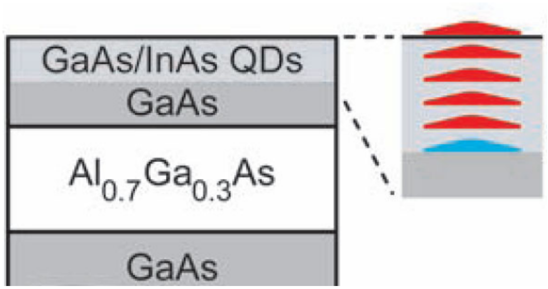
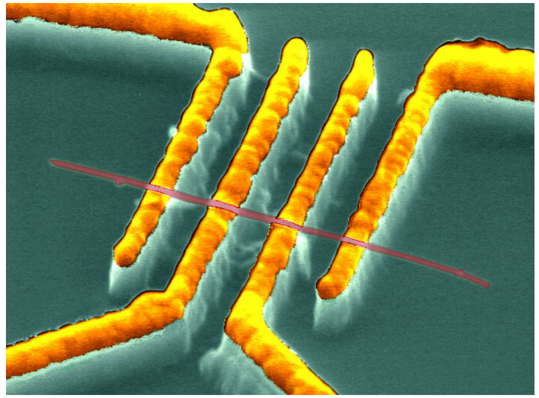
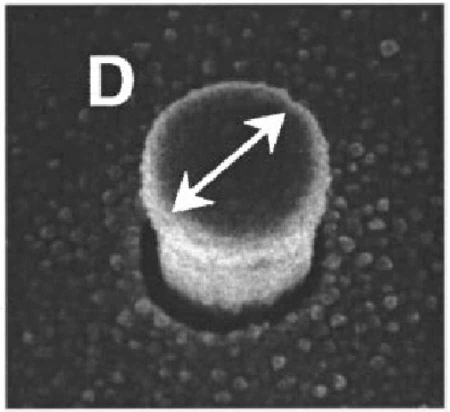
Quantum Dots



