Precision tomograpy of a three-qubit donor quantum processor in silicon

Nuclear spins were among the first physical platforms to be considered for quantum information processing[1,2], because of their exceptional quantum coherence[3] and atomic-scale footprint. However, their full potential for quantum computing has not yet been realized, owing to the lack of methods with which to link nuclear qubits within a scalable device combined with multi-qubit operations with sufficient fidelity to sustain fault-tolerant quantum computation. Here we demonstrate universal quantum logic operations using a pair of ion-implanted ^{31}P donor nuclei in a silicon nanoelectronic device. A nuclear two-qubit controlled-*Z* gate is obtained by imparting a geometric phase to a shared electron spin[4], and used to prepare entangled Bell states with fidelities up to 94.2(2.7)%. The quantum operations are precisely

The quantum operations are precisely
characterized using gate set tomography (GST)[5], yielding one-qubit average gate fidelities up to 99.95(2)%, two-qubit average gate fidelity of 99.37(11)% and two-qubit preparation/measurement fidelities of 98.95(4)%. These three metrics indicate that nuclear spins in silicon are approaching the performance demanded in fault-tolerant quantum processors[6]. We then demonstrate entanglement between the two nuclei and the shared electron by producing a Greenberger–Horne–Zeilinger three-qubit state with 92.5(1.0)% fidelity. Because electron spin qubits in semiconductors can be further coupled to other electrons[7–9] or physically shuttled across different locations[10,11], these results establish a viable route for scalable quantum information processing using donor nuclear and electron spins.

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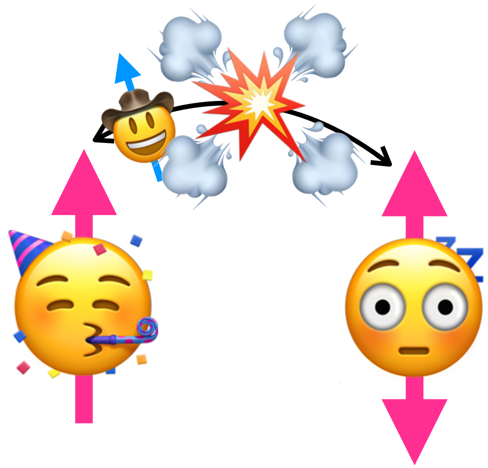
In a 🥜-shell

- **Nuclear spins (Ns):**
  - Long coherence / weak coupling to environment
- **Electron spins (Es):**
  - Shorter coherence / stronger coupling to environment
- **Combine the best of both:**
  - Bridge the gap between Ns through shared Es
  - Entangle electron as an ancilla for readout & operations



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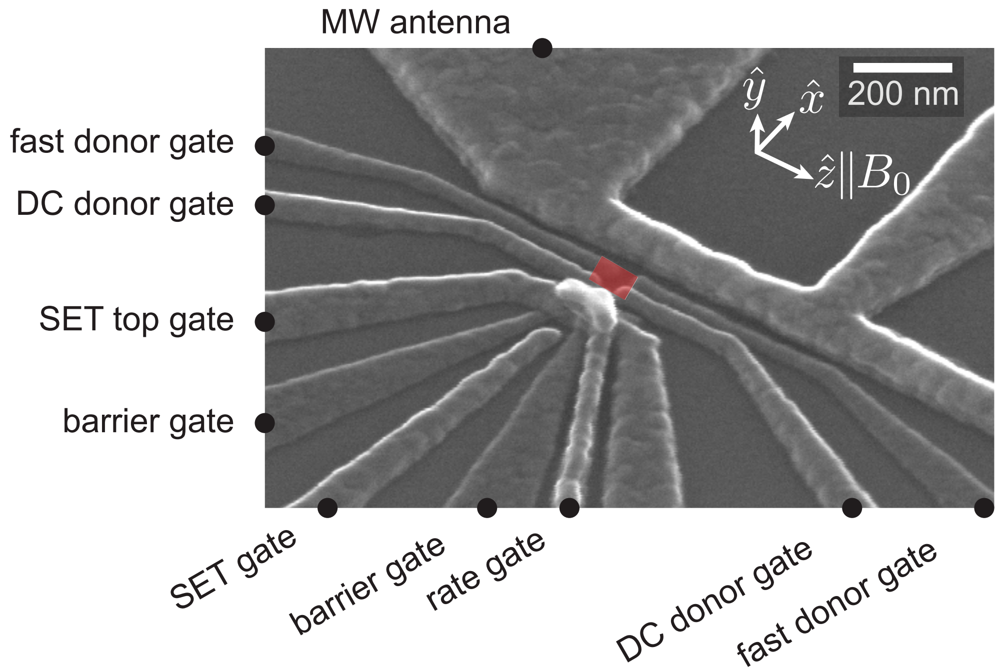
Outline

- **One electron - two nuclei quantum processor**
  - Device
  - Methods for readout & control of single qubit
  - Control & characterisation of both qubits
- **Nuclear two-qubit operations**
  - Circuit diagrams
  - Bell-states
- **Gate set tomography**
- **Three-qubit entanglement**
- **Outlook**

Device

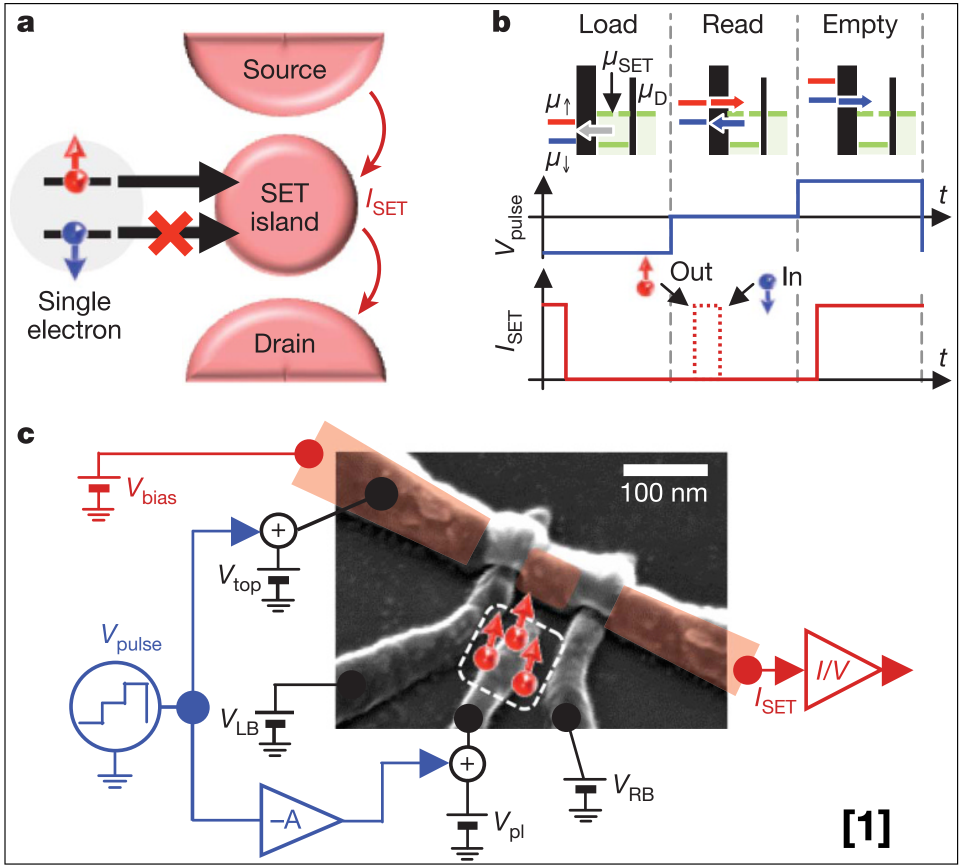
- **Standard MOS compatible processes:**
  - **p**-type **Si <100>**, 10-20 Ω cm
  - 900 nm epilayer of isotopically enriched ^{28}**Si**
  - 730 ppm **residual** ^{29}**Si**
  - doped n^+ / p for ohmics / leak prevention
  - **200 nm** SiO_2 with a 20 x 40 µm etch-window (HF) with **8 nm** SiO_2
  - **90 x 100 nm** area for P^+ **ion** implantation (10keV)
  - donor activation RTA 5” @ 1000° C
  - **Al metal gates** insulated by **native** Al_2O_3
  - Anneal 15’ @400° C to passivate interface traps
  - Static B_0 = 1.33 T



