta A}$ $\frac{\text{energy}}{\text{area}}$

energy arising due to presence of an interface between 2 bulk phases (liquid, gas here)
vapor

order of magnitude ...? thermodynamics argument

- microscopic view : intermolecular interaction ~ surface tension
- units ?

$\frac{\text{bond energy (between molecules)}}{\text{area (cross-section of molecule)}} \sim \sigma$

bond energy : $\sim k_B T = 25\,meV$ at R.T

cross section : $\sim 1\,nm = 10^{-9}\,m$

$\Rightarrow \sigma \sim 0.004 \frac{N}{m}$

$\sim 0.016 \frac{N}{m}$ for $(0.5 \cdot 10^{-9})^2$ cross section

$\sigma_{water} \simeq 0.07 \frac{N}{m}$; a bit higher ; bond energy > kT
0.0729

$\sigma_{water}(80°C) \simeq 0.0626 \frac{N}{m}$

$\sigma_{ethanol} \simeq 0.022 \frac{N}{m}$

exp : measure surf. tension (cf next page, description)

exp : needle, go back to previous page

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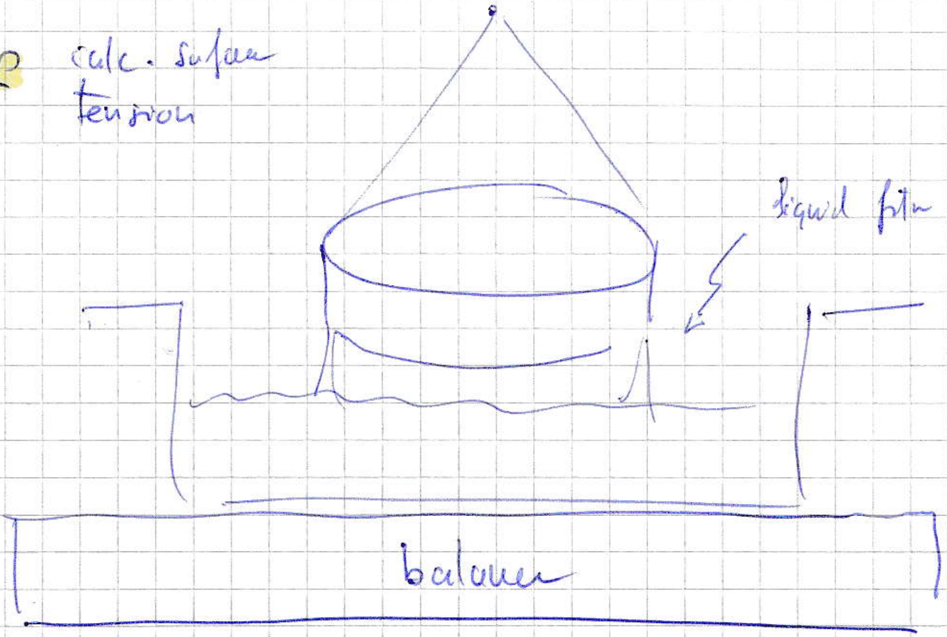
1°°

exp calc. surface tension



water : ~ 2.8 gr (?)

ethanol ~ 1.8 gr (?)

Calculate : ϕ ring 63 mm , $r = 31.5 \cdot 10^{-3}$ m

m of ring not important

$$2 \cdot 2\pi r \cdot \sigma = m \cdot g$$

↗ such film ⇒ 2 interfaces air-liquid

↑ measured on balance (zero'ed when ring in liquid)

$$\Rightarrow \sigma = \frac{m \cdot g}{4\pi \cdot r}$$

($g = 9.81 \frac{m}{s^2}$, gravitation acceleration)

(= m . 24.78)

$\sigma_{water} \approx 0.073 \frac{N}{m}$

$\sigma_{ethanol} \approx 0.022$

$m_{water} \approx 3$ gr $\Rightarrow \sigma_{H_2O} \approx 0.074 \frac{N}{m}$

$m_{ethanol} \approx 1.0$ gr $\Rightarrow \sigma_{ethanol} \approx 0.024 \frac{N}{m}$

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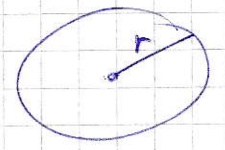
exp: surface tension with circle (piece of wire in soap film)
→ minimize energy → circle