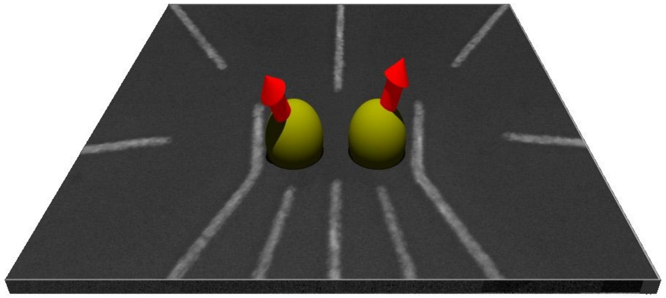
MBE grown



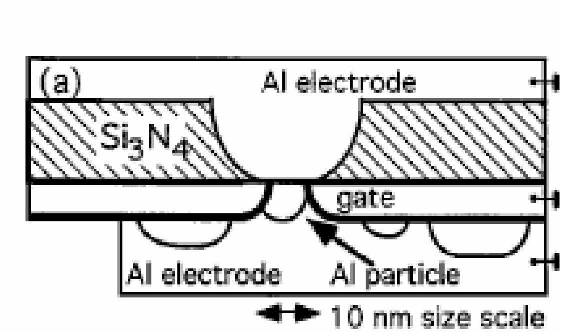
nanotube



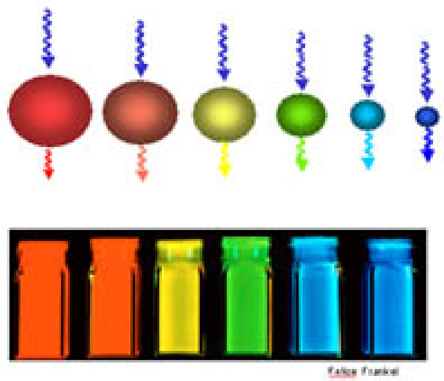
vertical dot



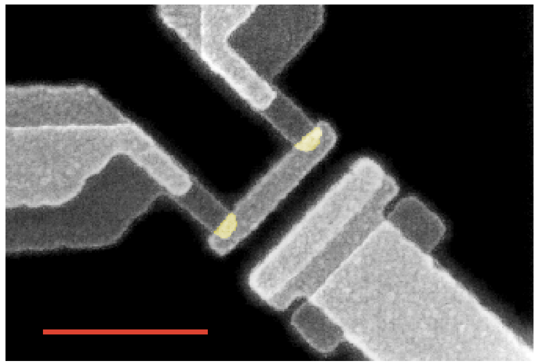
lateral



metal grain

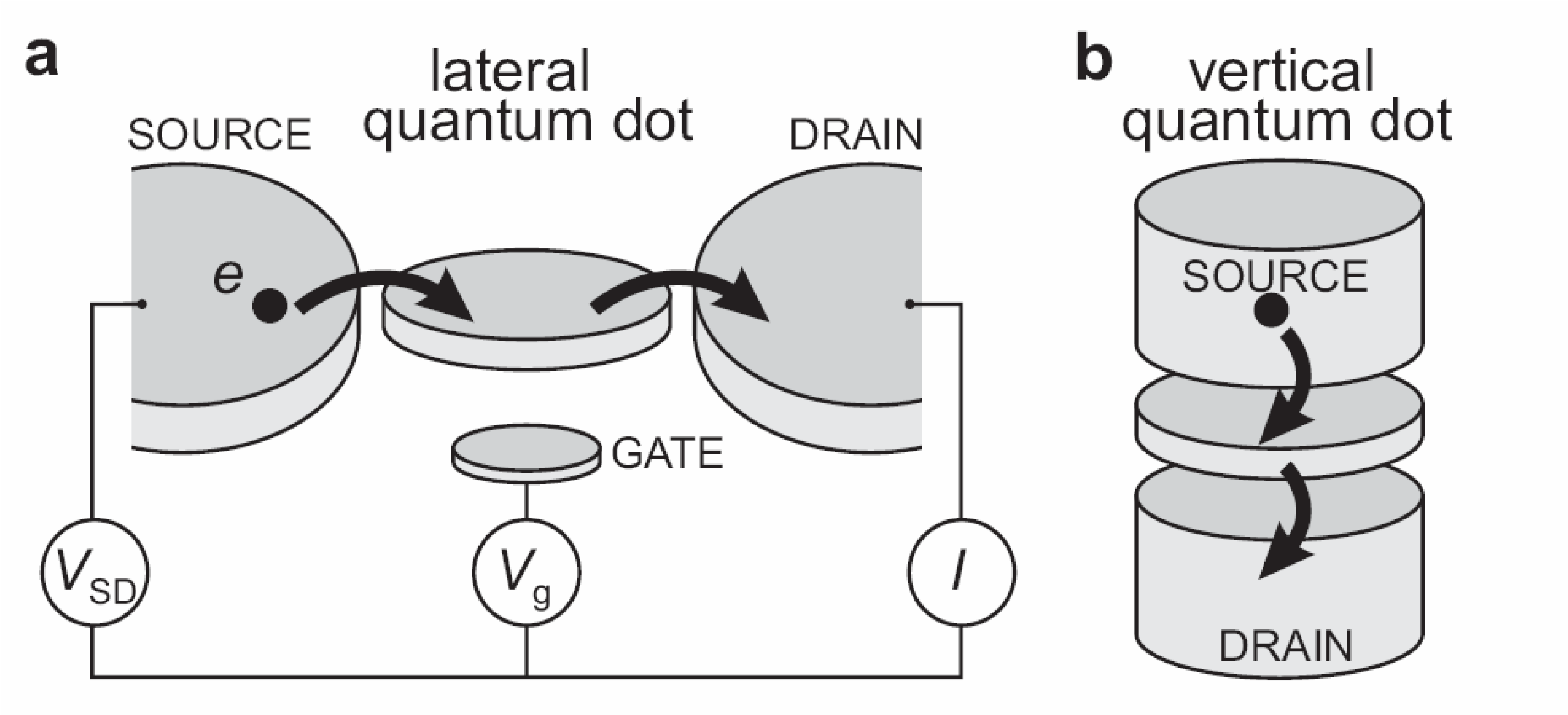


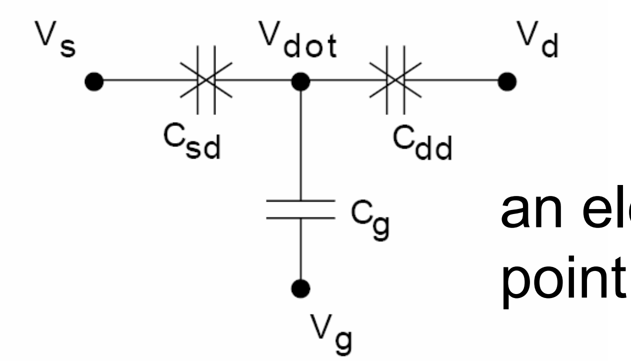
self assembled



metallic SET

lateral vs. vertical

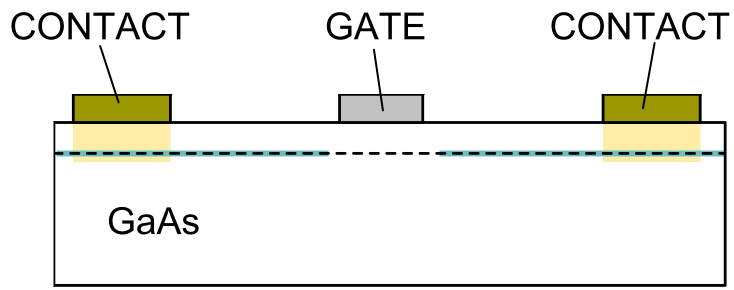


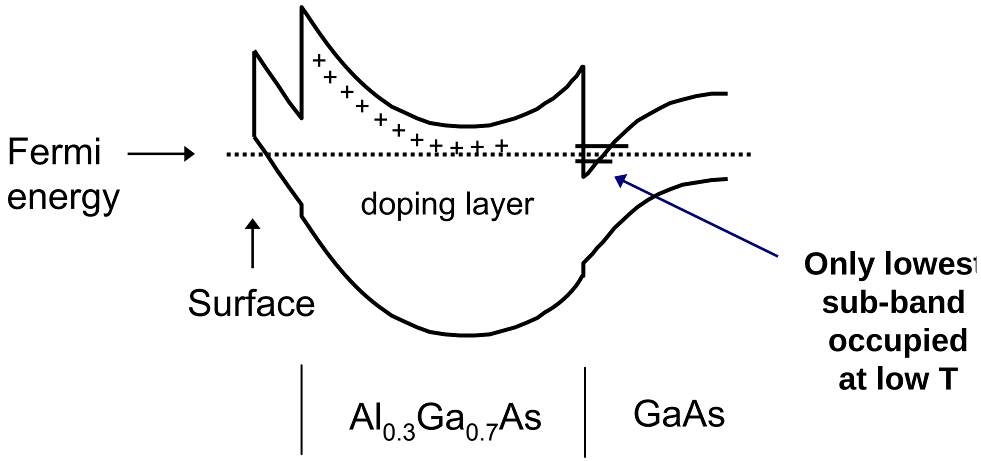


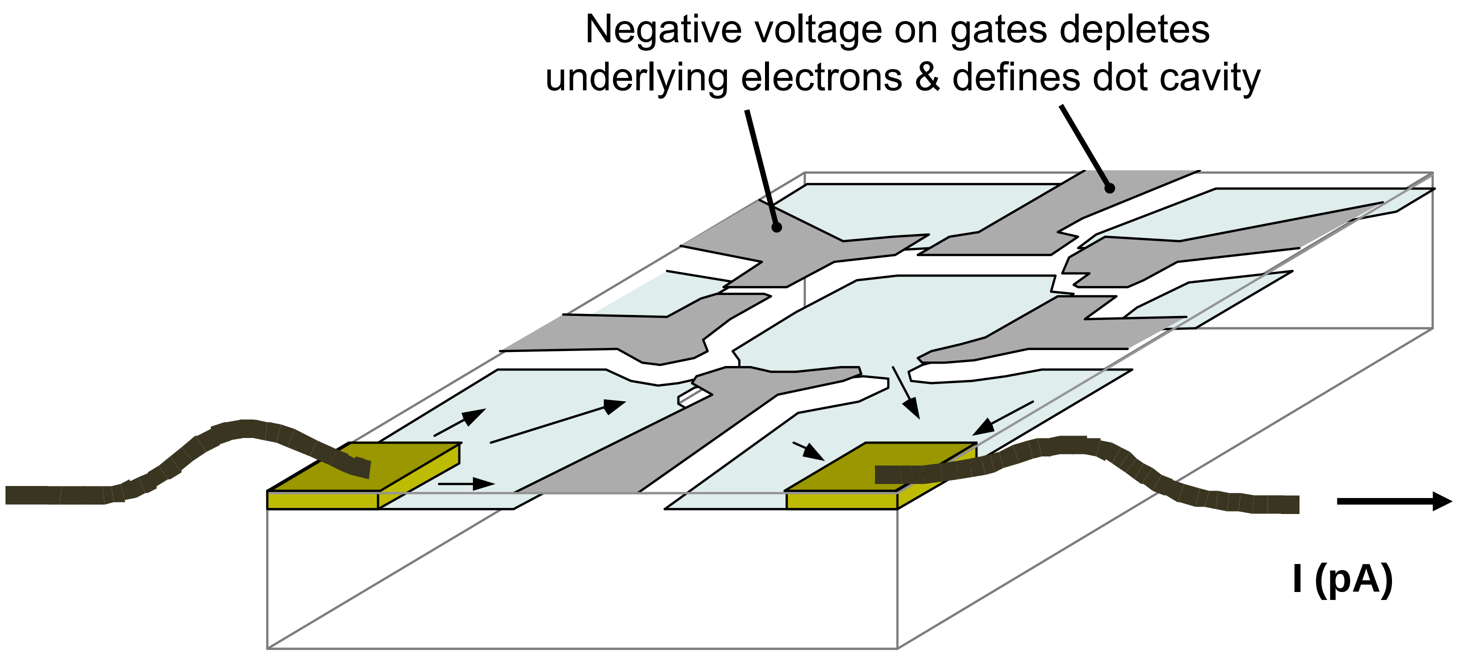
an electrical engineers
point of view

Lateral Dots: Formed in GaAs/AlGaAs 2DEG

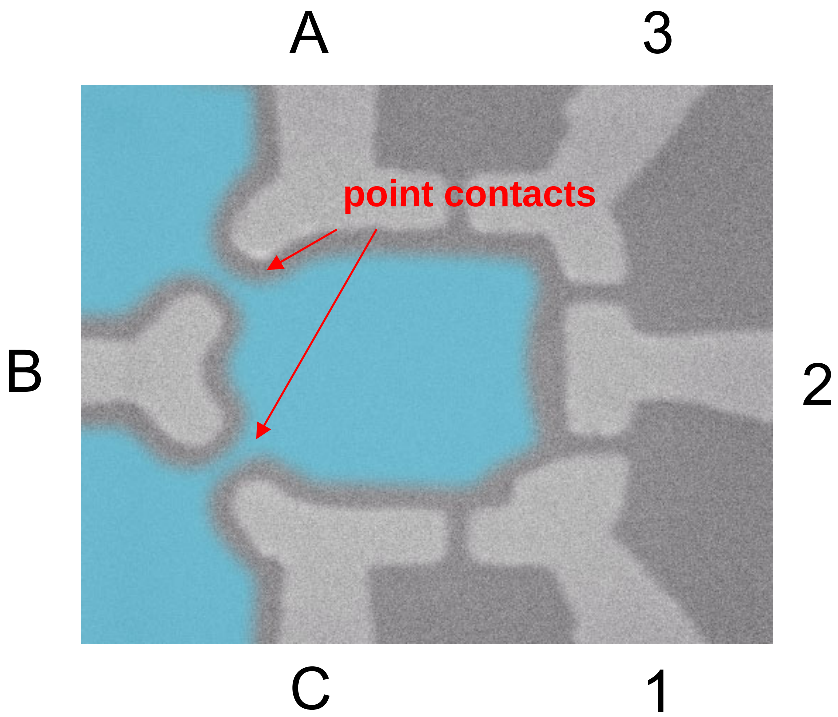
Electrons travel in sub-surface layer:

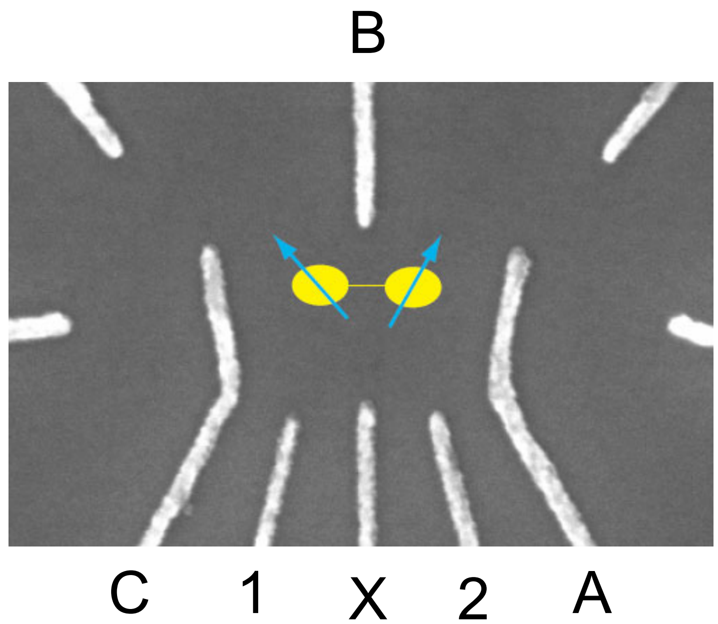






gate defined dots



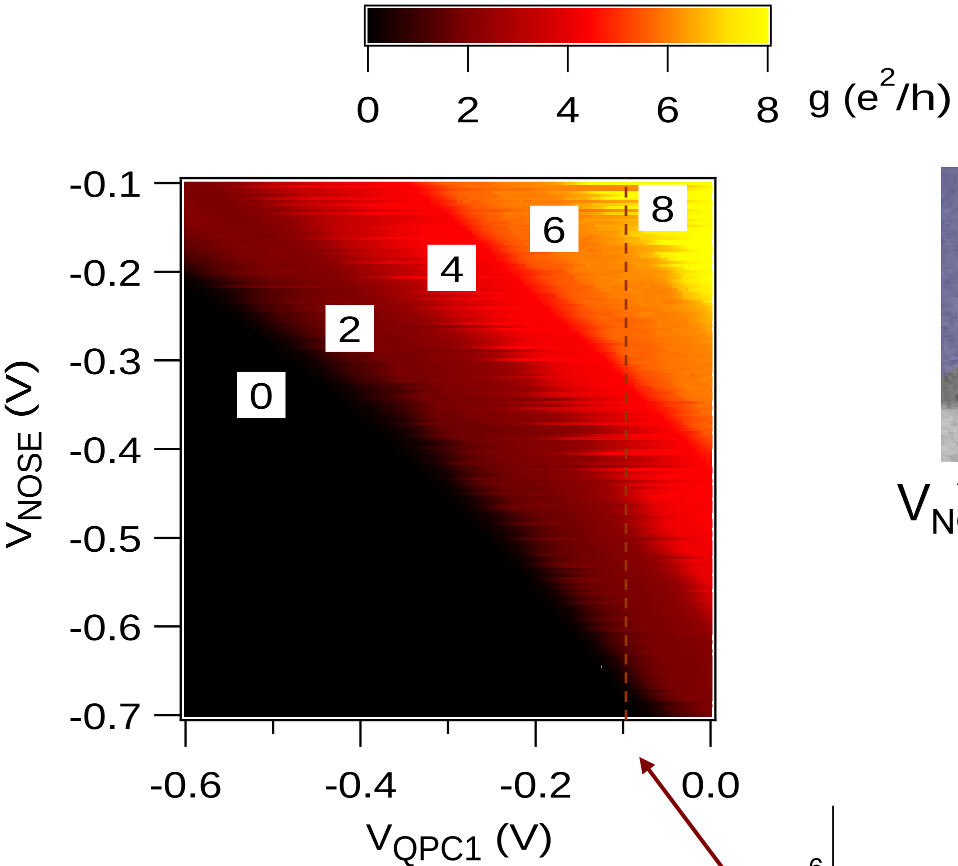


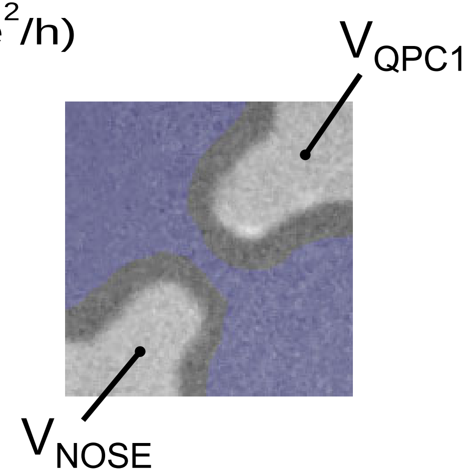
A,B,C : control quantum point contacts
transmission to reservoirs

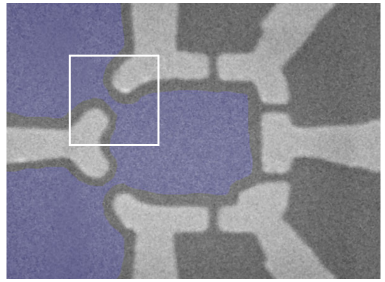
1,2,3: control confinement potential / energy levels only