Single Shot e^- - Spin Readout

- **Elzerman Protocol [2]**
  - QD coupled to QPC
  - $k_B T < \Delta E_Z < E_{ORB}$

- **Detector electrostatically & tunnel coupled to electron site**
  - Use a SET coupled to source & drain
  - B > 1 T , T_e ~ 200 mK



[1] A. Morello, Nature **467**, 687-691 (2010)
(**[2]** J.M. Elzermann, Nature **430**, 431-435 (2004))

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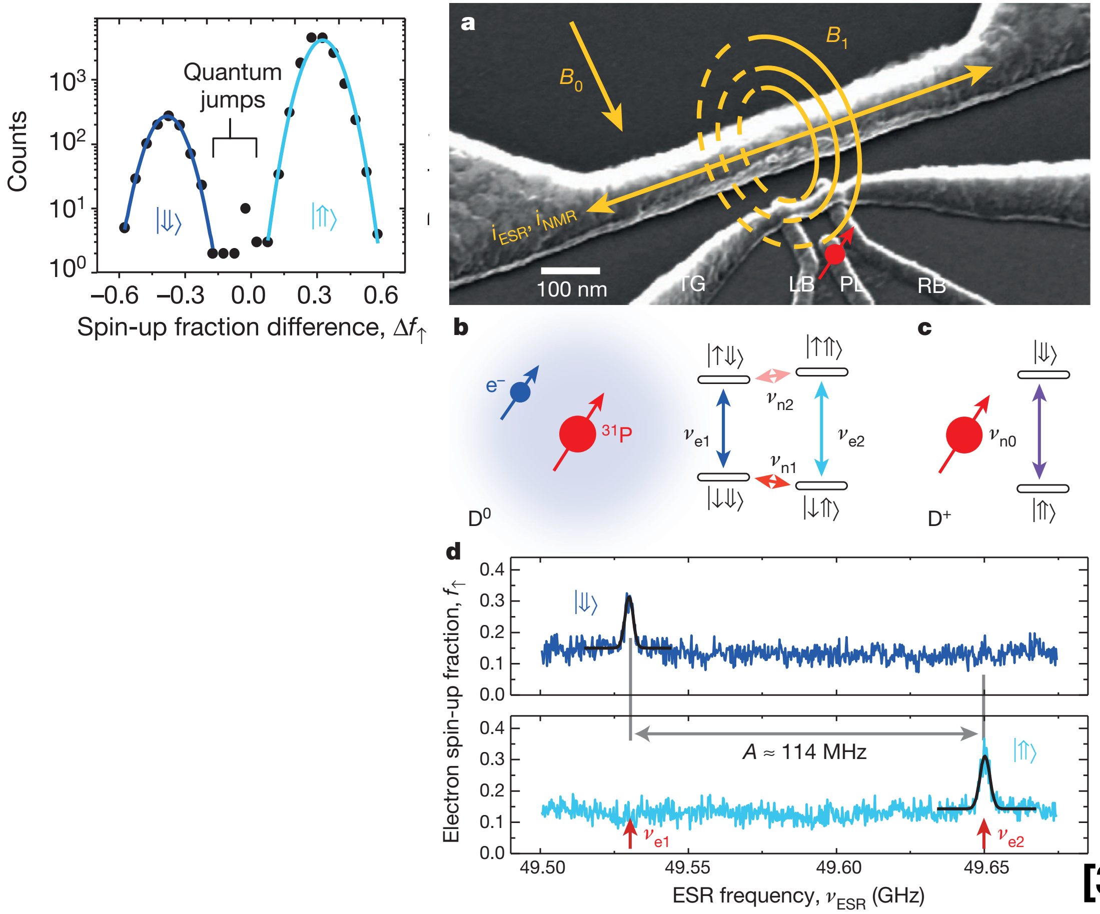
n^- - Spin Readout

- **Setup of 2-qubit system e & n**
  - ESR strip line + static B_0-field
  - ^{31}P donor island with bound S = 1/2 electron (D^0)
  - γ_n ~ 17 MHz/T, γ_e ~ 28 GHz/T, **A ~ 117 MHz**
- **For $\gamma_e B_0 >> A > 2\gamma_n B_0$:**
  - $| \downarrow \Uparrow \rangle, | \downarrow \Downarrow \rangle, | \uparrow \Downarrow \rangle, | \uparrow \Uparrow \rangle$
  - ionizing system to D^+ state —> $| \Uparrow \rangle, | \Downarrow \rangle$
- **ESR & NMR Resonances:**
  - $\nu_{e1,2} \approx \gamma_e B_0 \pm A/2$ (+/- for **n** up/down)
  - $\nu_{n1,2} \approx \gamma_n B_0 \pm A/2$ (+/- for **e** up/down)



[3]

1e⁻ 2n Quantum Processor

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- **2 ^{31}P donor atom nuclei I = 1/2 + 1 electron S 1/2**
  - $H = -\gamma_e B_0 \hat{S}_z - \gamma_e B_0 \,(\hat{I}_{1,z} + \hat{I}_{2,z}) + A_1\,\mathbf{S}\cdot\mathbf{I_1} \;+\; A_2\,\mathbf{S}\cdot\mathbf{I_2}$
  - A_1 = 95 MHz, A_2 = 9 MHz
  - hyperfine coupled e likely the third one (spin relax. time too short to be the firs one)

- **Effective mass calculation**
  - $A_{1,2}$ reproducible (calc.) assuming **6.5** nm donor-spacing
  - wide spacing -> less anisotropic hyperfine coupling
  - —> less chance of n-spin radomisation during readout ($P \sim 10^{-6}$ —> QND)