Droplets of liquid

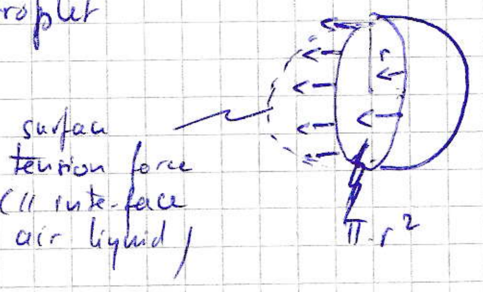


2 counteracting effects: pressure → "repulsion", "explode" droplet
surface tension → attraction, keep molecules of liquid together at interface

(~~[illegible]~~

relate surface tension and pressure:

Consider 2x 1/2 of the droplet



$$2\pi \cdot r \cdot \sigma = \pi r^2 \cdot p$$

↑ σ: surface tension (N/m) ↑ p: pressure (N/m²)

$$\Rightarrow \quad P = \frac{2\sigma}{r}$$

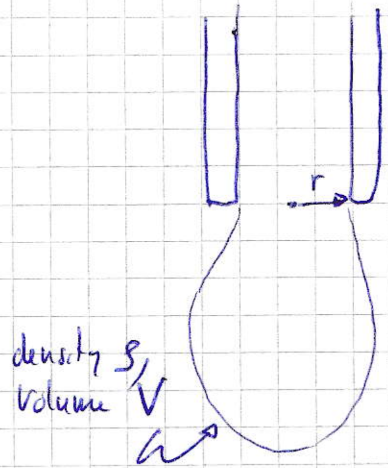
↑ P: pressure in droplet

exp: water drops in oil (paraffin)
formation of drops: see next page for calculation

2.11.2017

(1')

drop formation (gravitation)



in equilibrium: effect of gravity compensated by surface tension

$$F_g = F_{surf.}$$

$$m \cdot g = 2\pi r \cdot \sigma$$

$$\rho \cdot V \cdot g = 2\pi r \cdot \sigma$$

$$\Rightarrow \quad V = \frac{2\pi r \cdot \sigma}{\rho \cdot g}$$

(V: volume of droplet)

(exp: drops of ethanol / water } count drops ⇒ not same volume for same amount of drop

stalagmometer — 20 drops of each

<u>Capillary action</u> : results from adhesion and surface tension
(remember: effect of surfactant on needle)

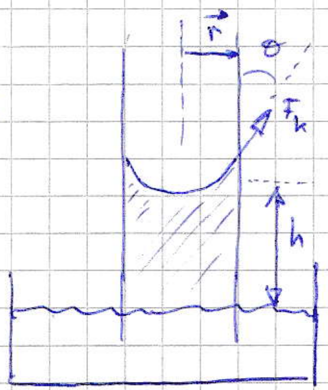
(slide) compare adhesion / cohesion ; $\sigma = \frac{\Delta E}{\Delta A}$ interface energy / surface tension $\left(\frac{N \cdot m}{m^2} = \frac{J}{m^2} \right)$

1) adhesion to walls of tube stronger than liquid cohesion (cohesive force between liquid molecules)

$F_{adh} > F_{coh}$

$0 < \theta < \frac{\pi}{2}$

<u>observed</u>
liquid "climbs" along walls

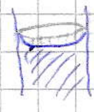


$F_g = F_k \cdot \cos\theta$
gravitation capillarity

$\rho \cdot (\pi r^2 \cdot h) \cdot g = \sigma \cdot (2\pi r) \cdot \cos\theta$

cylinder volume

surface tension acts on circular liquid/air edge

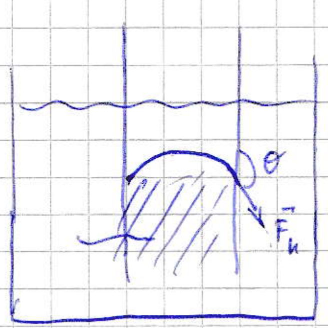


molecules of liquid more attracted to glass than to each other

$\Rightarrow \quad h = \frac{2 \cdot \sigma \cdot \cos\theta}{\rho \cdot g \cdot r}$

small diameter/radius, h ↗ larger

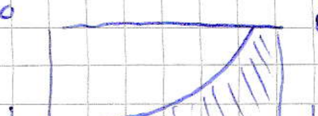
2) $F_{adh} < F_{coh}$



$\frac{\pi}{2} < \theta < \pi$

<u>exp</u> : capillarity show a) height in tube, effect of ⌀
H_2O, Hg b) meniscus shape (wetting / non wetting)

<u>exp</u> : 2 glass pieces with decreasing separation



d: interplate distance

NB importance of surface homogeneity

surfactants : amphiphilic molecules