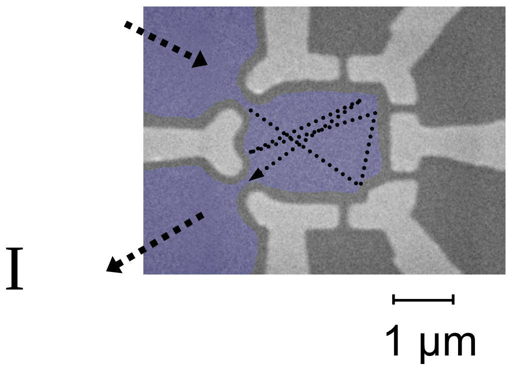
X control dot-internal tunneling rate

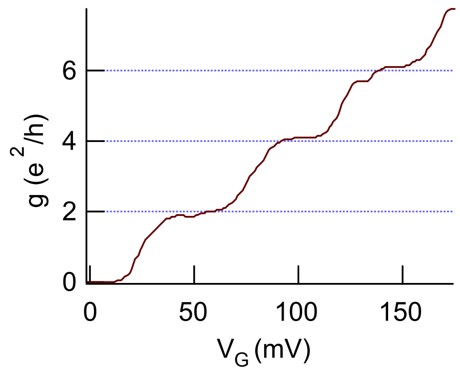
Quantum Point Contact Leads



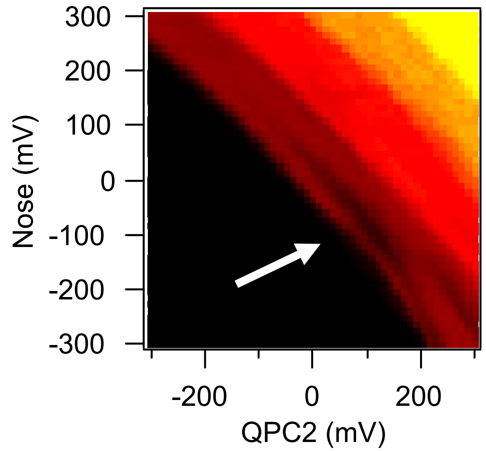








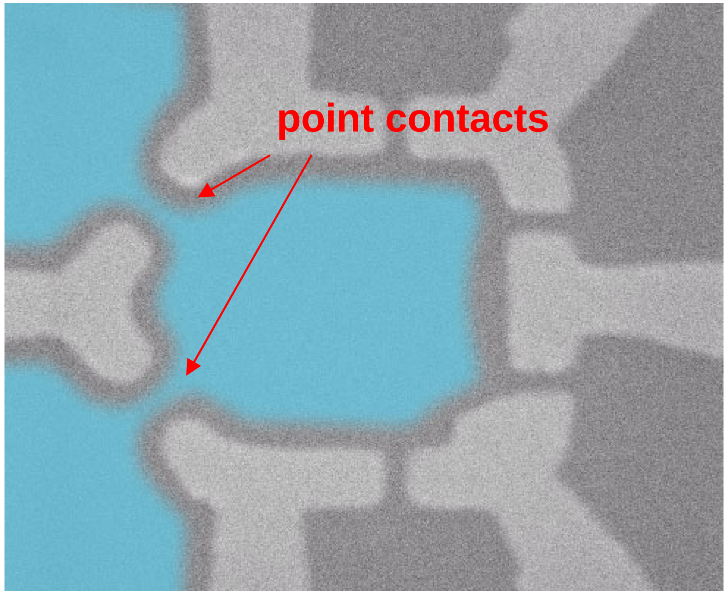
Not all QPCs are perfect:



slide from A. Huibers, Thesis (1999)

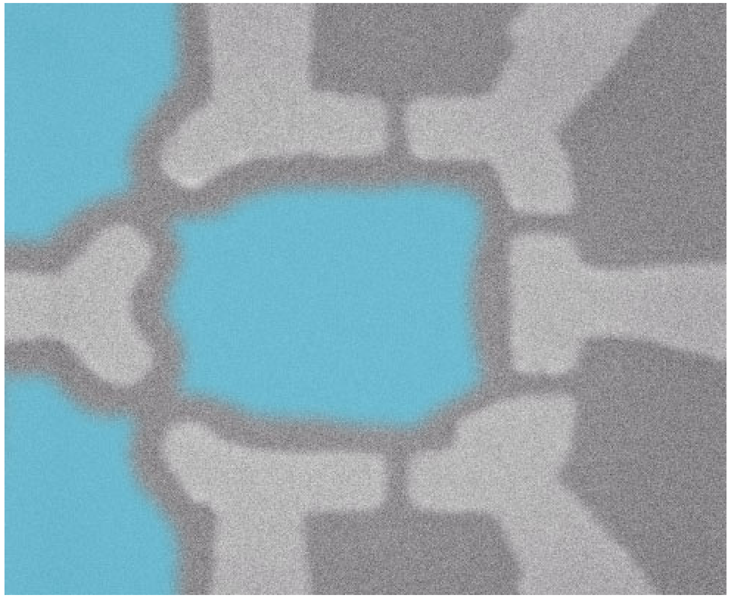
Open vs. Closed

Open Dot



- V_{gate} set to allow ≥ $2e^2/h$ conductance through each point contact
- Dot is well-connected to reservoirs
- Transport measurements exhibit CF and Weak Localization

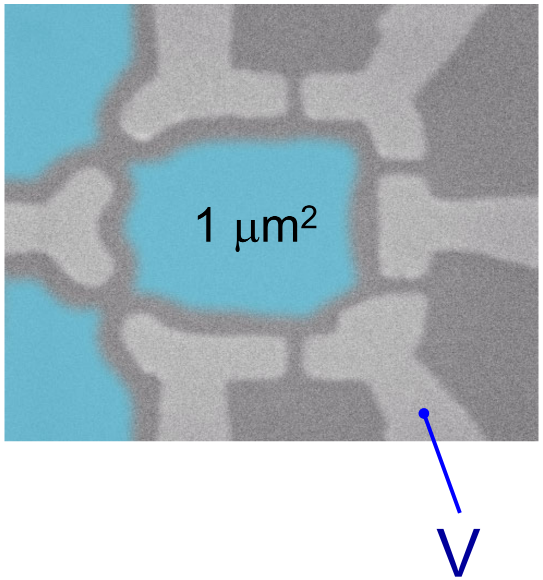
Closed Dot



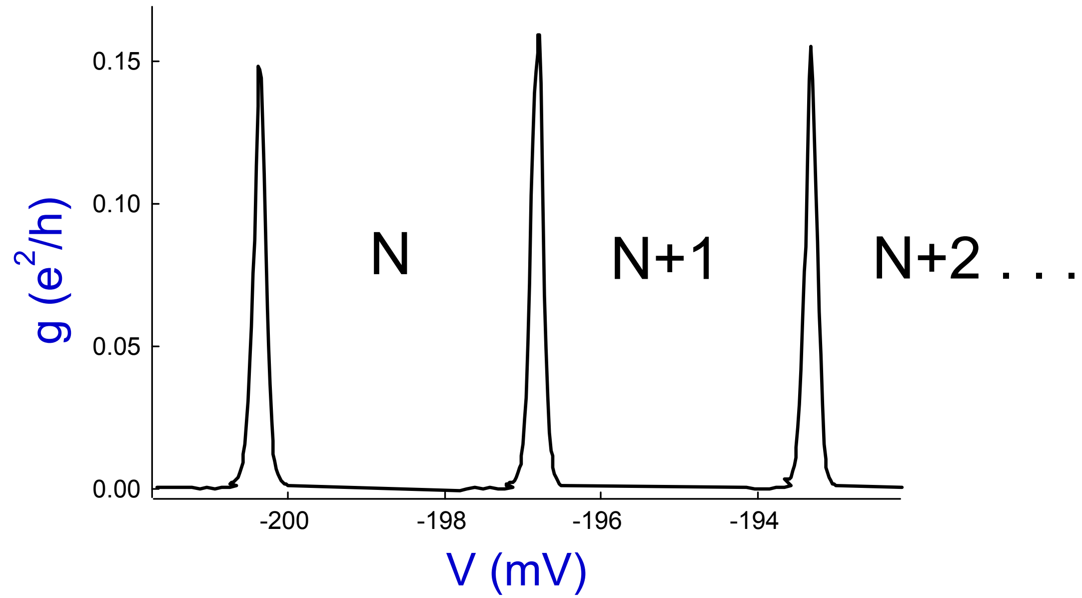
- V_{gate} set to require tunnelling across point contacts
- Dot is isolated from reservoirs, contains discrete energy levels
- Transport measurements exhibit Coulomb Blockade

Coulomb Blockade in Closed Dots

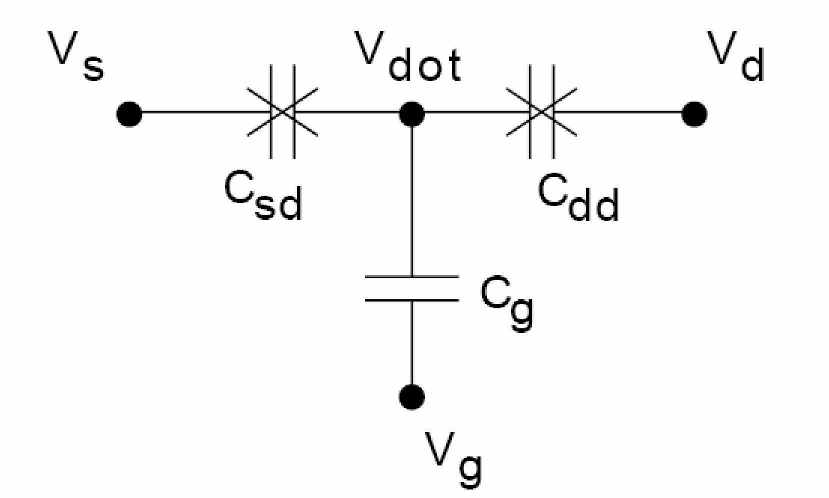
Finite energy $E_c = e^2/C_{dot}$ is needed to add an additional electron to the dot. When $kT \ll E_c$ charging blocks conduction in valleys.



Coulomb blockade peaks:
resonant transport through dot levels



Electrostatic Energy



$$C_\Sigma = C_{sd} + C_{dd} + C_{g1} + C_{g2} + \ldots$$

$$\alpha_i = \frac{C_i}{C_\Sigma}$$

apply voltages

what is potential on dot?

voltage divider...

$$V_{dot} = \sum_i \alpha_i V_i$$

can use V_g to shift dot energy!!