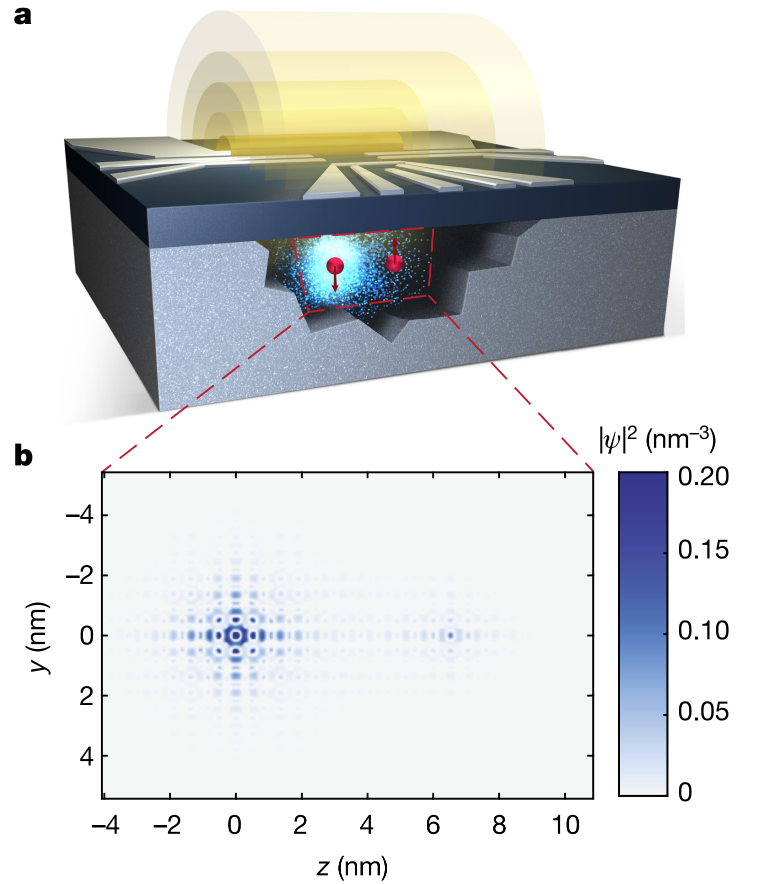

Fig. 1

1e⁻ 2n Quantum Processor

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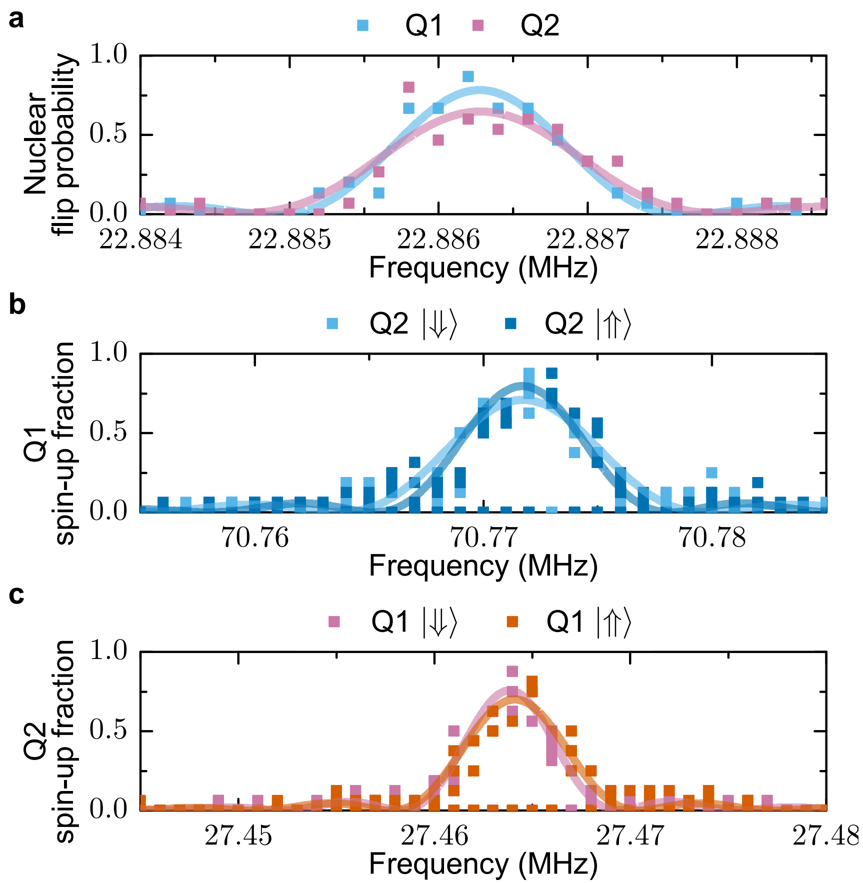
- **NMR of nuclei /wo no 3rd electron (2 in S = 0)**
  - $A_1 = A_2 = 0$
  - identical resonance frequencies

- **NMR of nuclei /w all electrons present**
  - spectator qubit either in $|\Uparrow\rangle$, $|\Downarrow\rangle$
  - no significant coupling between nuclei



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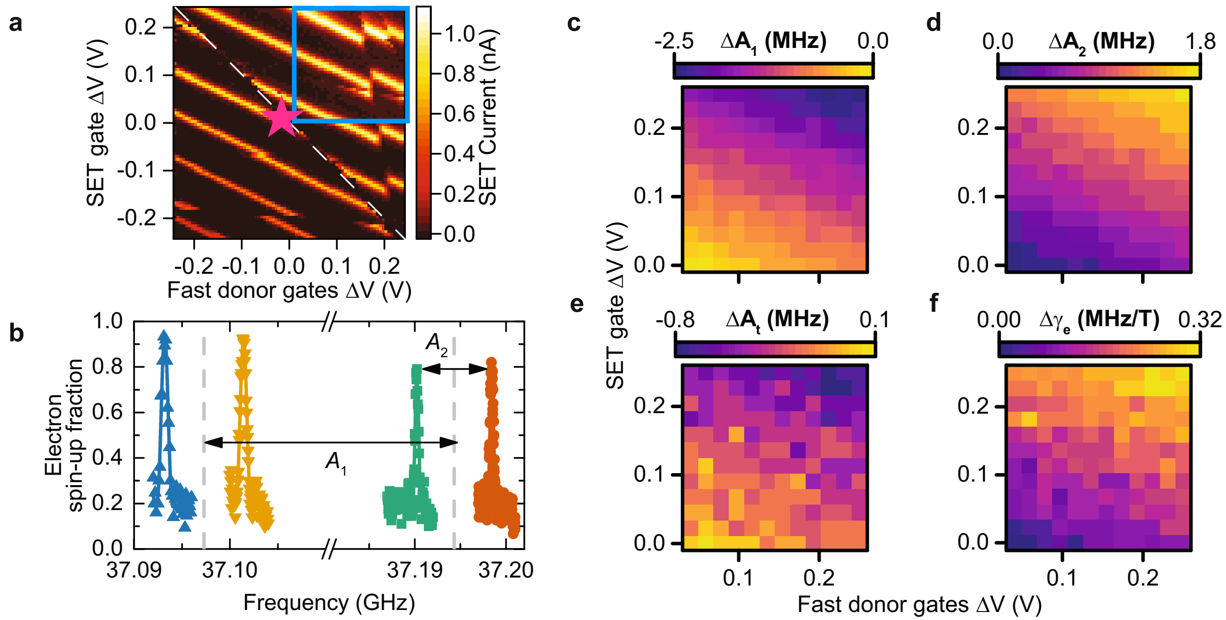
1e⁻ 2n Quantum Processor

