1. Practice at reading circuit diagrams.

A three qubit system can be written in the basis: $|000\rangle$, $|001\rangle$, etc.

a) How many basis states are there?

b) Consider the transformation performed on $|\Psi_{in}\rangle = |x_1 x_2 x_3\rangle$ by the circuit shown in figure 1. Write down the effective operator O_{123} in terms of the gates operations that have been presented in the lecture.

c) What is $|\Psi_{out}\rangle$ if $|\Psi_{in}\rangle = |110\rangle$?

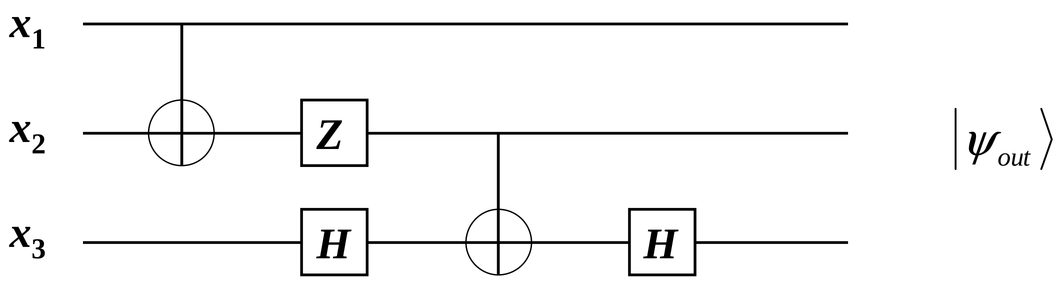

Figure 1: Three qubits are passed through the circuit depicted here, which consists of both two-qubit (*C-NOT*) and single-qubit (Z and H) gates. The incoming state can be any $|\Psi_{in}\rangle = |x_1 x_2 x_3\rangle$. The output is $|\Psi_{out}\rangle$.

2. Revisiting Entanglement.

Alice, Bob and Charlie walk into a bar. The bartender, Dominik, promises them they can have all the drinks they want for free if they win a game of his choosing. However, if they lose they have to work for him without pay every night for the next month (cleaning toilets, etc.). After hearing the problem Alice, Bob and Charlie accept the challenge...

It goes like this: D tells A, B and C to enter three different sound-proof rooms, in which their mobile phones have no service. He then gives each player a sheet of paper with "x" or "y" written on it. He has either given out three x's, or one x and two y's. That is, if we list the written letters given to the players in the order of abc (for A, B and C), the possible combinations are xxx, xyy, yxy or yyx. A, B and C only know what is on their own sheet. D then goes from room to room asking A, B and C to write "+1" or "-1" on their piece of paper. Finally, they all walk back to the bar to check the results.

How do they win? Well, D takes these three numbers and multiplies them together. A, B and C win if there are three x's and the product of the numbers is $+1$, or there are one x and two y's and the product is -1. They lose in all other cases.

a) D thinks he's brilliant because he is convinced it is impossible for A, B and C to win. Show that, in a classical world, he's correct (and therefore should have a very clean bar for the next month)?

b) Now suppose that - before being locked away - A, B and C took three qubits and prepared them in the following state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|000\rangle + |111\rangle \right) . \tag{1}$$

Afterward each of them brings one of these qubits to their sound-proof room. They agree to each measure their respective qubit in the $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ basis (eigenstates of the σ_X operator) if their sheet has an "x" on it, and the $|+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$, $|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$ basis (eigenstates of the σ_Y operator) if they have a "y". To see what effect the three measurements can have on the game, consider A and B to each measure their qubit before C does. What will C measure if (i) A and B had an "x" on their sheet of paper ($a = b = x$)?, (ii) $a = b = y$?, (iii) $a = x$ and $b = y$?

c) Now, amazingly enough, it will be possible for A, B and C to always win the game. What do they need to do in order for this to be the case?