Charging Energy

capacitance of dot to world = C

$$C = \epsilon_0 \epsilon \frac{A}{d}$$

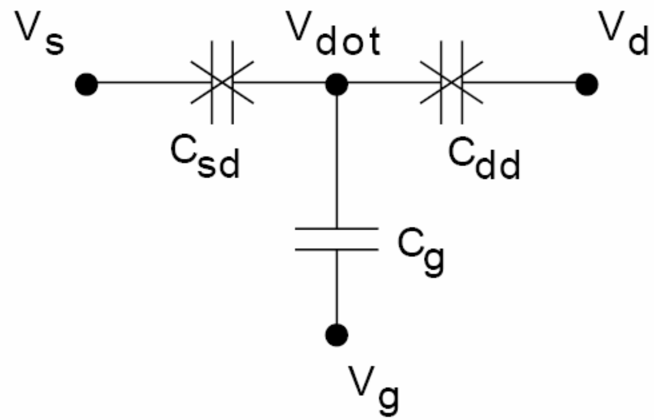
energy stored in capacitor

$$U = \int_0^Q V\,dq = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{C}$$

charging energy

$$E_C = \frac{e^2}{C_\Sigma}$$

can range from
~0 to many meV

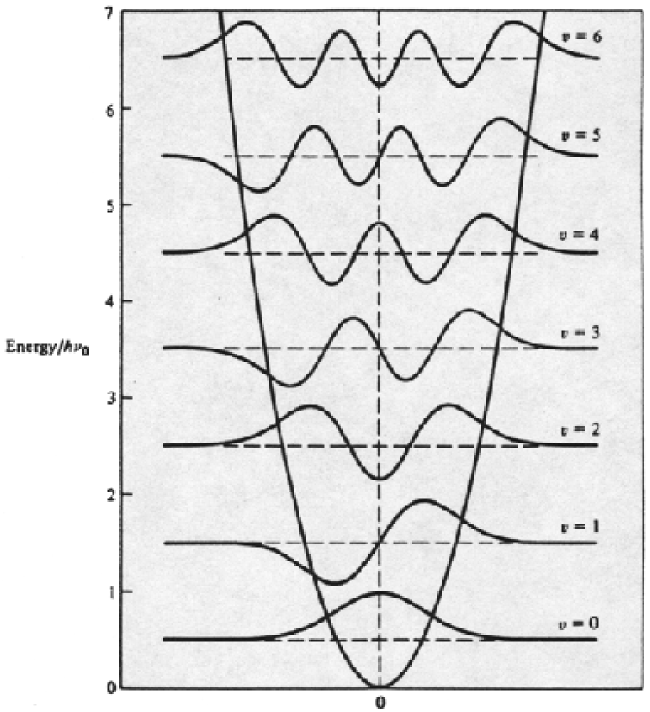


$$C_\Sigma \gtrsim 10\,\mathrm{aF}$$

Classical Effect, NOT quantum

Confinement Energy

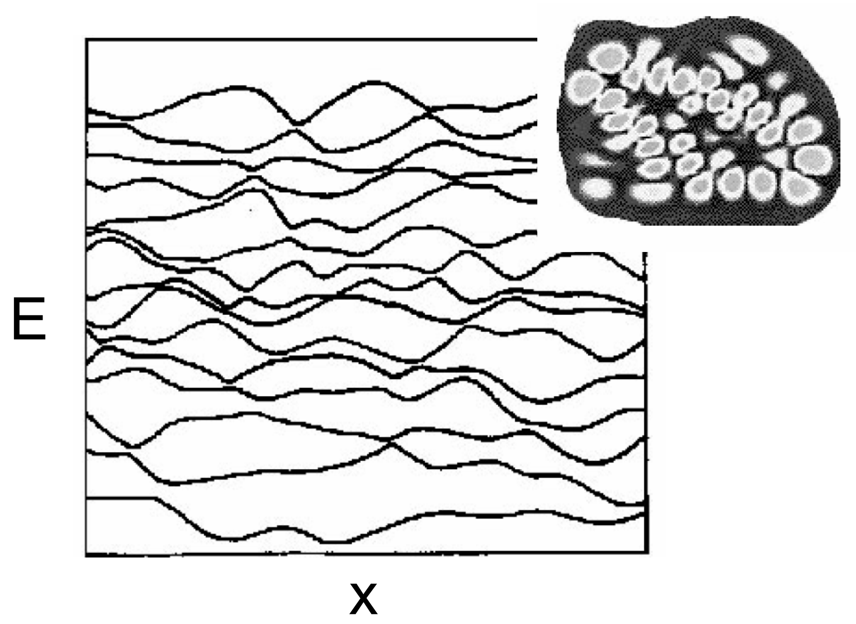
harmonic potential



$$E_n = \left[n + \frac{1}{2}\right]\hbar\omega$$

μeV to meV

complicated potential



average level spacing

$$\Delta = \frac{2\pi\hbar^2}{m^* A}$$

quantum mechanical effect!!

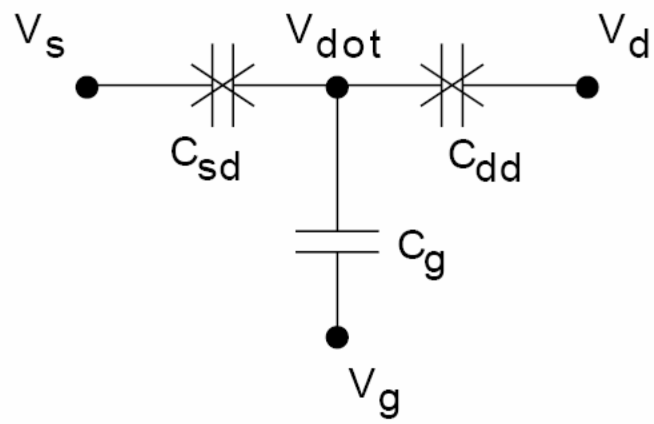
Capacitor Model

$$E(N) = [Q_{tot}]^2 / (2C_\Sigma) + \sum_{k=1}^{N} \epsilon_k$$

total dot energy

$$E(N) = \left[e(N - N_0) - \sum_{k=1}^{N} C_k V_k \right]^2 / (2C_\Sigma) + \sum_{k=1}^{N} \epsilon_k$$

offset charge



Constant Interaction Model

$$E_i = \sum_{k=1}^{N} q_k \phi_k$$

$q_k = -e$

ϕ_k : interaction of electron k with rest

constant inteaction: model ϕ_k with C_Σ

$$\phi_k = -(k-1)e/C_\Sigma$$

$$E_i = \frac{e^2}{C_\Sigma} \sum_{k=1}^{N} (k-1) = \frac{N(N-1)e^2}{2C_\Sigma}$$

$$E(N) = E_{\mathrm{QM}} + E_i + E_e$$

total dot energy

$$= \sum_{n=1}^{N} \epsilon_n + \frac{N(N-1)e^2}{2C_\Sigma} - Ne \sum_{i=1}^{6} \alpha_i \mathrm{V}_i$$

Chemical Potential / Addition Energy

$$\mu_{\mathrm{dot}}(N) \equiv E(N) - E(N-1)$$

energy to add one more electron

μ=0: change N current flows

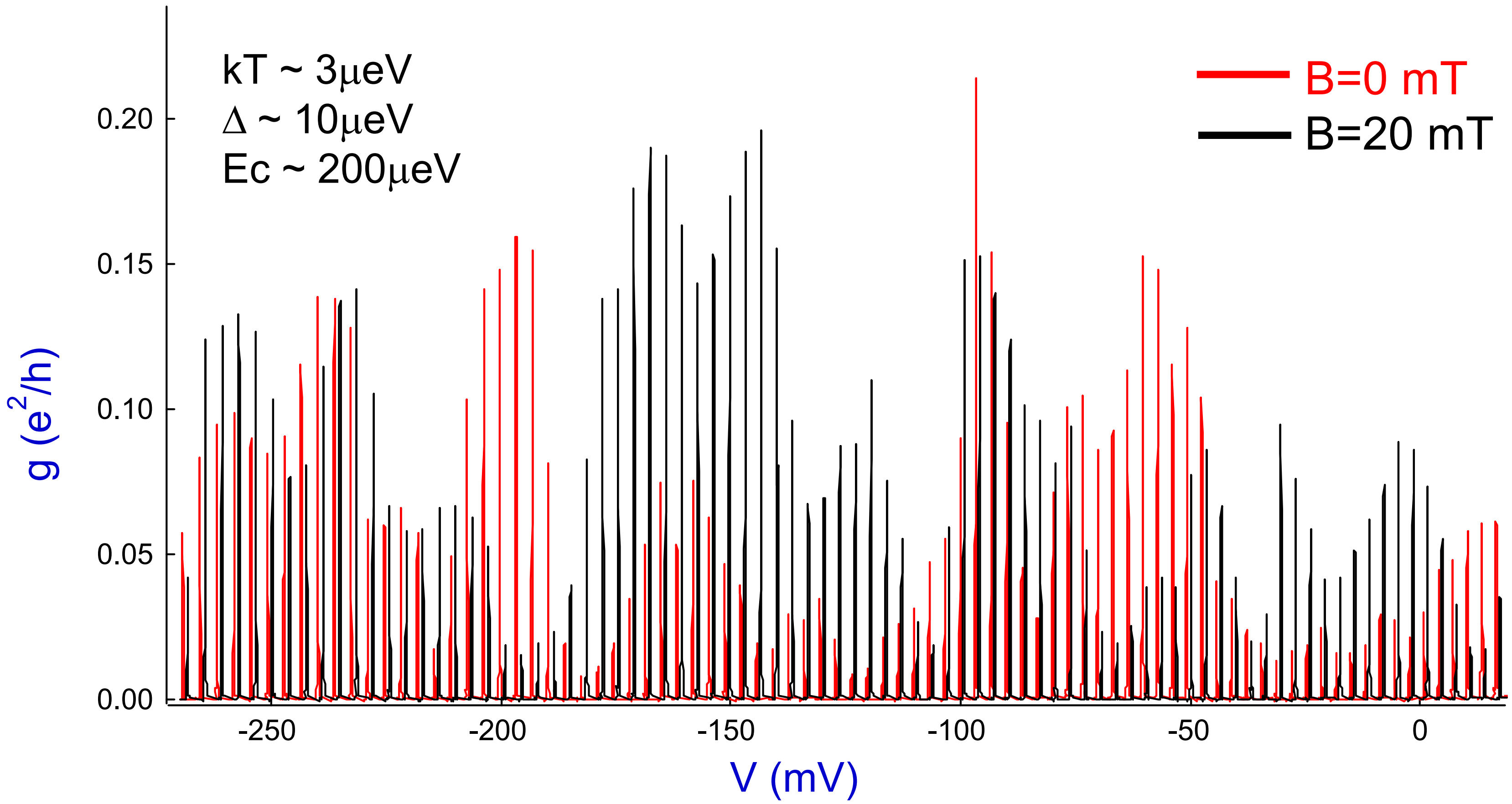
constant interaction model:

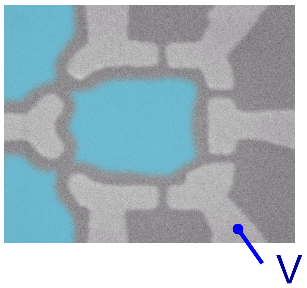
$$\mu_{dot}(N) = \epsilon_N + (N-1)\frac{e^2}{C} - e\sum_i \alpha_i V_i$$

addition energy

$$\begin{aligned}(\mu_{\mathrm{dot}}(N+1) - \mu_{\mathrm{dot}}(N))|_{\mathrm{fixed}\ \mathrm{V}_i} &= \epsilon_{N+1} - \epsilon_N + e^2/C_\Sigma \\ &\equiv \Delta\epsilon_{N\to N+1} + U\end{aligned}$$