- **4 ESR resonances**
  - Dependence on gate potentials (Stark shift)
  - $\Delta\nu_{e|\Uparrow\Uparrow} = 0.3\ \mathrm{MHzV}^{-1}$, $\Delta\nu_{\mathrm{e}|\Uparrow\Downarrow} = 5.2\ \mathrm{MHzV}^{-1}$, $\Delta\nu_{e|\Downarrow\Uparrow} = 7.6\ \mathrm{MHzV}^{-1}$, $\Delta\nu_{\mathrm{e}|\Downarrow\Downarrow} = 2.4\ \mathrm{MHzV}^{-1}$
  - $A_1 = (\nu_{e|\Uparrow\Downarrow} + \nu_{e|\Uparrow\Uparrow})/2 - (\nu_{e|\Downarrow\Downarrow} + \nu_{e|\Uparrow\Downarrow})/2$
  - $A_2 = \nu_{e|\Uparrow\Uparrow} - \nu_{e|\Uparrow\Downarrow}$
  - pink star, readout



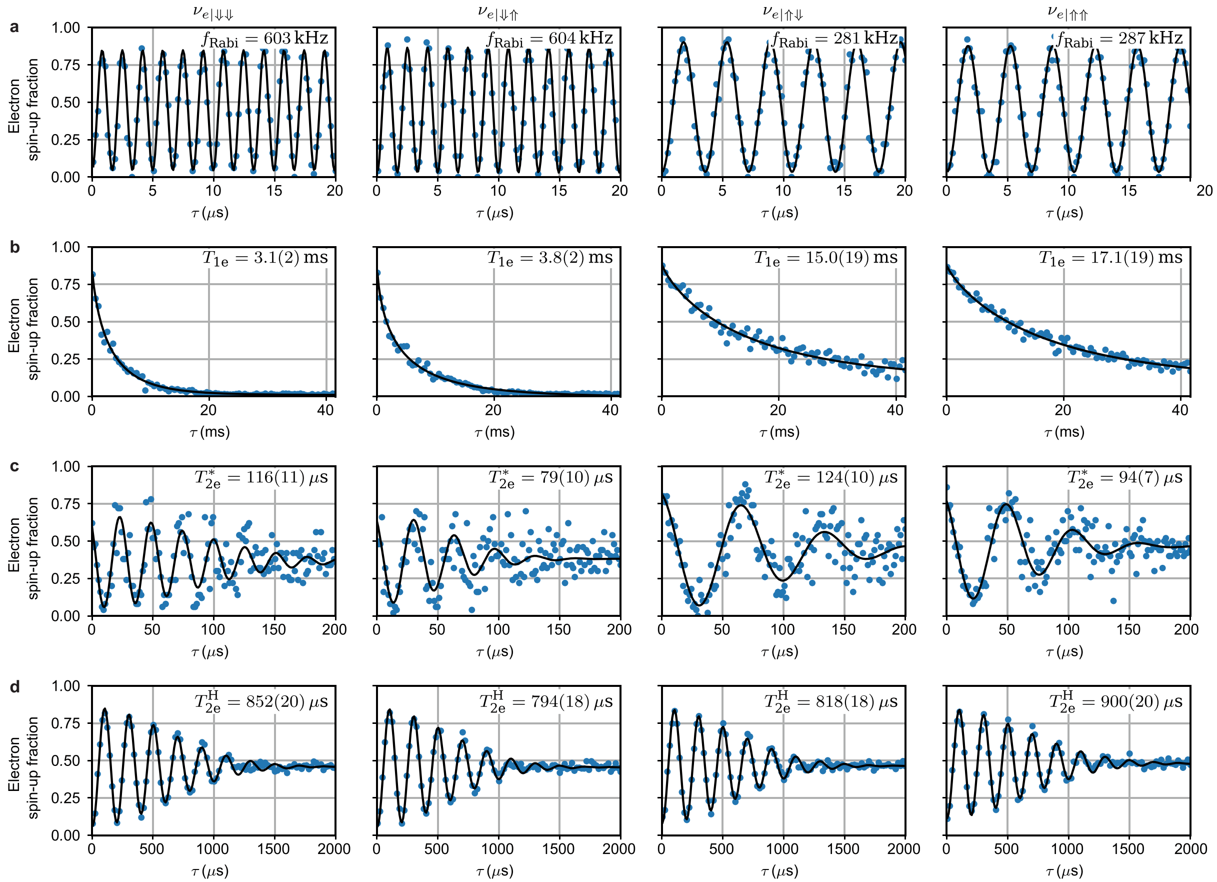
1e⁻ 2n Quantum Processor

Rabi

Init in $|\uparrow\rangle$, vary τ

Ramsey

Hahn Echo

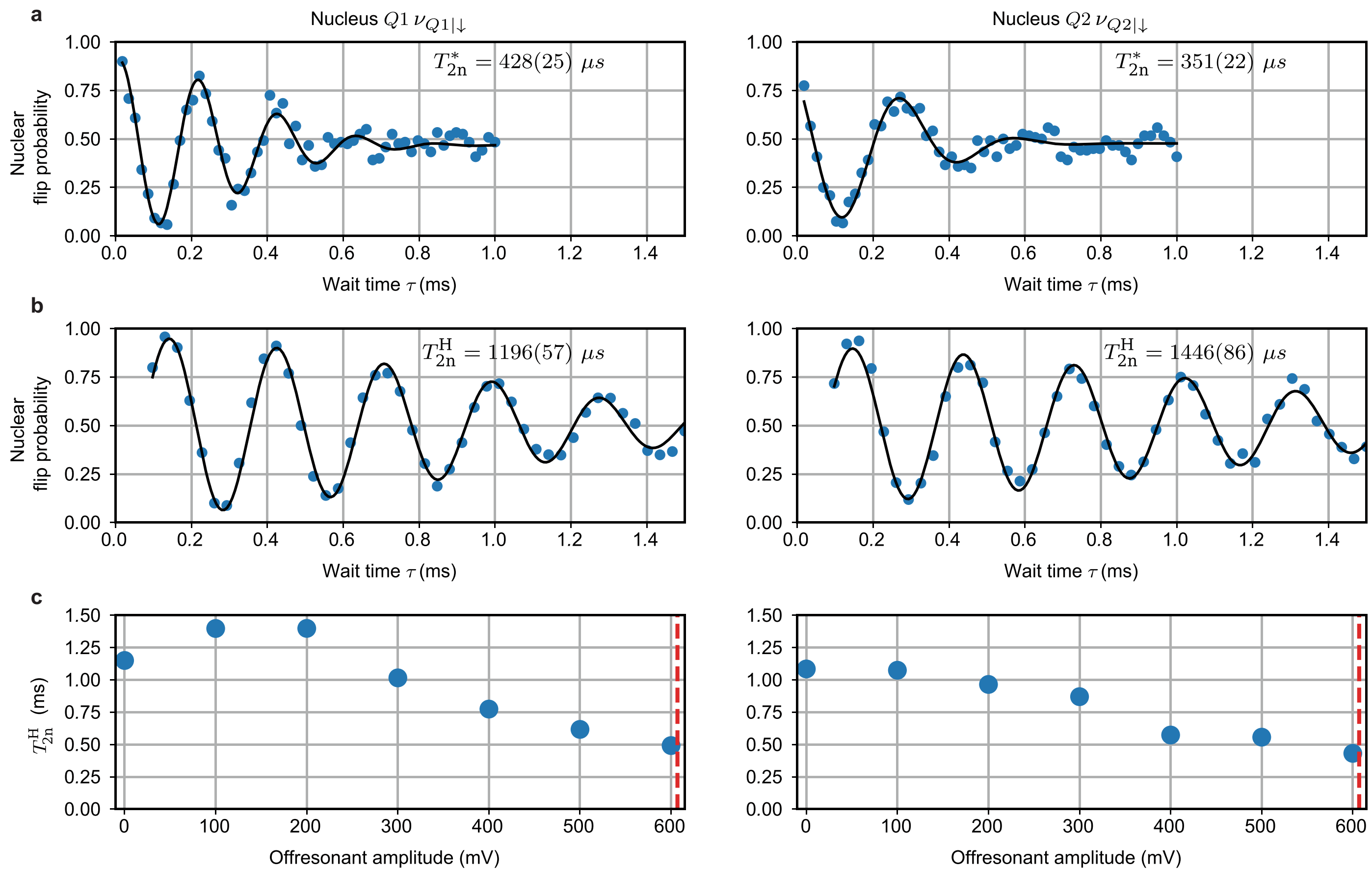


1e- 2n Quantum Processor

Ramsey

Hahn Echo

Check if left (idle) is affected by pulse on other qubit



1e⁻ 2n Quantum Processor

