1. Emission / Absorption von Photonen

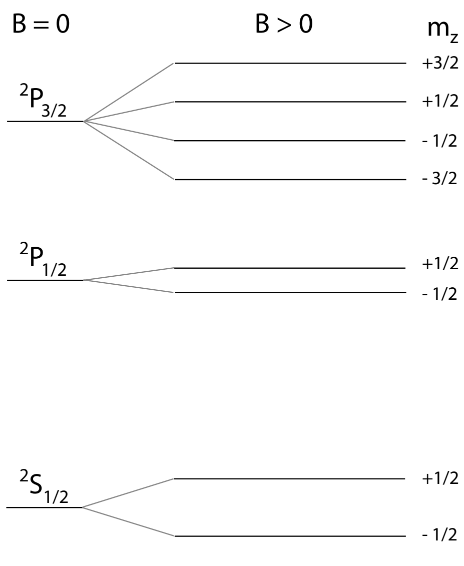
a) Ein Atom habe Zustände bei Energien E_1, E_2. Bei welchen Photonen-Energien kann dieses Atom prinzipiell Licht aussenden? (1 Punkt)

b) Gegeben seien Wellenfunktionen ψ_1 und ψ_2 eines Atoms, zwischen denen ein Dipolübergang stattfindet. Wie hängt die spontane Emissions-wahrscheinlichkeit von den Wellenfunktionen ab? (1 Punkt)

c) ψ_1 und ψ_2 sollen gerade Parität haben. Können Photonen in einem Dipolübergang $1 \to 2$ emittiert werden? Und wenn es ein Quadrupolüber-gang ist? Begründe. (2 Punkte)

d) Unten ist das Termschema von Na in einem Magnetfeld abgebildet. Gib die relevanten Auswahlregeln an, zeichne für $B > 0$ die optischen Übergänge im Termschema unten ein, gib für jeden Übergang jeweils die Polarisation an und zeichne qualitativ die Wellenlängen der Übergänge auf der λ - Achse ein. Zwei Übergänge mit den grössten Wellenlängen λ sind schon eingetragen, ergänze die restlichen Übergänge. (4 Punkte)

e) Sind optische Übergänge zwischen Singulett und Triplett Zuständen möglich? Erkläre. (2 Punkt)



polarisation:

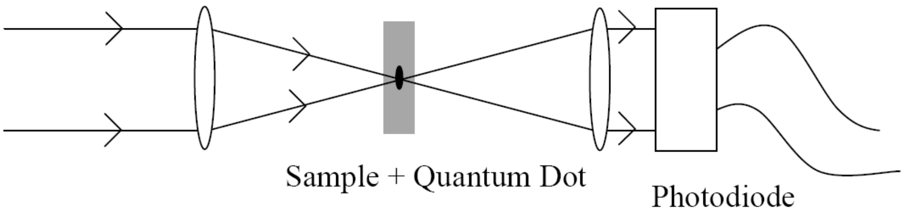
2. Selection Rules

With the help of the hydrogen atom wavefunctions listed in the table below, explicitly verify that $\Delta l = \pm 1$ for $n = 2 \rightarrow n = 1$ dipole transitions in the hydrogen atom. Start by formulating a mathematical condition for dipole transitions to be forbidden between to arbitrary states $|\psi_A\rangle$ and $|\psi_B\rangle$. Then individually evaluate this condition for the hydrogen states given below.

n	l	m_l	$\psi(r, \theta, \phi)$
1	0	0	$\frac{1}{\sqrt{\pi}\, a_0^{3/2}} e^{-r/a_0}$
2	0	0	$\frac{1}{4\sqrt{2\pi}\, a_0^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$
2	1	0	$\frac{1}{4\sqrt{2\pi}\, a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \cos\theta$
2	1	± 1	$\frac{1}{8\sqrt{\pi}\, a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \sin\theta\, e^{\pm i\phi}$

3. Absorption Spectroscopy on a Quantum Dot

Quantum dots are man-made structures with atomic like behavior. Often, one can treat a quantum dot to a good approximation as a two level system. In optical transmission spectroscopy measurements on quantum dots, a monochromatic beam of photons is focused on the sample containing the quantum dot and the transmitted photons are detected using a photodiode behind the sample (see figure below).



a) In absence of a quantum dot, all the photons in the beam are detected by the photodiode (assuming negligible reflection and absorption due to the sample). Now add the quantum dot: At what frequency do you expect to see a change in photodiode signal? Take E_g and E_e as the ground state and excited state energies of the quantum dot.

b) If then quantum dot is in the excited state, it can emit a photon either through spontaneous emission or stimulated emission. How does each of these mechanisms affect the photodiode signal?

c) Write down the expressions for the photon absorption rate and for the photon emission rate in terms of the following parameters: Einstein's A and B coefficients, the incoming photon rate N (number of photons hitting the quantum dot per second), the probability of the ground state of the dot being occupied, P_g, or the excited state being occupied P_e. Note that $P_g + P_e = 1$.

d) How do the rates of absorption and emission compare in steady state? Using the results of part c), calculate P_e as a function of N.

e) In light of your answer to part b), write down the expression for the amplitude of the absorption signal. The absorption signal is the change in the total signal detected by the photodiode due to the interaction of photons with the quantum dot.

f) As one increases the rate of incoming photons N one expects the absorption signal to increase. However, this is only true up to a certain photon rate, after which the signal remains constant and is saturated. Using the expression you derived in part d) calculate how the absorption signal changes with photon rate at the limit of very low photon rate (low power beam). How is the situation different for the limit of very high photon rate (high power beam)? Make a qualitative plot of the absorption signal as a function of the photon rate, mark the low and high power regimes. Explain in words why the absorption signal saturates.