Quantum Coulomb Blockade

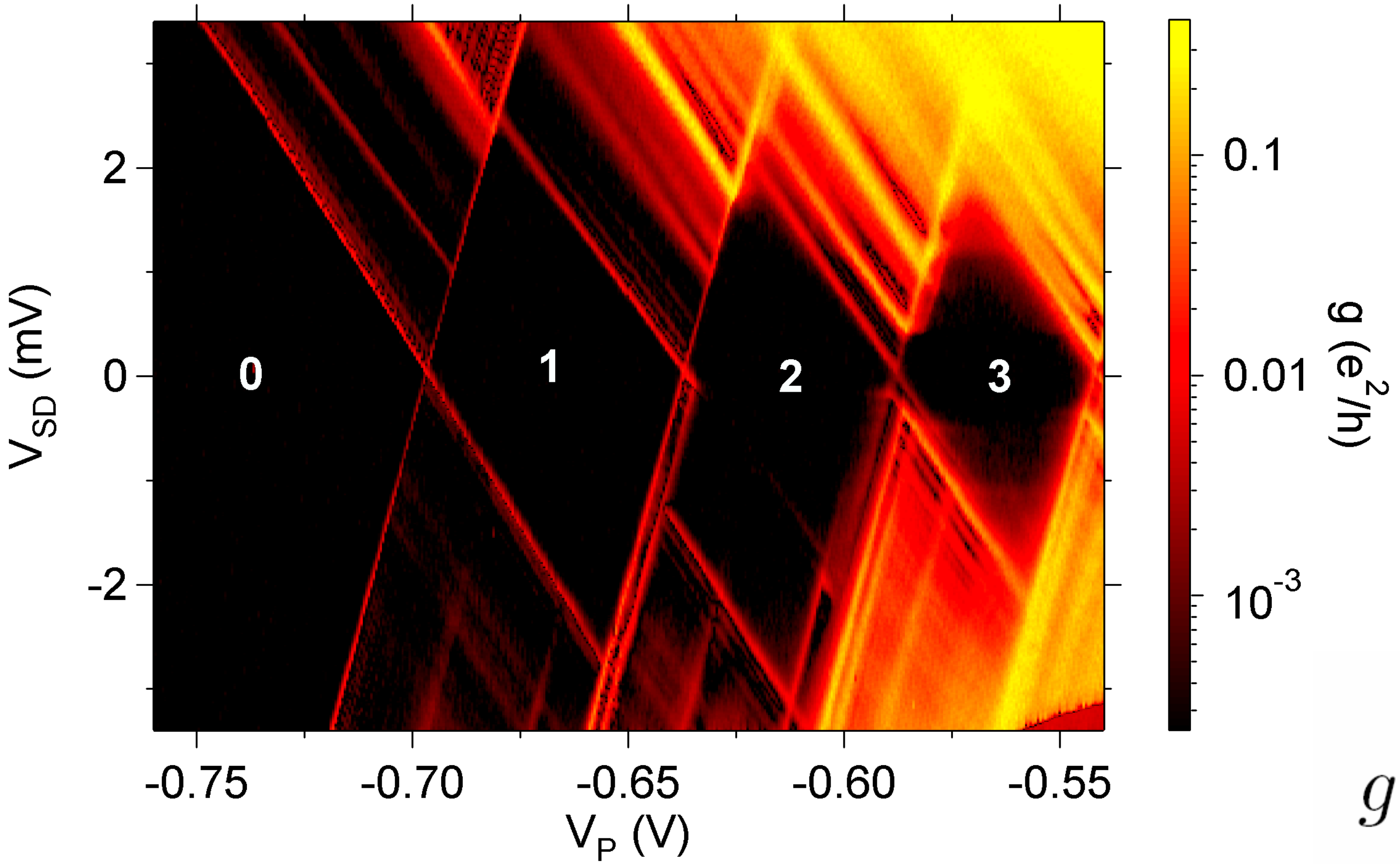




For kT < Δ, each peak describes tunnelling into a single eigenstate. Wavefunction amplitude fluctuations lead to peak height fluctuations.

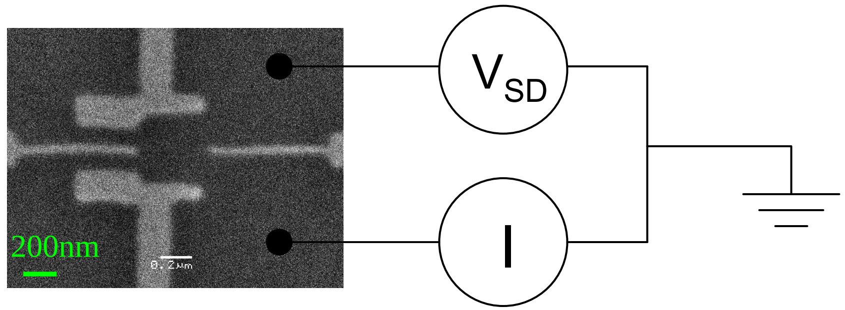
slide from J. Folk (2002)

Coulomb Diamonds



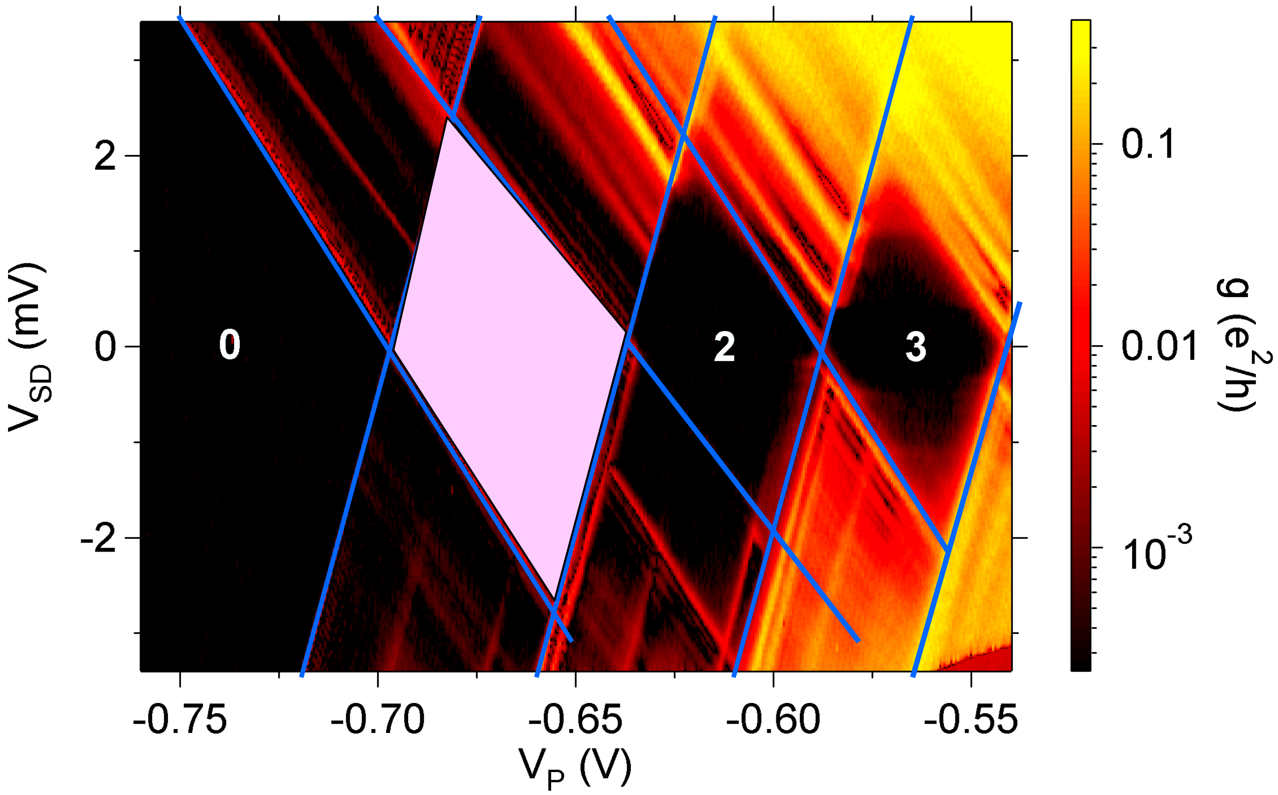
kT ~ 1.5μeV
Δ ~ 1000μeV
Ec ~ 2000μeV

$$g = \frac{dI}{dV}$$

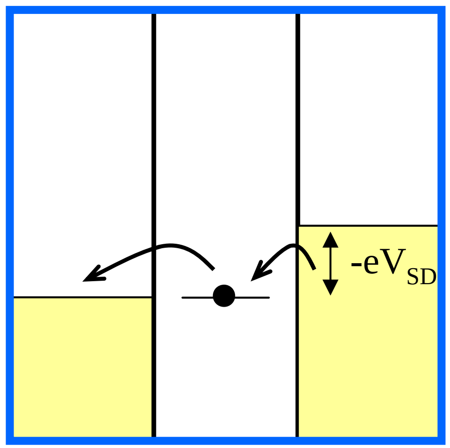


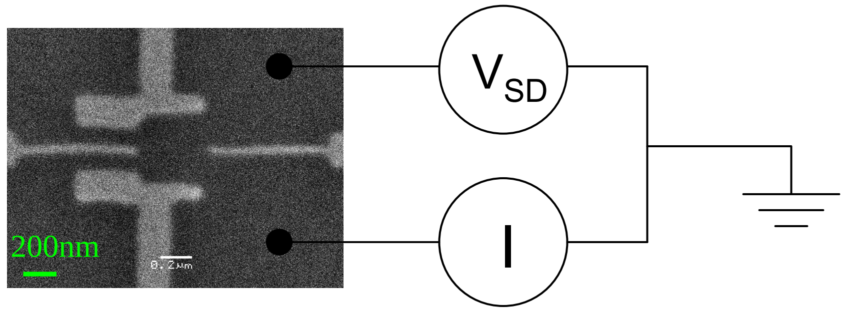
differential conductance: peaks when current through dot is changing

Coulomb Diamonds

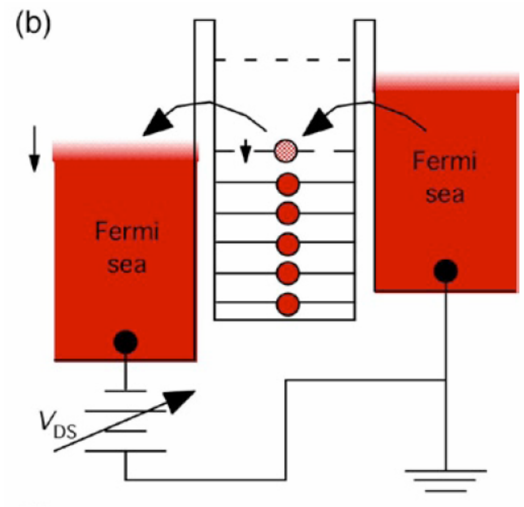


peaks in g appear when dot level aligned with either source or drain chemical potential