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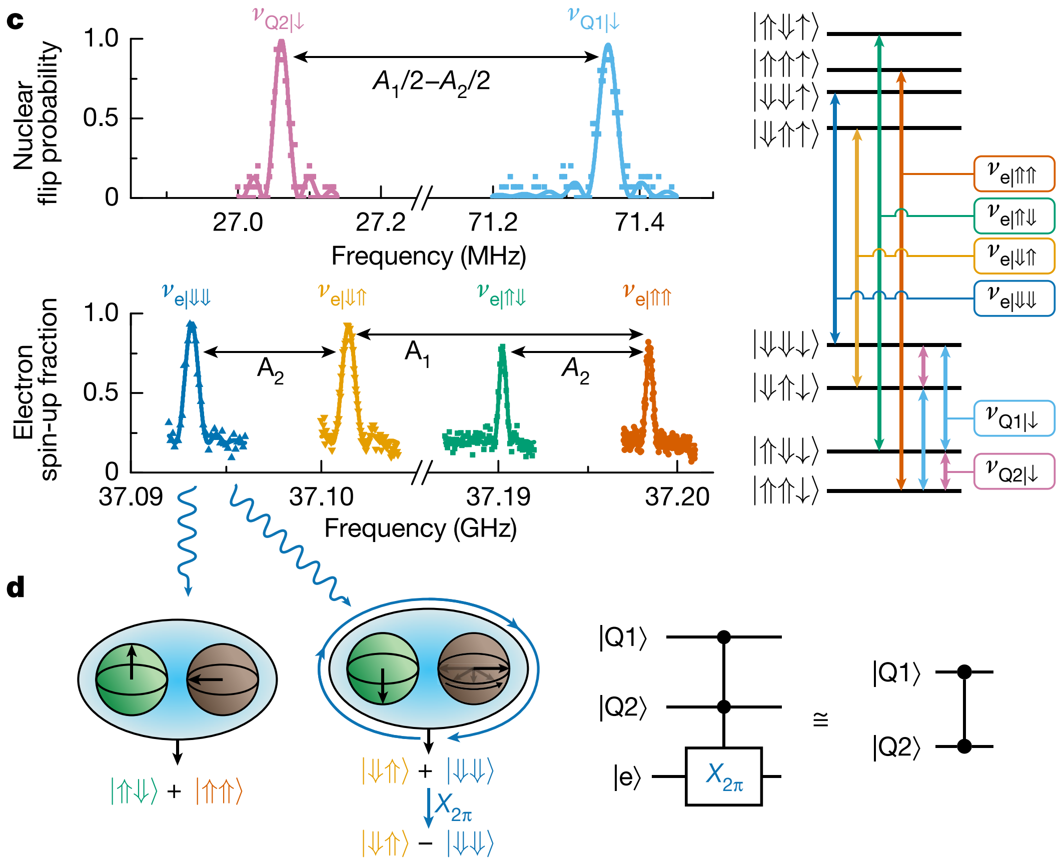
- **Single qubit ops. by NMR ✅**

- **Two qubit ops.: exploit hyperfine e.g. CZ**
  - 2π pulse on electron to acquire phase -1
  - $|\Downarrow\rangle \otimes (|\Downarrow\rangle + |\Uparrow\rangle)/\sqrt{2} \equiv (|\Downarrow\Downarrow\rangle + |\Downarrow\Uparrow\rangle)/\sqrt{2}$ and apply $X_{2\pi}$ at $\nu_{e|\Downarrow\Downarrow}$,
    $(-|\Downarrow\Downarrow\rangle + |\Downarrow\Uparrow\rangle)/\sqrt{2} \equiv |\Downarrow\rangle \otimes (-|\Downarrow\rangle + |\Uparrow\rangle)/\sqrt{2}$



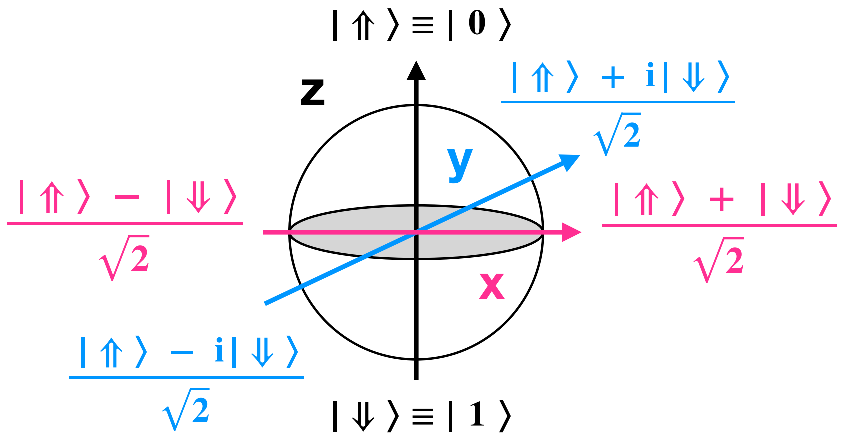
Nuclear 2-Qubit Operations

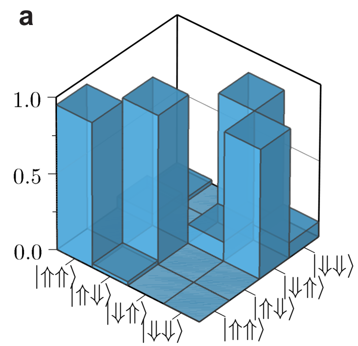
- **CNOT: Q1 as control of Q2**
  - $|\Downarrow\rangle \otimes |\Uparrow\rangle$ apply $Y_{-\pi/2}$, $|\Downarrow\rangle \otimes |\Uparrow\rangle - |\Downarrow\rangle$

