Physik IV FS 2009

Blatt 11 Abgabe: **Montag 25.05.2009, 12:15 Uhr**

1. Measurements on qubits.

A three qubit system is in the state:

$$|\Psi\rangle = \frac{i}{2}|000\rangle + \frac{12+5i}{26}|001\rangle - \frac{1}{2}|101\rangle + \frac{3}{10}|110\rangle - \frac{4i}{10}|111\rangle. \tag{1}$$

a) Verify that $|\Psi\rangle$ is normalized.

b) Compute the probability of the following outcomes: (i) measurement on the first qubit returns 0; (ii) measurement on the first two qubits gives 00; (iii) measurement on the last two qubits gives 11.

c) Following an H transformation of the third qubit, what is the probability that a measurement on all three qubits gives 111?

d) After the H transformation mentioned above, what is the state of the system if the second qubit is measured to be 1?

2. Bell's inequality.

Two spins can be described by the product of their individual states, $|\Psi\rangle = |x_1\rangle|x_2\rangle = |x_1 x_2\rangle$. Entangled states can be written in the Bell basis:

$$\begin{aligned}
|\beta_{00}\rangle &= \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)\\
|\beta_{01}\rangle &= \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)\\
|\beta_{10}\rangle &= \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)\\
|\beta_{11}\rangle &= \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right).
\end{aligned}$$

Let $|\Psi\rangle = |\beta_{11}\rangle$. Show that $\langle QS\rangle = \langle RS\rangle = \langle RT\rangle = 1/\sqrt{2}$ and $\langle QT\rangle = -1/\sqrt{2}$, such that $\langle QS\rangle + \langle RS\rangle + \langle RT\rangle - \langle QT\rangle = 2\sqrt{2} > 2$! [$Q$, R, S and T are defined in the lecture notes.]

3. Practice at reading circuit diagrams.

A three qubit system can be written in the basis: $|000\rangle$, $|001\rangle$, etc.

a) How many basis states are there?

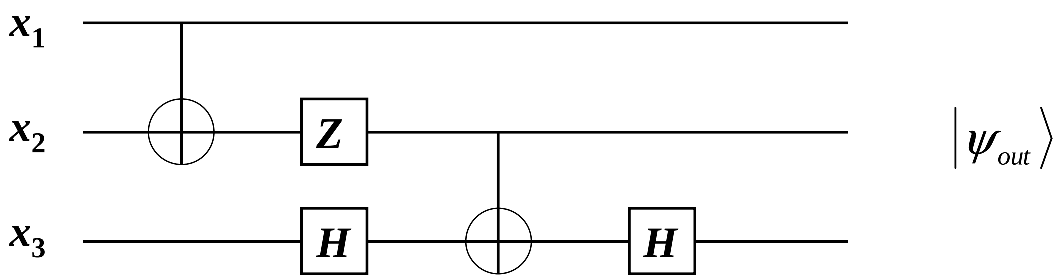

Abbildung 1: Three qubits are passed through the circuit depicted here, which consists of both two-qubit (*C-NOT*) and single-qubit (Z and H) gates. The incoming state can be any $|\Psi_{in}\rangle = |x_1 x_2 x_3\rangle$. The output is $|\Psi_{out}\rangle$.

b) Consider the transformation performed on $|\Psi_{in}\rangle = |x_1 x_2 x_3\rangle$ by the circuit shown in figure 1. Write down the effective operator O_{123} in terms of the gates operations that have been presented in the lecture.

c) What is $|\Psi_{out}\rangle$ if $|\Psi_{in}\rangle = |110\rangle$?

d) What is $\langle O_{123}\rangle$ when $|\Psi_{in}\rangle = |011\rangle$?