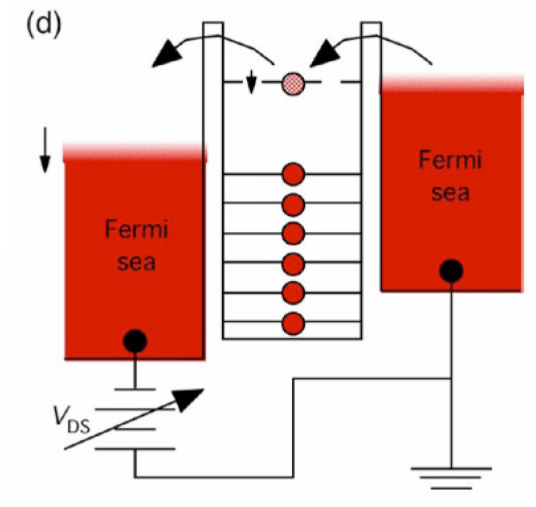


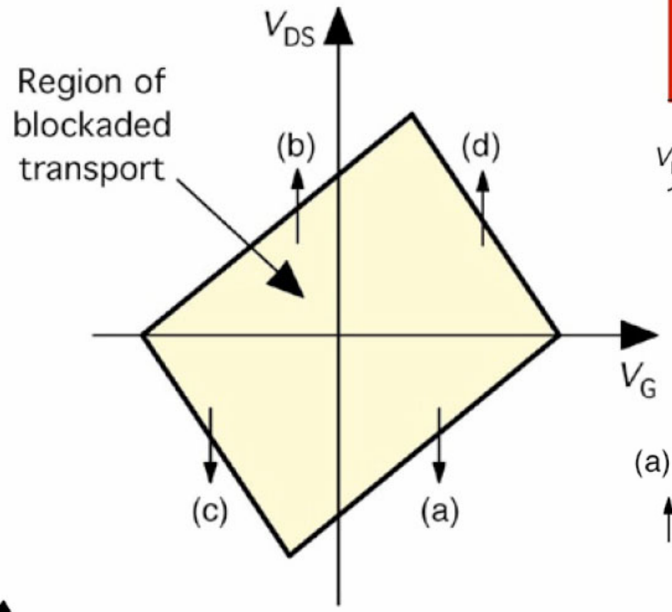


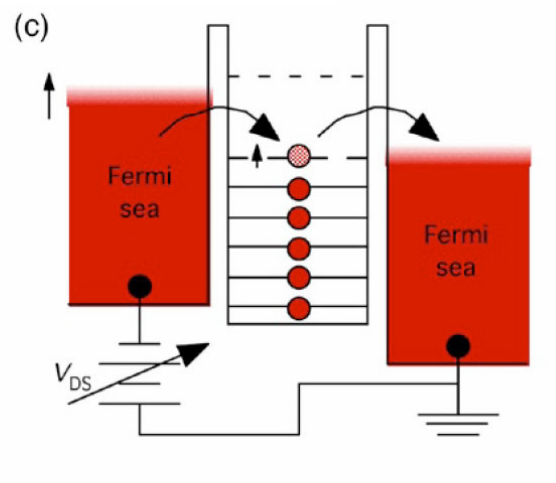
Coulomb Diamonds, Sequential Tunneling Transport



electrons tunnel on / off dot
one by one
(charging energy)

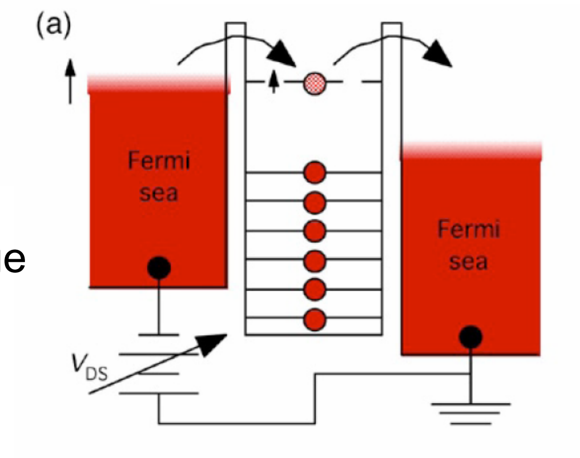




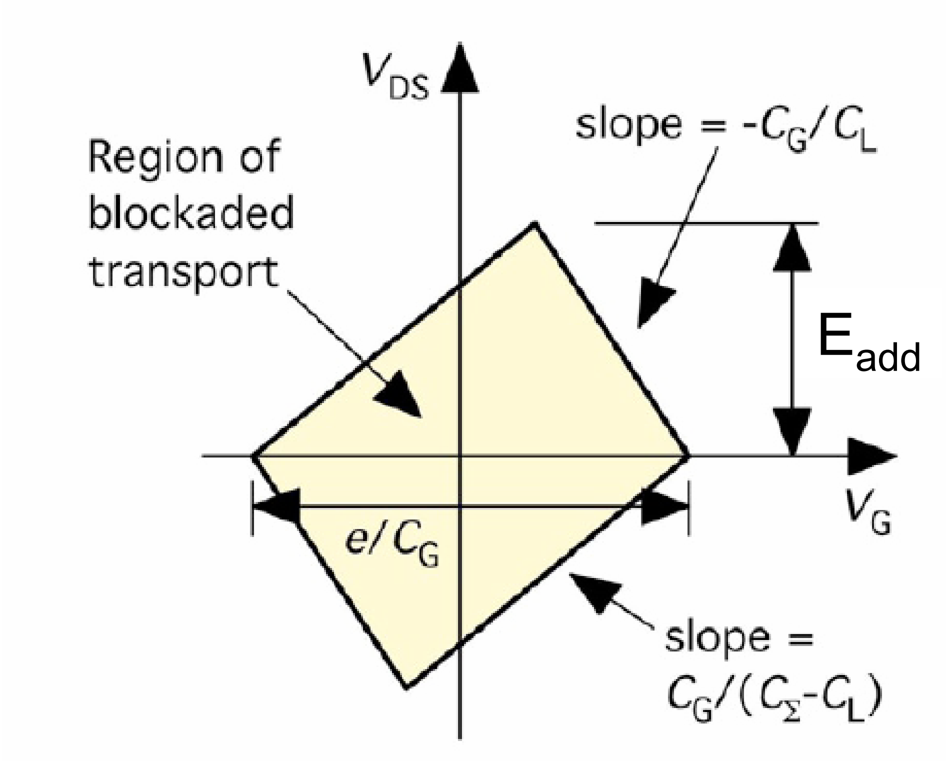


electrons do not change
energy when tunneling

dot energy unchanged

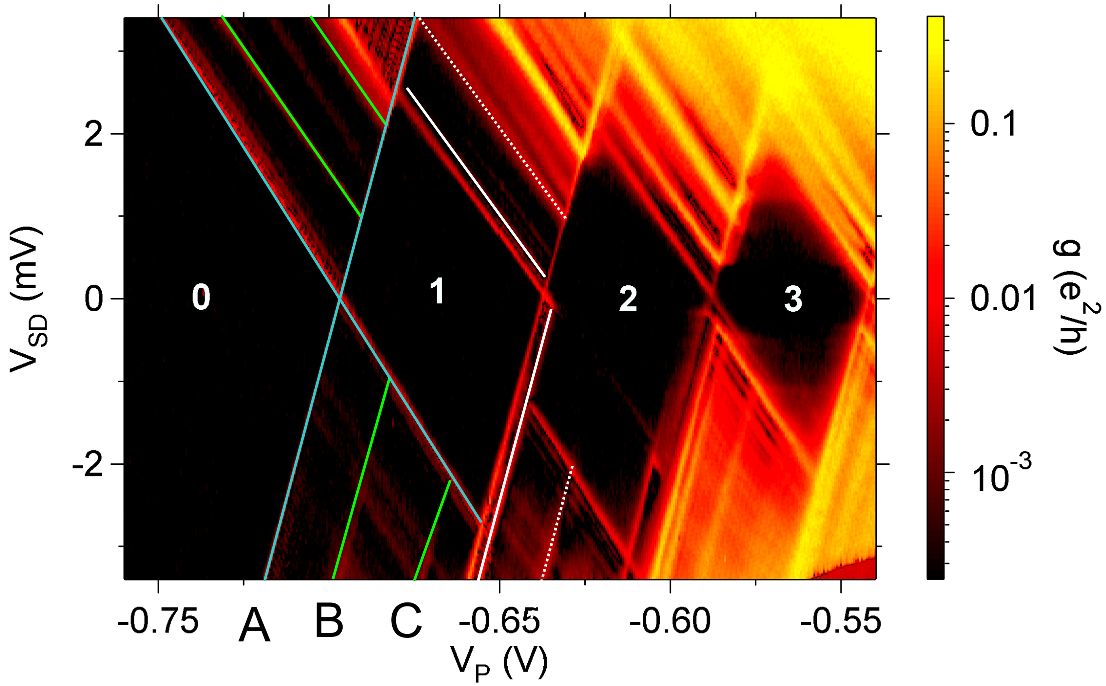


Coulomb Diamonds



two slopes, each associated with its respective dot-lead capacitance

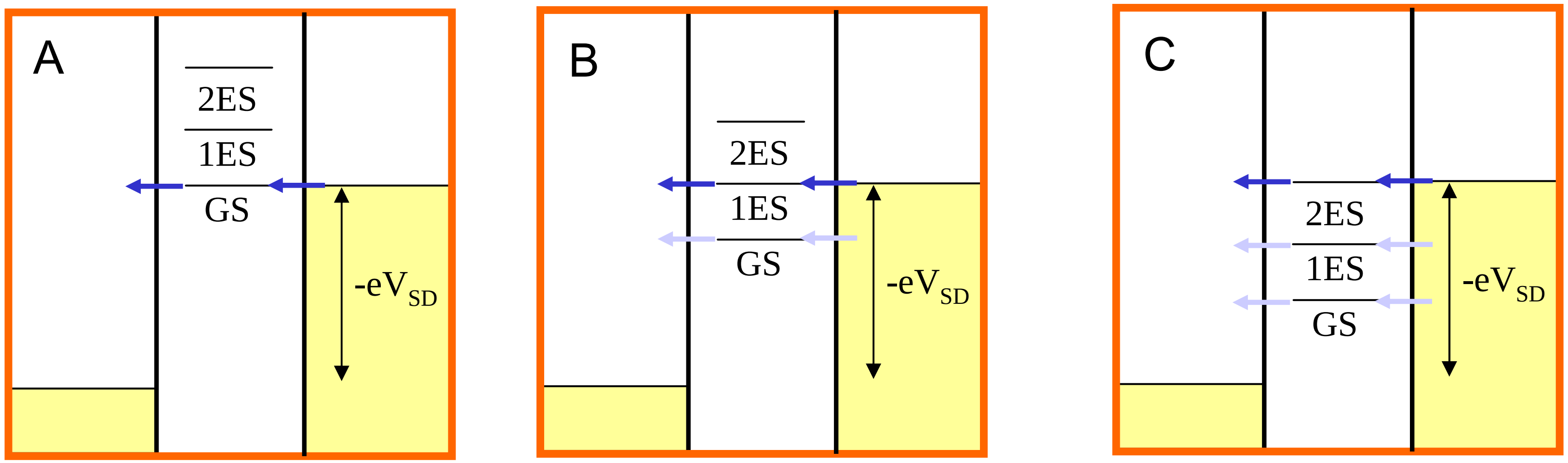
Excited State Spectroscopy: Sequential Transport



lab to investigate quantum levels in device!!