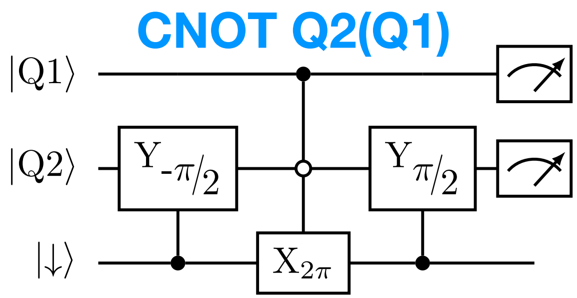
    $|\Downarrow\Uparrow\rangle - |\Downarrow\Downarrow\rangle$ apply $\mathbf{X}_{2\pi}$ at $\nu_{e|\Downarrow\Downarrow}$,

    $|\Downarrow\Uparrow\rangle + |\Downarrow\Downarrow\rangle\, |\Downarrow\rangle \otimes |\Uparrow\rangle + |\Downarrow\rangle$ apply $Y_{+\pi/2}$

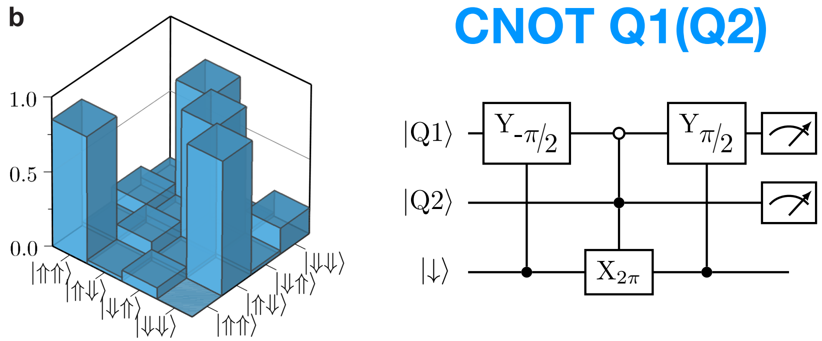
    $|\Downarrow\rangle \otimes |\Downarrow\rangle$







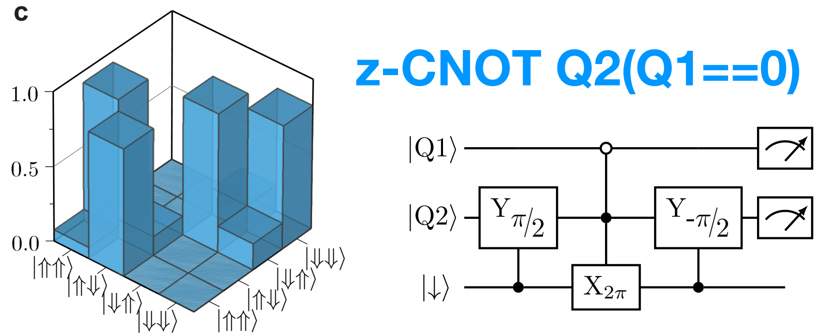
Nuclear 2-Qubit Operations



- **zCNOT**
  - $|\Uparrow\rangle \otimes |\Uparrow\rangle$ apply $\mathbf{Y}_{-\pi/2}$, $|\Uparrow\rangle \otimes |\Uparrow\rangle - |\Downarrow\rangle$

    $|\Uparrow\Uparrow\rangle - |\Uparrow\Downarrow\rangle$ apply $\mathbf{X}_{2\pi}$ at $\nu_{e|\Uparrow\Downarrow}$, $|\Uparrow\Uparrow\rangle + |\Uparrow\Downarrow\rangle$

    $|\Uparrow\rangle \otimes |\Uparrow\rangle + |\Downarrow\rangle$ apply $\mathbf{Y}_{+\pi/2}$ $|\Uparrow\rangle \otimes |\Downarrow\rangle$



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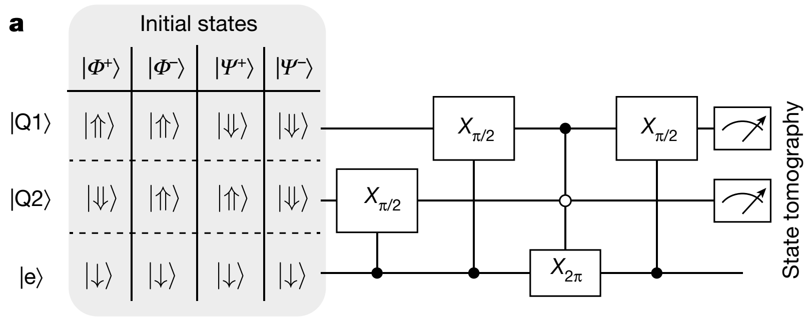
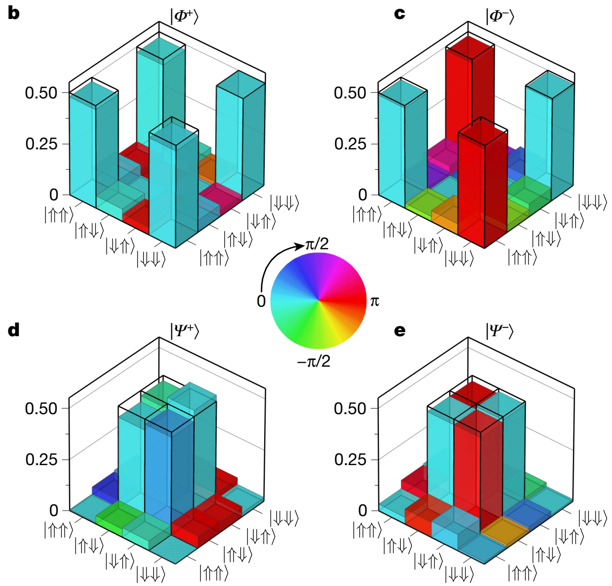
Nuclear Bell State Tomography

- **Apply universal gate to produce max. entangled Bell states**
  - $|\Phi^{\pm}\rangle = (|\Downarrow\Downarrow\rangle \pm |\Uparrow\Uparrow\rangle)/\sqrt{2}$
  - $|\Psi^{\pm}\rangle = (|\Downarrow\Uparrow\rangle \pm |\Uparrow\Downarrow\rangle)/\sqrt{2}$