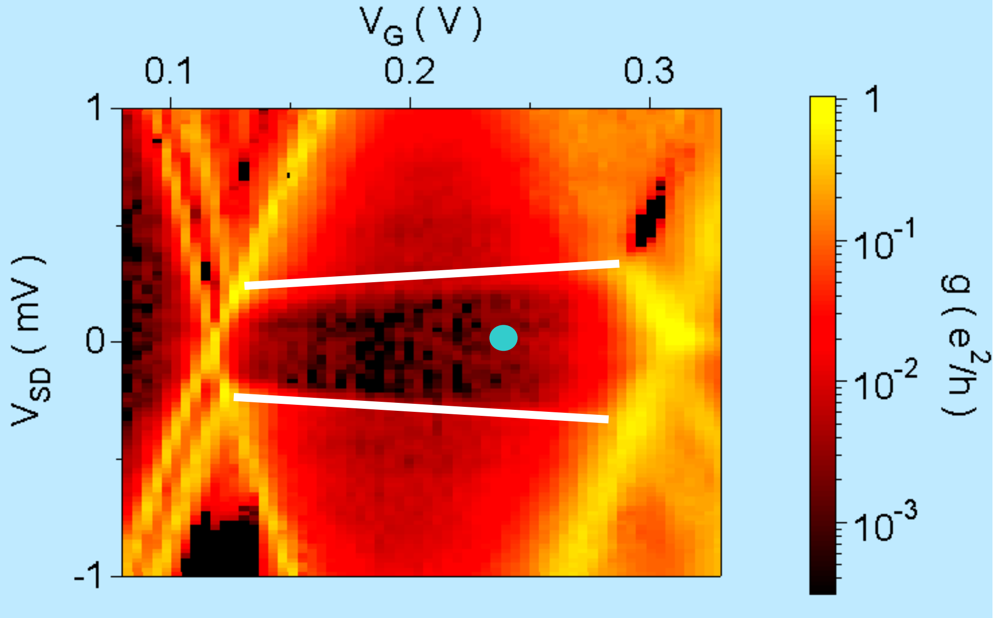
quantum confinement energies

internal excitations (spin)



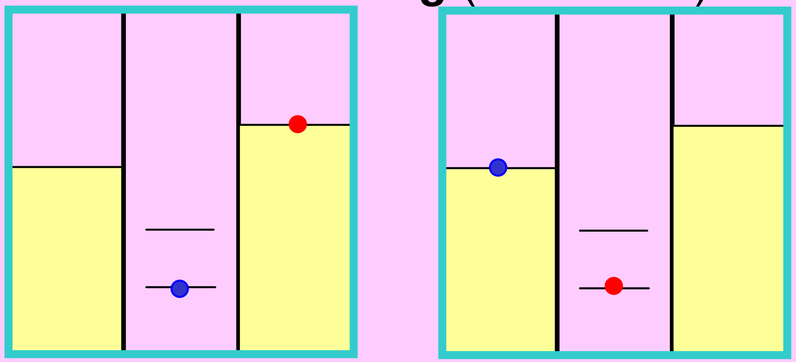
only one excess electron can be on dot (charging energy)

Cotunneling Transport



higher order process:
two electrons tunnel and change energy

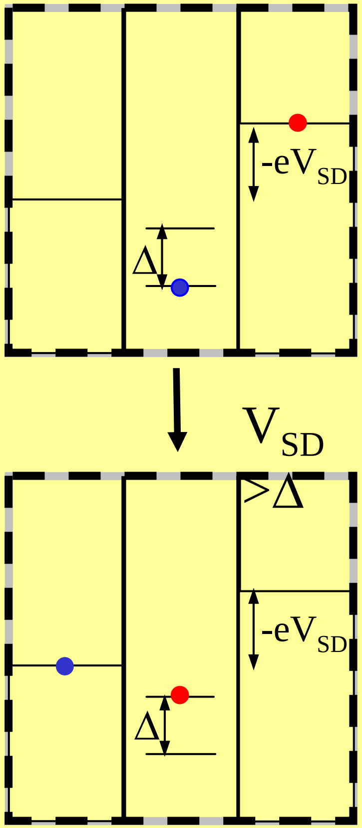
elastic cotunneling (blue circle)



before tunneling after

dot energy unchanged

inelastic cotunneling
(white lines)
dot energy changes
only possible for $V_{SD} > \Delta$



before tunneling

after

dot now in excited state

Temperature Regimes

$$\Delta, \frac{e^2}{C} \ll kT$$

no charging effects, no Coulomb blockade

$$g_\infty = \left(\frac{1}{g_L} + \frac{1}{g_R}\right)^{-1}$$

$$\Gamma, \Delta \ll kT \ll \frac{e^2}{C}$$

classical Coulomb blockade (metallic CB)
temperature broadened
transport through several quantum dot energy levels

$$g \sim \frac{g_\infty}{2} \cosh^{-2}\left(\frac{\epsilon}{2.5kT}\right)$$

peak conductance idependent of T
FWHM ~ 4.35kT

$\Gamma = \Gamma_L + \Gamma_R$

escape broadening (tunneling rates)

Temperature Regimes

$$\Gamma \ll kT \ll \Delta \ll \frac{e^2}{C}$$

quantum Coulomb blockade
temperature broadened regime
resonant tunneling
transport through only one dot level

$$g \sim \frac{e^2}{h}\frac{\gamma}{4kT}\cosh^{-2}\left(\frac{\epsilon}{2kT}\right)$$

peak conductance 1/T
FWHM ~ 3.5kT

$$\gamma = \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R}$$

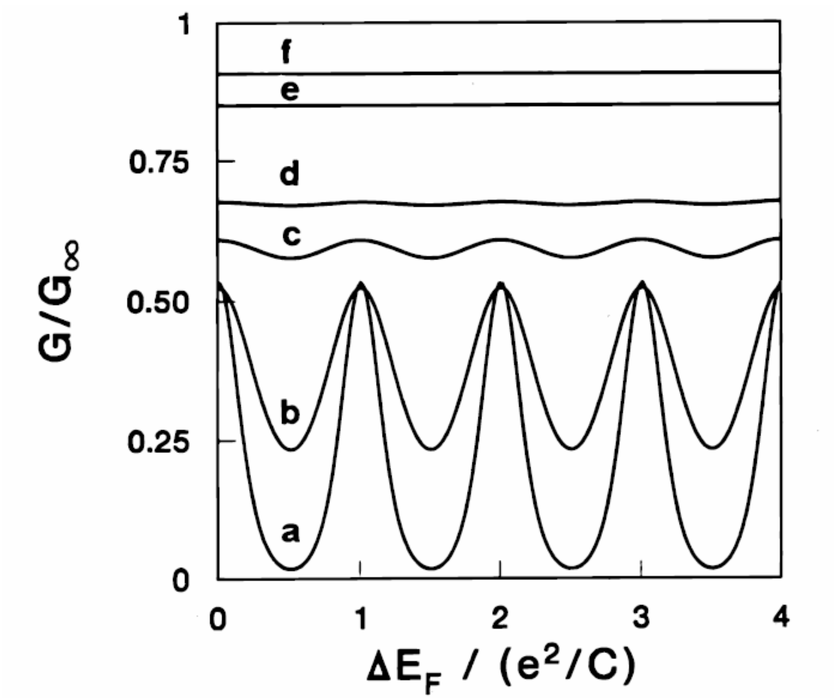
$$kT \ll \Gamma, \Delta \ll \frac{e^2}{C}$$

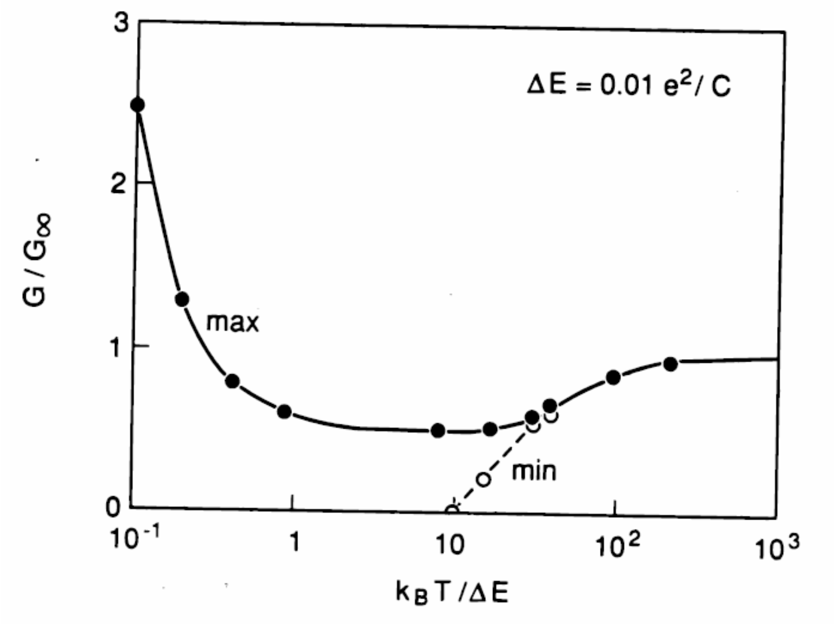
quantum Coulomb blockade
lifetime broadened regime
transport through only one dot level

$$g_{BW} \sim \frac{e^2}{h}\frac{\gamma\Gamma}{(\epsilon/\hbar)^2 + (\Gamma/2)^2}$$