f

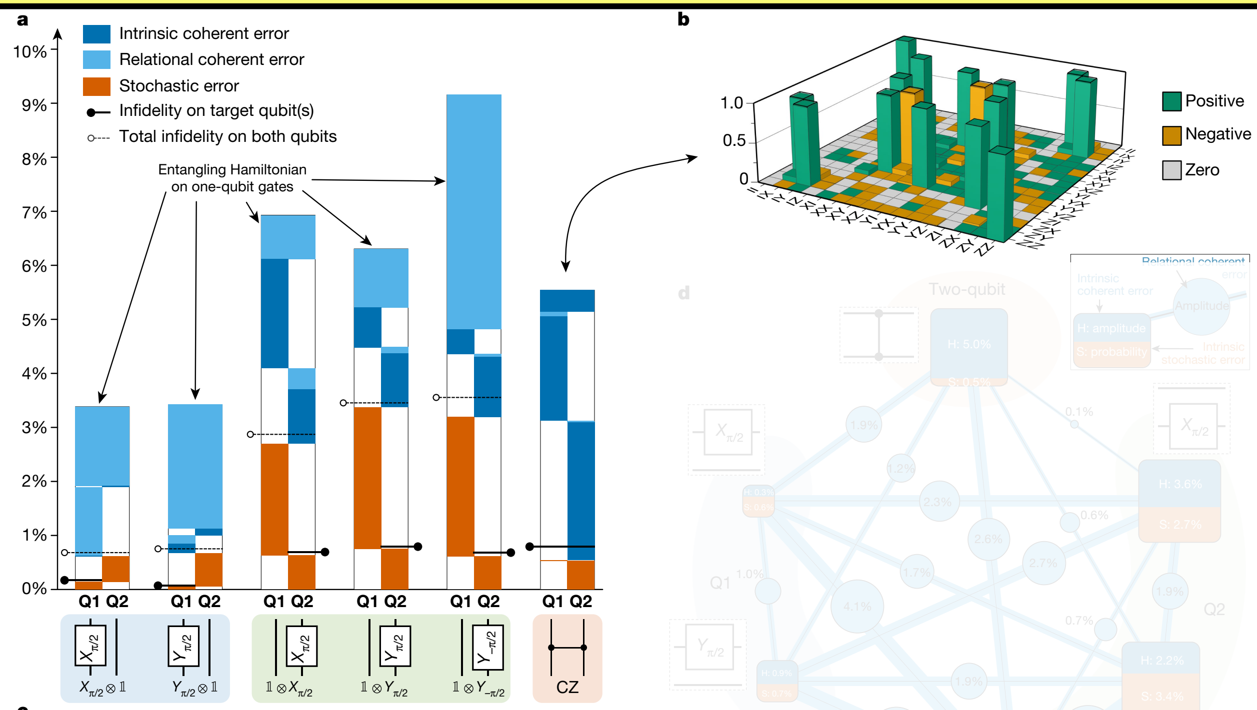
State	Fidelity	Concurrence
Φ^+	94.2(2.7)	0.91(5)
Φ^-	93.9(5.3)	0.89(10)
Ψ^+	94.1(2.3)	0.93(4)
Ψ^-	91.5(2.3)	0.87(4)

- **Reconstruct density matrices /w max. likelyhood QST**
  - Errors in state prep. fidelity calc. /w Monte Carlo bootstrap resampling

Gate Set Tomography

- **Randomised benchmarking does not reveal the nature of errors**

- **GST distinguishes stochastic & coherent errors, and separates local from crosstalk errors**
  - GST estimates a process matrix for each logic operation $(\mathbb{L}_i) : G_i = e^{\mathbb{L}_i}\mathbb{G}_i$ (error generator)
  - Behaviour of each gate described by linear combination of 13-14 elementary errors (coeffs. in $\mathbb{L}_i$, rate of error build up per gate)



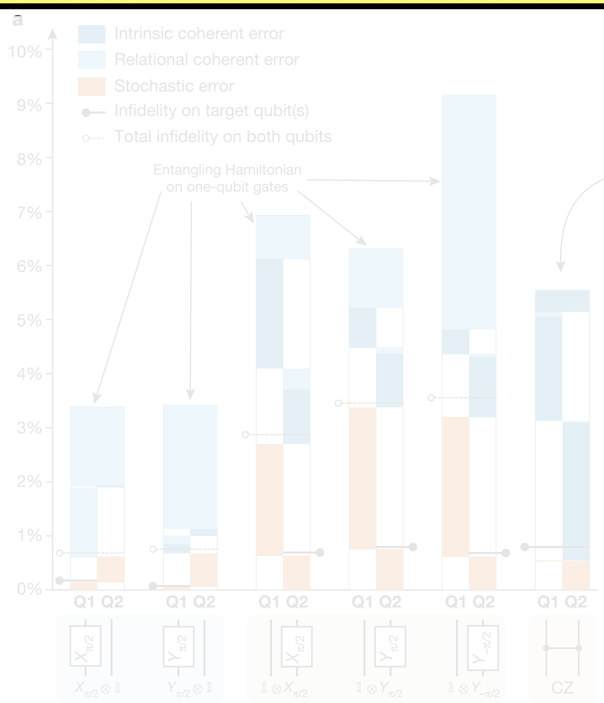
c

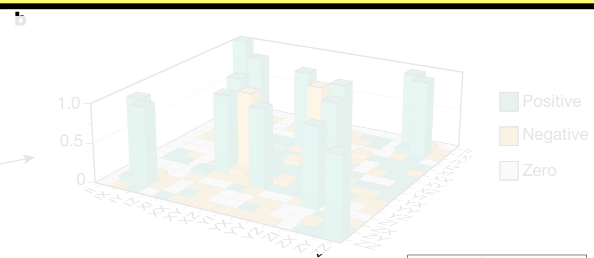
Gate	On-target average gate fidelity	Generator infidelity On Q1	Generator infidelity On Q2	Generator infidelity Total	Stochastic error	Intrinsic coherent error	Relational coherent error	Total error On Q1	Total error On Q2	Total error Total
$X_{\pi/2} \otimes \mathbb{1}$	**99.87(2)%**	**0.17(3)%**	0.47(5)%	0.68(6)%	0.60(6)%	0.34(14)%	2.8(3)%	2.0(3)%	0.71(15)%	3.4(3)%
$Y_{\pi/2} \otimes \mathbb{1}$	**99.95(2)%**	**0.07(3)%**	0.61(6)%	0.75(6)%	0.67(6)%	0.9(3)%	2.6(3)%	1.0(2)%	1.2(4)%	3.4(3)%
$\mathbb{1} \otimes X_{\pi/2}$	**99.48(3)%**	2.14(5)%	**0.69(4)%**	2.86(7)%	2.69(6)%	3.6(3)%	2.2(3)%	5.0(3)%	3.05(17)%	6.9(3)%
$\mathbb{1} \otimes Y_{\pi/2}$	**99.41(5)%**	2.63(8)%	**0.79(6)%**	3.44(10)%	3.36(10)%	2.3(4)%	1.9(4)%	4.1(6)%	2.6(2)%	6.3(5)%
$\mathbb{1} \otimes Y_{-\pi/2}$	**99.49(5)%**	2.60(13)%	**0.68(7)%**	3.54(16)%	3.18(15)%	3.1(4)%	5.1(5)%	4.2(7)%	3.2(3)%	9.1(5)%
CZ	**99.37(11)%**	0.11(10)%	0.66(11)%	**0.79(14)%**	0.54(13)%	4.9(3)%	0.7(3)%	3.2(4)%	4.1(3)%	5.5(4)%

Fig. 3

Gate Set Tomography

- **Process matrices are not unique**
  - PMs are conjugated $G_i \rightarrow M G_i M^{-1}$
  - some errors unaffected by gauge (intrinsic) others are shifted by gauge (relational)





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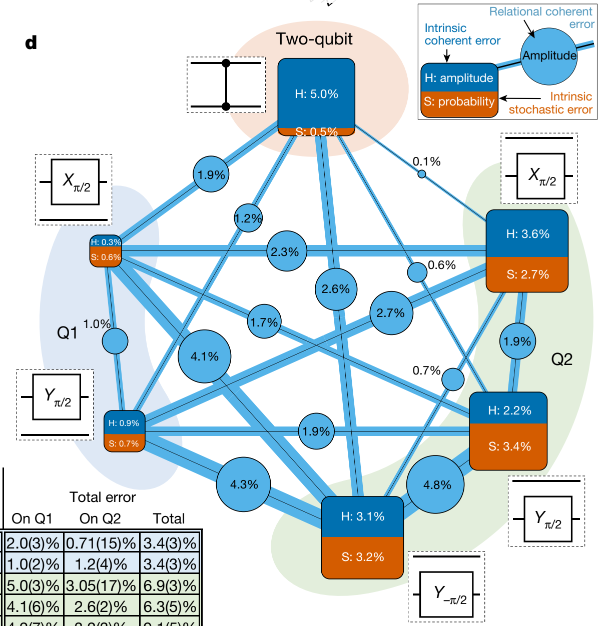
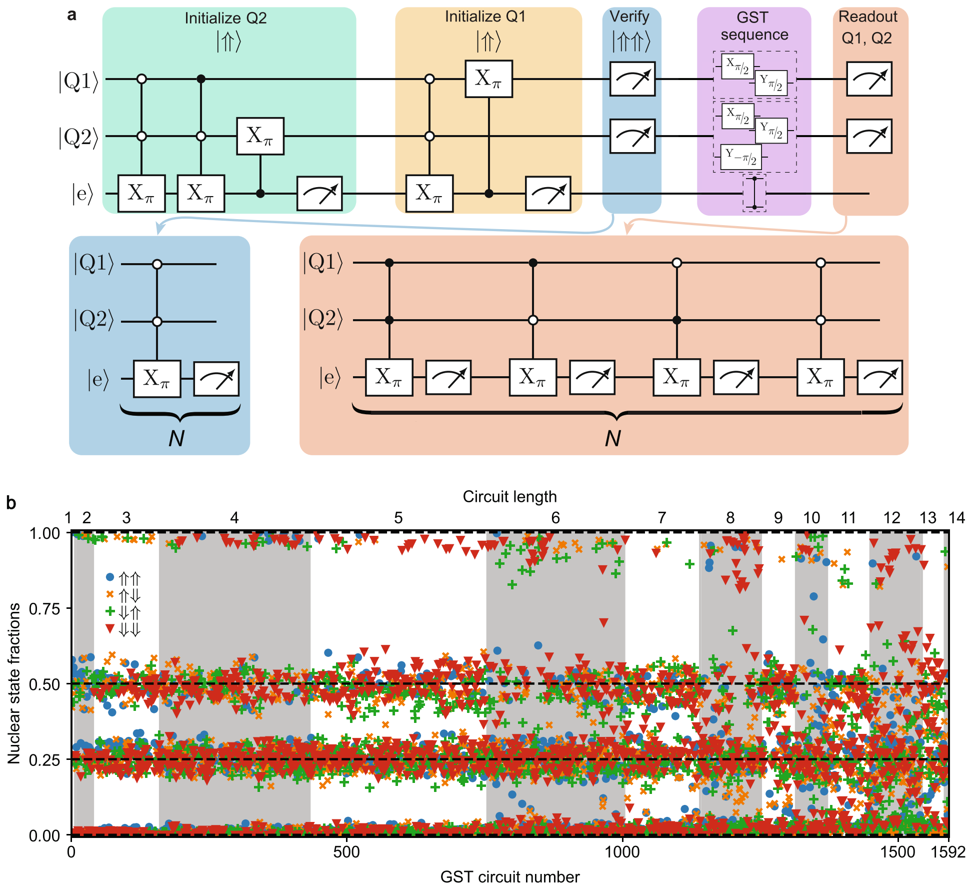


Gate Set Tomography

- **2-qubit GST circuit**
  - Init. Q1, Init. Q2, QND verify, GST seq., QND Readout

- **first 145 circuits estimate prep. & meas. fiducials**

- **at the end of each circuit, the population is spread over 1, 2 or 4 states**



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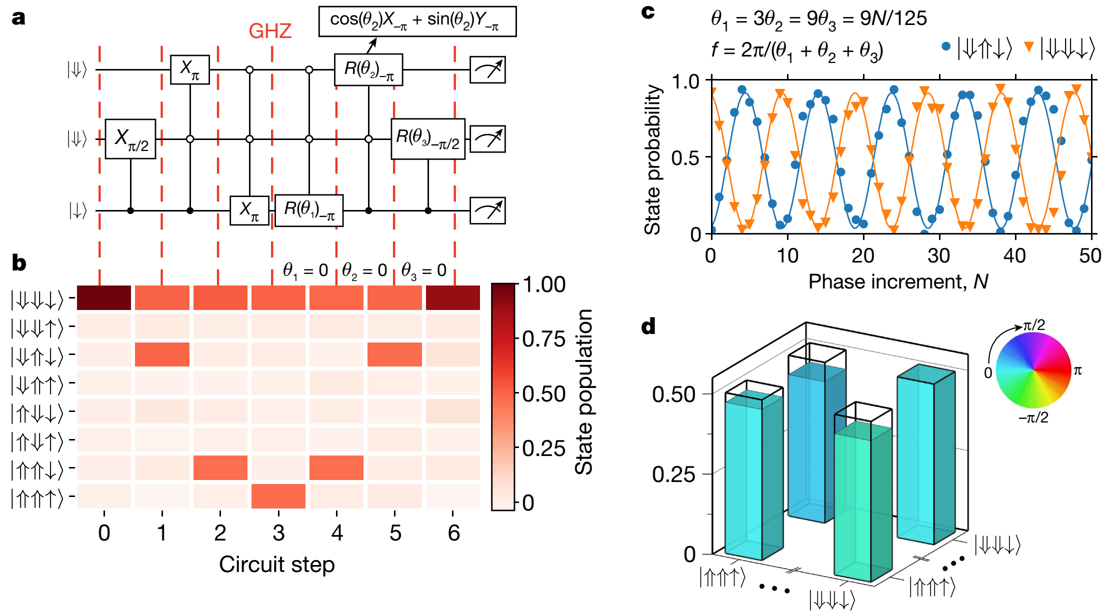
Three-Qubit Entanglement

- **Demonstrate max. entangled Greenberger-Horne-Zeilinger state** $(|\Uparrow\Uparrow\uparrow\rangle + |\Downarrow\Downarrow\downarrow\rangle)/\sqrt{2}$
- **Problem: GHZ state dephases too quickly as to measure it in different bases required for tomography**
- **Solution: Repeat reversal of GHZ state N=100 times /w different phase shifts on the axes of the reversal pulses**
  - Amplitude and phase of oscillations yield coherence $\langle\Downarrow\Downarrow\downarrow|\rho_{\mathrm{GHZ}}|\Uparrow\Uparrow\uparrow\rangle = \rho_{18}$ sufficient together with ρ_{11}, ρ_{88}, to determine GHZ fidelity of 92.5%



Summary & Outlook

- Demonstrated 1-qubit, 2-qubit and SPAM errors at or below 1%

- Demonstrated max. entangled GHZ-state

- Replacing P donors with $^{123}Sb(I = 7/2)$ or $^{209}Bi(I = 9/2)$ could provide a larger Hilbert space in which to encode Q.I.: (2Sb ~ 6P+1e)

- Heavier group-V donors also enable el. control of nucler spins, combined with e-nuclear flipflop transition

- Recent experiments on e-spin qubits in Si with fidelities >99% suggest the electron fidelity will no linger constitute a bottle neck