peak conductance e^2/h indep. of T
FWHM ~ Γ

Temperature Dependence: Theory





Δ= 0.01 e^2/C

kT / e^2C

a 0.075

b 0.15

c 0.3

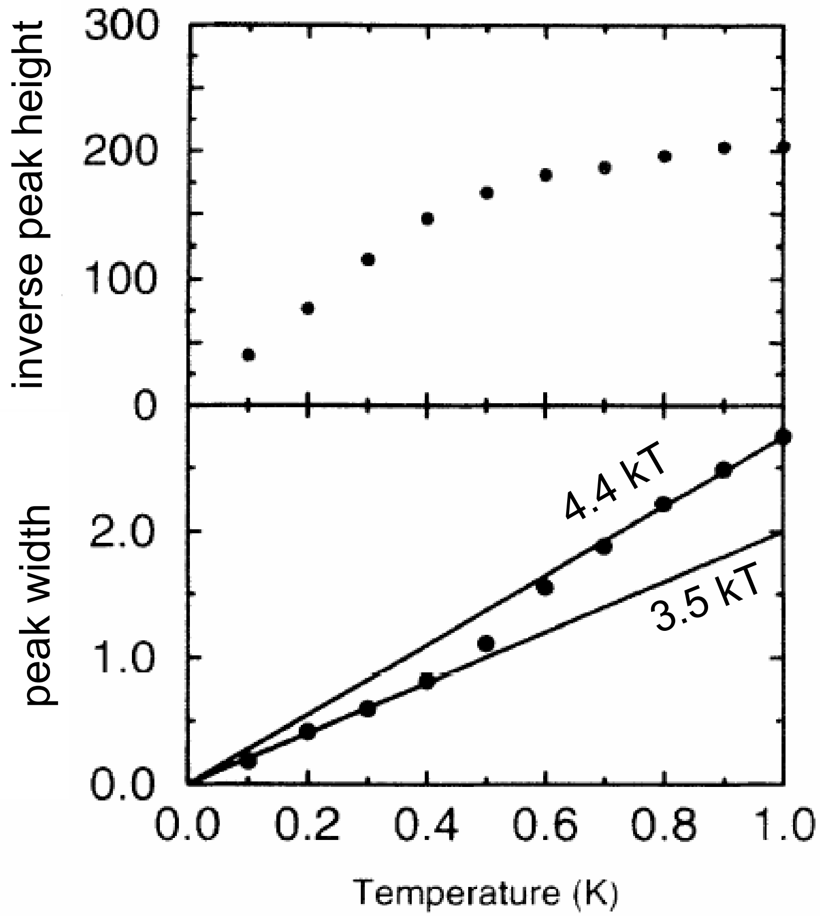
d 0.4

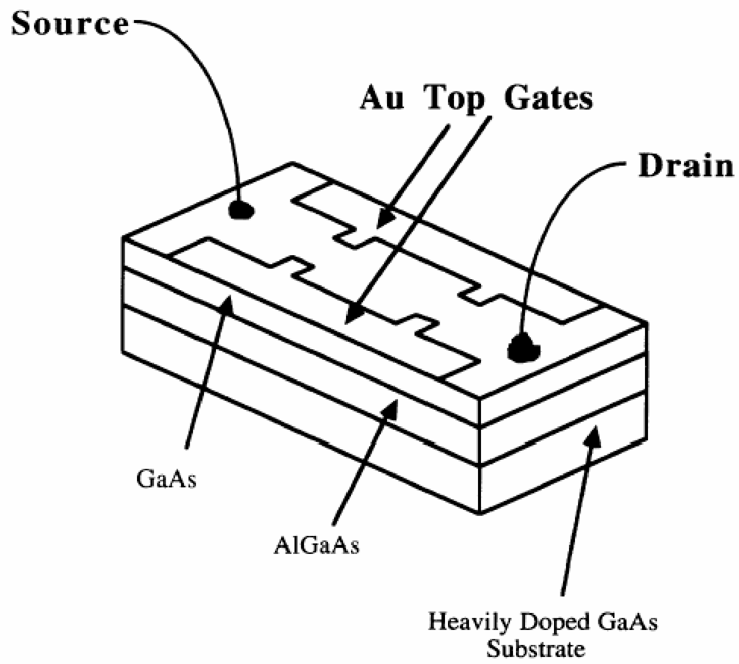
e 1

f 2

van Houten, Beenakker & Staring, NATO ASI Series

Temperature Dependence: Experiment



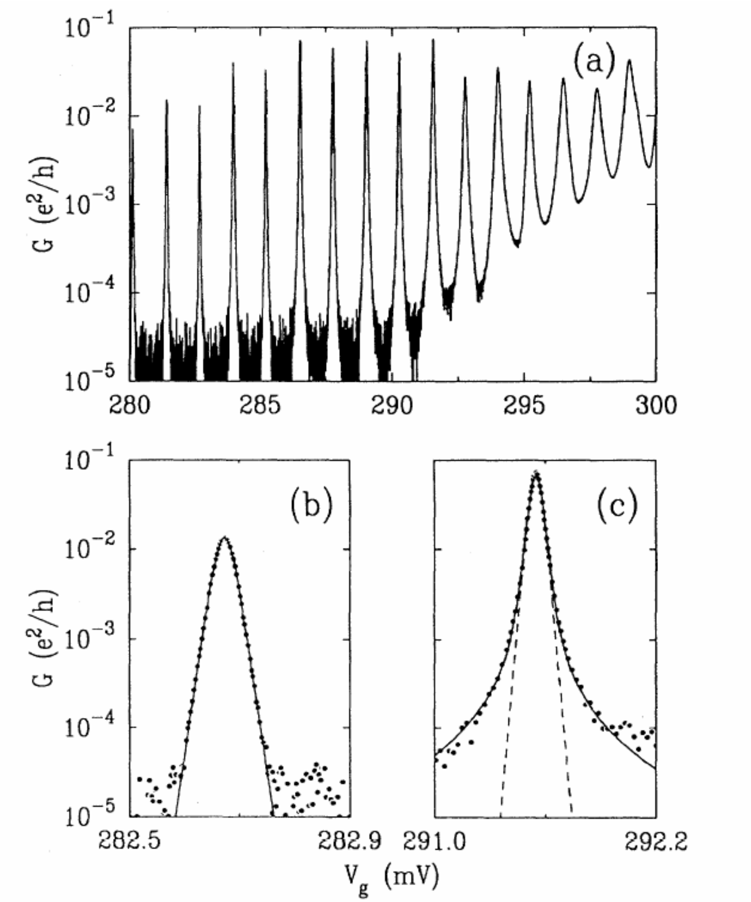


crossover 3.5 to 4.3kT peak width

peak g
1/T dependence: quantum regime
T independent: cassical regime

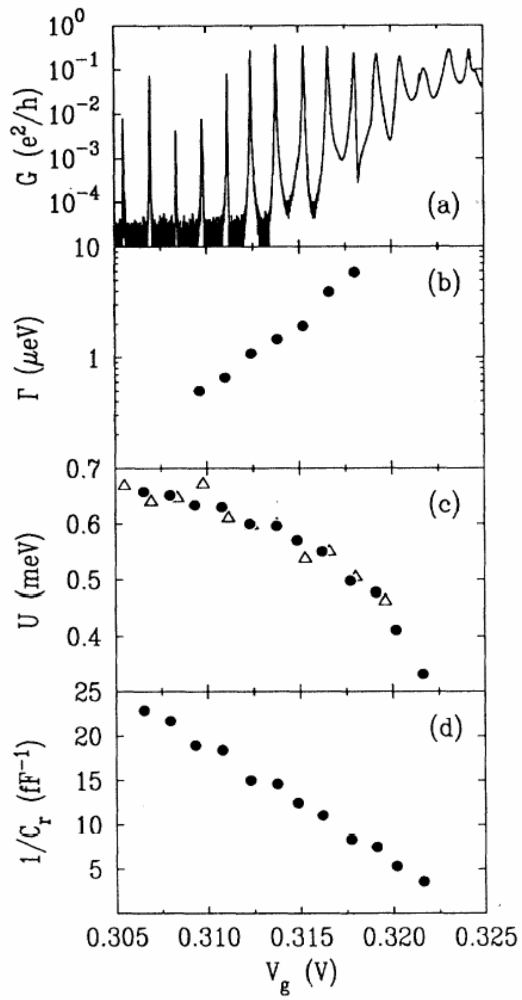
Foxman et al., PRB50, 14193 (1994)

Line Shapes: Experiments



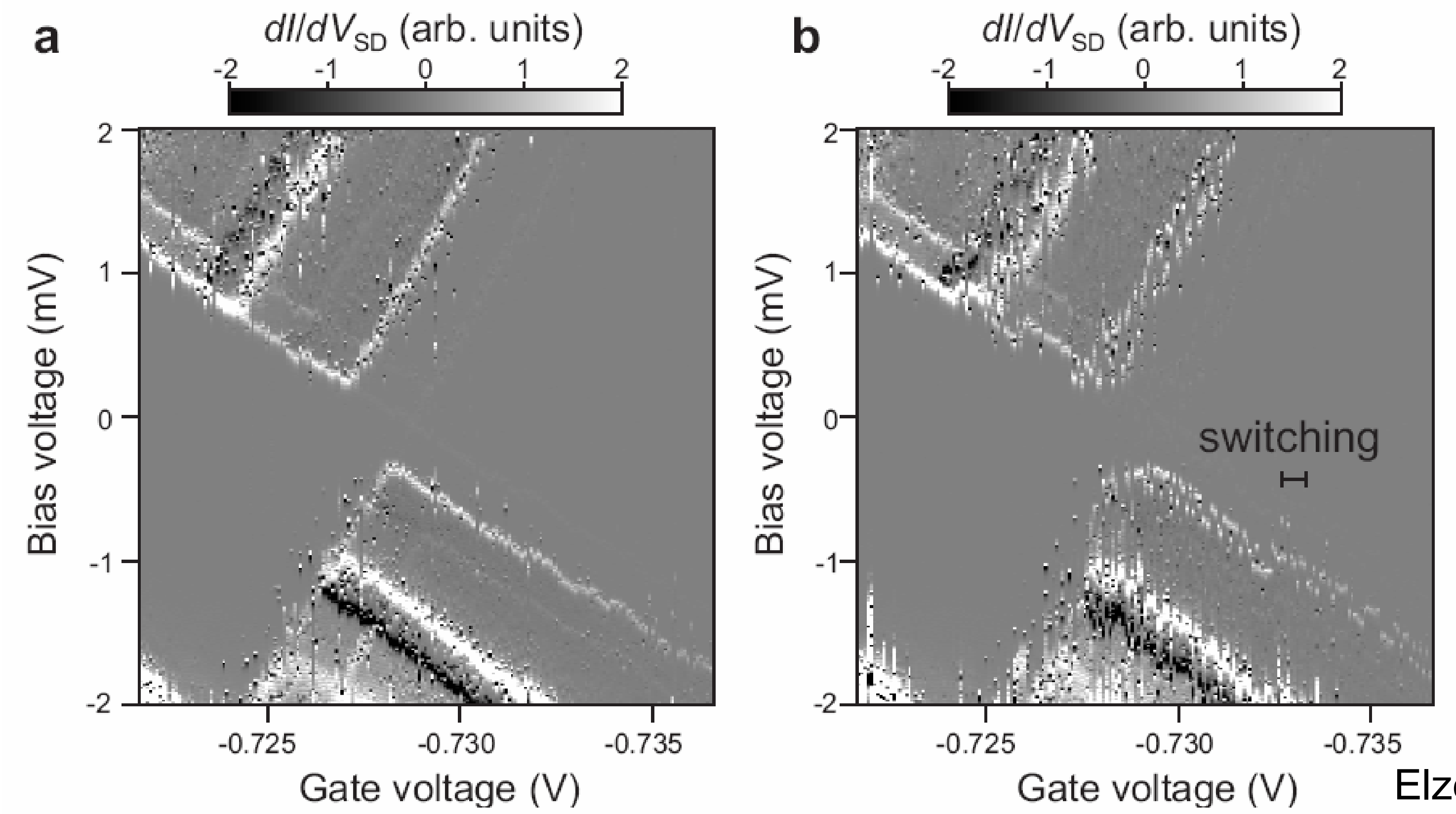
T-broadened

lifetime broadened



Foxman et al., PRB47, 10020 (1993)

Charge Switching / Telegraph Noise



Elzermann, 2003